# Dynamic response evaluations of truss spar structure due to wave force interaction

Min-su Park \*, Kenji Kawano \*\*

\* Graduate Student, Dept. of Ocean Civil Eng., Kagoshima University, 1-21-40, Korimoto, Kagoshimasi, 890-0065 \*\* Dr. of Eng., Professor, Dept. of Ocean Civil Eng., Kagoshima University, 1-21-40, Korimoto, Kagoshimasi, 890-0065

It is known that great possibilities to develop ocean spaces which may be used as resident areas, airports, power stations, etc. can be provided by offshore structures with a large deck area. The dynamic response properties on an offshore structure of truss spar type are significantly depended on the wave force evaluation. In the present study, under the assumption of potential flow and linear wave theory, the diffraction in each fluid region can be expressed by an eigenfunction expansion method. By using the wave force interaction obtained by the method, the dynamic response of truss spar structure is examined and compared with the case of using Morison equation. It is suggest that the effect on interaction based on diffraction theory has important roles on the reliable evaluation of the dynamic response for the truss spar structure.

Key Words: diffraction theory, wave force interaction, eigenfuntion expansion method, Morison equation, truss spar structure, dynamic response.

# 1. Introduction

The wave force is one of the most important loads on design of offshore structure. The dynamic response evaluation due to wave forces has significant roles on the reliable design of the offshore structure. The wave forces on an offshore structure are usually obtained by diffraction theory and Morison equation. The wave diffraction problem about a vertical circular cylinder is a typical problem with exact analytical solution in ocean engineering. The analytical solution was proposed by MacCamy & Fuchs (1954). Under the assumptions of potential flow and linear wave theory, a semi-analytical solution is obtained by an eigenfunction expansion approach first proposed for impermeable cylinders by Spring & Monkmeyer(1974), and latter simplified by Linton & Evans(1990) for N bottom-mounted circular cylinders. They had divided the fluid domain into N+1regions. The diffraction in each fluid region was expressed by an eigenfuncition expansion method.

The offshore structure such as truss spar structure has great possibility for developing the offshore structure with a large deck area. While the truss spar structure has been examined to the application in the deep water sea, it has great advantages on developing offshore structure in relatively shallow water by the drastic reduction of reaction forces on the base foundation. Since the truss spar structure has multi-cylinders with relatively large diameter, the wave force evaluation has important roles on the reliable design of the structure.

In the present study, the eigenfunction method is applied to evaluate the wave force to the offshore structure with Nbottom-mounted cylindrical structure. To verify the present method, the results obtained by the numerical evaluation are compared with these results such as MacCamy & Fuchs' analytic solution(1954) and Linton & Evans' numerical results(1990). For an idealized truss spar structure, the dynamic response evaluation is carried out using wave forces which can be evaluated with the diffraction theory and Morison equation. Applying the wave force by the diffraction method to the truss spar structure, the dynamic response is carried out using modal analysis that can be solved by step-by-step integration such as Newmark  $\beta$  method (Kawano, et al. (1990)). The wave force interaction effects due to multi-spar structure are examined by comparison of these responses. It is suggest that the dynamic response of the present truss spar structure can be availably evaluated with the Morison equation, if the situation can be negligible the wave force interaction with respect to truss spar structure. In order to perform a reliable evaluation of the dynamic response of truss spar structure, it would be necessary to examine the influence of interaction between wave and structure by effective method such as the present evaluation.

### 2. Formulation

### 2.1 Wave force interaction with N bottom-mounted cylinders

It is assumed that the fluid is inviscid and incompressible, its motion is irrotational and the fluid motion is small. The geometry of the problem is shown in Fig. 1. An arbitrary array of *N* bottom-mounted vertical circular cylinders of radius  $a_j$  (j=1,2,3,...,N), is situated in water of uniform depth *h*. The global Cartesian coordinate system is defined with an origin located on the still-water level with the *z*-axis directed vertically upwards. The center of each cylinder at  $(x_j, y_j)$  is taken as the origin of a local polar coordinate system  $(r_j, \theta_j)$ , where  $\theta_j$  is measured counterclockwise from the positive *x*-axis. The center of the *k*th cylinder has polar coordinates  $(R_{jk}, \alpha_{jk})$  relative to the *j*th cylinder. The coordinate relationship between the *j*th and *k*th cylinders is also shown in Fig. 1.



Fig. 1. Definition sketch of an array of cylinders

The array is subjected to a train of regular surface waves of height *H* and angular frequency  $\omega$  propagating at an angle  $\beta$  to the positive *x*-axis. The uniform geometry of the array members in the vertical allows the depth dependency in the solution to be factored out as follow:

$$\Phi(x, y, z) = R_{e}[\phi(x, y)f(z)e^{-i\omega t}]$$
(1)

where, Re[] denotes the real part of a complex expression and

$$f(z) = -\frac{ig(H/2)\cosh\kappa(z+h)}{\omega\cosh\kappa h}$$
(2)

In equation (2), g is the acceleration of gravity and the wave number k is the positive real root of the dispersion relation  $\omega^2 = gk \tanh kh$ .

The fluid domain is divided into N+1 regions: a single exterior region and N boundary regions. The velocity potential of incident wave with an angle  $\beta$  to the positive *x*-axis is presented as follow:

$$\phi_{I} = I_{i} e^{i\kappa r_{j} \cos(\theta_{j} - \beta)}$$
(3)

where,  $I_j$  (= $e^{ik(x_j\cos\beta + y\sin\beta)}$ ) is a phase factor associate with cylinder *j*.

Therefore, equation (3) is represented as follow:

$$\phi_{I} = I_{j} \sum_{n \to -\infty}^{\infty} J_{n} \left( \kappa r_{j} \right) e^{in(\pi/2 - \theta_{j} + \beta)}$$
(4)

in which  $J_n$  denotes the Bessel function of the first kind of order *n*. (Gradshteyn & Ryzhik (1965)).

It can be shown that two-dimensional scattered velocity potentials from *j*th vertical circular cylinder must satisfy a Helmholtz equation and the usual radiation boundary condition.

$$\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2} + \kappa^2\right)\phi_s^j = 0 \qquad (5)$$

$$(\kappa r)^{1/2} \left(\frac{\partial}{\partial r} - i\kappa\right) \phi_s^j \to 0 \quad as \quad \kappa r \to \infty$$
 (6)

Following boundary problems, the general form for the scattered wave emanating from cylinder *j* can be written as:

$$\phi_s^j = \sum_{n \to \infty}^{\infty} A_n^j Z_n^J H_n \left( \kappa r_j \right) e^{i n \theta_j}$$
<sup>(7)</sup>

where,  $Z_n^j (= Z_{-n}^j) = J'_n (\kappa a_j) / H'_n (\kappa a_j)$ 

$$H_n(\kappa r_i) = J_n(\kappa r_i) + iY_n(\kappa r_i)$$

The simplification will be obtained by the introduction of the factor  $Z_n^j$ . If, instead of  $A_n^j Z_n^j$ , we put  $B_n^j$  in equation (7) then it can shown that  $Z_n'(ka_j)=0$  implies  $B_n^j=0$  and so no restrictions are being added by the inclusion of the factor  $Z_n^j$ . Clearly the value of  $A_n^j$  is irrelevant if  $Z_n'(ka_j)=0$  and so it is assumed that this is not the case in the following analysis. The total potential can thus be written as follow.

$$\phi = \phi_i + \sum_{j=1}^N \phi_s^j$$

$$= e^{ikr\cos(\theta - \beta)} + \sum_{j=1}^N \sum_{n=-\infty}^\infty A_n^j Z_n^j H_n(\kappa_j) e^{in\theta_j}$$
(8)

Using Graf's addition theorem for Bessel functions (Gradshteyn & Ryzhik(1965)) equation (8) can be expressed in terms of the coordinates  $(r_k, \theta_k)$  and applying the boundary conditions as follow.

$$\frac{\partial \phi}{\partial r_k} = 0 \quad on \quad r_k = a_k, \ k = 1, 2, 3, \dots N. \tag{9}$$

Some algebra leads to the following infinite systems of equations.

$$A_{m}^{k} + \sum_{j=1 \atop \neq k}^{N} \sum_{n=-\infty}^{\infty} A_{n}^{j} Z_{n}^{j} e^{i(n-m)\alpha_{jk}} H_{n-m} (\kappa R_{jk}) = -I_{k} e^{im(\pi/2-\beta)}$$
(10)

in which,  $k=1,2,3,\ldots,N$ ,  $-\infty < m < \infty$ 

It is important to note that Bessel functions of  $(r_j, \theta_j)$  in terms of the coordinates can be related with  $(r_k, \theta_k)$  in using the addition theorem for Bessel functions. It must be  $r_k < R_{jk}$ . This is certainly true on the boundary of the *k*th cylinder for all *j* and thus equation (10) is valid. The expression obtained for  $(r_k, \theta_k)$  is in fact as follows

$$\phi(\mathbf{r}_{k},\boldsymbol{\theta}_{k}) = \sum_{n=-\infty}^{\infty} \left[ I_{k} J_{n} \left( \kappa \mathbf{r}_{k} \right) e^{in(\pi/2-\theta_{k}+\beta)} + A_{n}^{k} Z_{n}^{k} H_{n} \left( \kappa \mathbf{r}_{k} \right) e^{in\theta_{k}} \right]$$

$$+ \sum_{j=1}^{N} \sum_{n=-\infty}^{\infty} A_{n}^{j} Z_{n}^{j} \sum_{m=-\infty}^{\infty} J_{m} \left( \kappa \mathbf{r}_{k} \right) H_{n+m} \left( \kappa R_{jk} \right) e^{im(\pi-\theta_{k})} e^{i(n+m)\alpha_{\mu}}$$

$$(11)$$

And this expression is valid if  $r_k < R_{jk}$  for all *j*. This is therefore an expansion valid near to cylinder *k*. Replacing *m* by -*m* in the final term of equation (11) allows us to write this term as

$$\sum_{m=-\infty}^{\infty} \left( \sum_{\substack{j=l\\j\neq k}}^{N} \sum_{m=-\infty}^{\infty} A_n^j Z_n^j H_{n-m} \left( \kappa R_{jk} \right) e^{i(n-m)\alpha_{jk}} \right) J_m \left( \kappa r_k \right) e^{im\theta_k}$$
(12)

The group of terms contained within the brackets can now be substituted for using the infinite system of equations (10). The resulting simple formula is

$$\phi(r_{k},\theta_{k}) = \sum_{n=-\infty}^{\infty} A_{n}^{k} (Z_{n}^{k}H_{n}(\kappa r_{k}) - J_{n}(\kappa r_{k}))e^{in\theta_{k}}$$

$$if \quad r_{k} < R_{jk} \quad (\forall j)$$
(13)

This expression can be provided by an extremely simple formula for the velocity potential near any cylinder. In particular the velocity potential on the *k*th cylinder reduces to

$$\phi(a_k, \theta_k) = -\frac{2i}{\pi \kappa a_k} \sum_{n=-\infty}^{\infty} \frac{A_n^k}{H_n'(\kappa a_k)} e^{in\theta_k}$$
(14)

where Wronskian relations for Bessel functions have been used.

In order to evaluate the constants  $A_n^j$  the infinite system (10) is truncated to an N(2M+1) terms.

$$A_{m}^{k} + \sum_{j=1 \atop \neq k}^{N} \sum_{n=-M}^{M} A_{n}^{j} Z_{n}^{j} e^{i(n-m)\alpha_{jk}} H_{n-m} \left( \kappa R_{jk} \right) = -I_{k} e^{im(\pi/2-\beta)}$$
(15)

in which, *k*=1, 2, 3, ...., *N*, *m*=-*M*, ...., *M* 

By increasing *M* terms greater accuracy can be achieved with the expense of computing time. From examinations it is assumed that, except when the cylinders are very close together, taking M=10 could produce accurate results to all following calculations.

The first-order force on the *j*th cylinder ( $Re\{F^{j}e^{i\omega t}\}$ ) is givens by integrating the pressure over the surface of the cylinder as follows

$$F^{j} = -\frac{\rho g (H/2) a_{j}}{\kappa} \tanh \kappa h \int_{0}^{2\pi} \phi (a_{j}, \theta_{j}) \begin{cases} \cos \theta_{j} \\ \sin \theta_{j} \end{cases} d\theta_{j}$$
(16)

in which the upper elements of a bracketed pair refer to the force in the *x*-direction and the lower elements to that in the *y*-direction. It can be also expressed with using equation (14) as follows

$$F^{j} = -\begin{cases} i \\ 1 \end{cases} \frac{2\rho g (H/2) \tanh \kappa h}{\kappa^{2} H_{1}' (\kappa a_{j})} \left( A_{-1}^{j} \left\{ -\right\} A_{1}^{j} \right)$$
(17)

#### 2.2 Dynamic response due to wave forces

The dynamic response properties of truss spar structure due to wave force are expected to be significantly depended on the wave force evaluation. It is known that the diffraction theory is more suitable for the wave force evaluating of structure with relatively large diameter. Otherwise, the wave force evaluation of many offshore structures is performed with the Morison equation. In the present study, the dynamic responses of truss spar structure are examined in order to evaluate interaction effects between the structure and the wave force. The wave force can be evaluated with the water particle motion such as velocity and acceleration equation. For an idealized truss spar structure as shown in Fig.2, the governing equation of motion can be expressed with the finite element method as follows:

$$[M]{\ddot{x}} + [C]{\dot{x}} + [K]{x} = {F}$$
(18)

in which [M], [C] and [K] denote the mass matrix, damping matrix and stiffness matrix, respectively, and  $\{x\}$  denotes the displacement vector. Using the Morison equation, the external force  $\{F\}$  denotes the wave forces expressed with the drag force and inertia force.

$$\{F\} = \left[\overline{C}_{M}\right]\!\!\left\{\!\vec{v}\right\}\!-\!\left[\overline{C}_{m}\right]\!\!\left\{\!\vec{x}\right\}\!+\!\left[\overline{C}_{D}\right]\!\!\left\{\!\left(\!\vec{v}-\vec{x}\right)\!\!\left|\!\vec{v}-\vec{x}\right|\!\right\}\!\right\}$$
(19)

in which

$$\begin{bmatrix} \overline{C}_{_{M}} \end{bmatrix} = \begin{bmatrix} \cdot . \rho C_{_{M}} V^{\cdot} \cdot . \end{bmatrix}$$

$$\begin{bmatrix} \overline{C}_{_{m}} \end{bmatrix} = \begin{bmatrix} \cdot . \rho (C_{_{M}} - 1) V^{\cdot} \cdot . \end{bmatrix}$$

$$\begin{bmatrix} \overline{C}_{_{D}} \end{bmatrix} = \begin{bmatrix} \cdot . \rho C_{_{D}} \frac{A}{2} \cdot . \end{bmatrix}$$
(20)

where { $\dot{v}$ } and { $\ddot{v}$ } denote the velocity and acceleration of the water particle. For the structure subjected to the diffraction and the wave force interaction, the wave force can be evaluated with the acceleration of water particle. Obtaining the acceleration of the water particle, the wave force on the structure can be determined with the equivalent nodal force such as expressed in Morison equation.

To apply the wave force interaction to the modified Morison equation, the wave force interaction can be expressed with

$$F^{j} = C_{M} \rho \frac{\pi D^{2}}{4} \ddot{v}_{M} \tag{21}$$

Using equation (17), the acceleration of the water particle can be expressed with

$$\ddot{v}_{M} = \frac{2\rho g H}{k} \frac{\cosh k(z+h)}{\cosh kh} \frac{1}{H_{1}'(ka_{j})} \cos(\omega t - \alpha) \frac{1}{C_{M} \rho \pi a^{2}}$$
(22)

here, the difference of  $C_M$  is very small for D=10m. So that  $C_M$  is not modified in this study.

If the dynamic response is restrained within linear response, the governing equation of motion can be effectively determined with the modal analysis.

$$\left[\widetilde{M}\right]\!\!\left[\ddot{x}\right]\!+\left[\widetilde{C}\right]\!\!\left[\dot{x}\right]\!+\left[K\right]\!\!\left[x\right]\!=\left\{\widetilde{F}\right\}$$
(23)

in which

$$\begin{bmatrix} \widetilde{M} \\ \widetilde{P} \end{bmatrix} = \begin{bmatrix} M \end{bmatrix} + \begin{bmatrix} \overline{C}_m \end{bmatrix} \\ \begin{bmatrix} \widetilde{C} \\ \widetilde{P} \end{bmatrix} = \begin{bmatrix} C \end{bmatrix} + \begin{bmatrix} \overline{C}_D \end{bmatrix} \\ \begin{bmatrix} \widetilde{F} \\ \widetilde{F} \end{bmatrix} = \begin{bmatrix} \overline{C}_M \\ \widetilde{V} \\ \widetilde{F} \end{bmatrix} + \begin{bmatrix} \overline{C}_D \\ \widetilde{V}_M \end{bmatrix}$$
 for Morison equation 
$$\{ \widetilde{F} \} = \begin{bmatrix} \overline{C}_M \\ \widetilde{V}_M \end{bmatrix}$$
 for Interaction 
$$(24)$$

Applying the eigen value analysis for equation (23), it can be expressed with the following equation.

$$[I]\{\ddot{q}\} + \left[ \cdot \cdot \left(2\beta_{j}\omega_{j} + \widetilde{C}_{D}\right) \cdot \cdot \right]\{\dot{q}\} + \left[ \cdot \cdot \omega_{j}^{2} \cdot \cdot \right]\{q\} = \left\{\widetilde{f}\right\}$$
(25)

in which

$$\{x\} = [\Phi] \{q\}$$

$$[I] = [\Phi]^{T} [\widetilde{M}] \Phi]$$

$$[ \cdot .(2\beta_{j}\omega_{j} + \widetilde{C}_{D}) \cdot .] = [\Phi]^{T} [\widetilde{C}] \Phi ]$$

$$[ \cdot .\omega_{j}^{2} \cdot .] = [\Phi]^{T} [K] \Phi ]$$

$$\{\widetilde{f}\} = [\Phi]^{T} \{\widetilde{F}\}$$

$$(26)$$

Equation (25) can be solved by step-by-step integration such as Newmark  $\beta$  method.



Fig.2 Analytical model of a truss spar structure

# 3. Numerical results and discussions

## 3.1 The wave interaction with structure

To verify the wave forces evaluation, the present method is compared with these results such as MacCamy & Fuchs(1954) and Linton & Evans(1990). Comparison is made for the ratio between the water depth to a radius of the cylinder, h/a=5.

Fig.3 shows that the wave forces are non-dimensionalized by  $\rho gHa^2$ . The abscissa denotes the nondimensional wave number.

It is known that there is no difference for the nondimensional wave number under 2.5.



Fig.3 Comparison of dimensionless wave forces on a single circular cylinder for *h/a=5.0* 



Fig.4 Comparison of dimensionless amplitude of the wave forces on four cylinders for a/h=1/2, R/h=2,  $\beta = \pi/4$ 

The wave force interaction acting on arrays of vertical circular cylinders is examined by comparison with the results of Linton & Evans(1990). The results can be obtained from equation(21). The wave force interaction with structure is examined about four cylinders arranged at the vertices of a square of side distance(R). The various parameter are a/h=1/2, R/h=2 and  $\beta = \pi/4$ . The cylinders are numbered clockwise 1-4 and are situated at (-2a, 2a), (2a, 2a), (2a, -2a) and (-2a, -2a) respectively, so that the forces in the direction of wave advance on cylinders 1 and 3 are identical. It is noted that the wave force by the present method gives the good agreement to the results of Linton & Evans(1990). The curve shows that interaction effects can be extremely important in determining the amplitude of the wave forces. Therefore, the present method on wave force evaluation is very useful to accurate the wave forces acting on arrays of vertical circular cylinders.



Fig.5 Comparison of dimensionless wave forces on four spars for h/a=17,  $\beta=0$ 

The dynamic response characteristics of truss spar structure as shown in Fig.2 are carried out using the present method for wave force evaluation. The truss spar structure is composed of two parts. One part is the upper structure that is composed of a middle size spar(D=10m). The upper structure can support the deck weight by having buoyancy and the wave force can be evaluated with Morison equation and wave force interaction. Another part is the lower structure that is composed of a small size pipe member(D=0.3m). The lower structure can support 10% of the deck weight and the wave force be influenced by Morison equation. The spars are numbered by clockwise 1-4. So that the

nodal 1 is located on the top of spar 4. The incident wave acts on the right direction through spar 4 and spar 1. The structure has the characteristics that unit weight is  $77.0(kN/m^3)$ , stiffness coefficient is  $2.1 \times 10^8 (kN/m^2)$  and shear stiffness coefficient is  $8.1 \times 10^7 (kN/m^2)$ .



Fig.6 Comparison of dimensionless wave forces on four spars for h/a=17,  $\beta = \pi/4$ 

Fig.5 shows comparison of dimensionless wave forces on four spars. Here the various parameter are a=5m, h=85m and  $\beta=0$ . The wave forces are presented for a relative cylinder distance 40m, 80m and 120m, and wave period 9sec, 10sec and 11sec. It is noted that a majority of the waves considered in the range of 7-16sec give a value of ka less than 0.5. In the case of less than nondimensional wave number ka=0.5, there is exactly existed the

variation of wave force by the interaction with truss spar structure. The wave force interaction of spar 4 is much larger than the spar 2, because the spar4 is located in front of the spar2. The wave force of spar 4 is decreased gradually by increasing the distance and wave period. It is understood that the interaction effect is depended on the distance among spars and wave period. While the wave forces of spar 2 have a similar value at the distance about 70m, there is slightly difference for each distance of spar.

Fig.6 shows the wave forces of spar 4 and spar 2 for  $\beta = \pi/4$ . The wave forces of spar 4 are nearly the similar values at R=60m, and in the cases of T=10sec and T=11sec, the wave force of spar 2 has a similar value when the distance is less than 100m. It is noted that in case of  $\beta = \pi/4$ , the variation of wave force becomes drastic than  $\beta=0$ , because the interaction effect of spar 1 and spar 3 contributes to other spars for changing the incident wave angle. It is understood that the wave forces of spar 4 is considerably influenced by incident wave angle, and the wave force of spar2 is slightly influenced.



Fig.7 Comparison of dimensionless wave forces on four spars for h/a=17, R=80m,  $\beta=0$ 

It is known that the wave force due to the diffraction wave theory could provide the exact evaluation for the structure with large diameter. Otherwise, the Morison equation is widely applied for the wave force evaluation with the offshore platform with the member of relatively small diameter. While the wave force by the Morison equation is represented with the inertia force and the drag force, the inertia force is dominated for increasing the diameter of the member. For the wave force evaluation such as the truss spar structure with the relative large diameter member, it is necessary to examine comparison between the wave force interaction and the Morison equation. Fig.7 shows comparison due to these wave force evaluation. The wave force due to the Morison equation sharply increases until the wave number is 0.15, and has a similar value after that. The wave force of spar 2 by the interaction has similar results to Morison for the wave number under ka=0.25(T=9sec) and gradually becomes more increased than the Morison equation after that. The wave force of spar 4 has a similar value of the

Morison for the wave number under ka=0.12(T=13sec) and has considerable variation after that. To inspect the effects of wave force interaction of spar 4, the dynamic response of truss spar structure is desired to examine in three cases such as ka=0.16(T=11sec), ka=0.2(T=10sec) and ka=0.25(T=9sec). It corresponds that the wave force of spar 4 has a maximum wave force at T=11sec, a similar wave force at T=10sec and a minimum wave force at T=9sec. If the nondimensional wave number becomes larger than 0.3, the wave period becomes less than 7sec.



Fig.8 Time histories of displacement responses



Fig.9 Time histories of bending stress responses

Taking into account for the interaction effects on the wave force, it is expected to have important roles on the dynamic response evaluation of the truss spar structure. Fig. 8 shows time histories of displacement response at the nodal point 1 for the wave height, *7m* and the wave period, *11sec*. It is observed that the displacement response for the wave force interaction is slightly higher than the Morison, as noted that the wave force interaction has larger than the Morison at the period, *11sec*, as shown in Fig.7.

Fig.9 shows the time histories of the bending stress at the nodal point 8. The bending stress response, which the wave force

interaction acts on structure, is also slightly higher than the Morison equation. It is noted that while the difference of displacement response and bending stress response are small, the wave force interaction gives considerable effects on the dynamic responses of truss spar structure.





Fig.10 Relations between maximum displacement responses and wave height



Fig.11 Relations between maximum bending stress response and wave height

Fig.10 and Fig.11 show relations between maximum displacement response and wave height, and relations between bending stress response and wave height, respectively. The four lines correspond to the wave height from *3m* to *10m* and the wave period, *11sec*. Comparing the responses to the wave force interaction with the Morison equation, the former becomes slightly larger response than the latter. It is noted that the influence of the wave force interaction should be enlarged as increase of wave height. It is understood that the dynamic response of truss spar structure may be very susceptible of increasing wave force.

Fig.12 and Fig.13 show relations between maximum displacement response and wave period, and relations between maximum bending stress response and wave period at spar 4,



respectively. By comparing all cases, the response of the wave

force interaction is lager than the Morison equation except for

T=9sec. It is understood that the wave force of spar 4 is lower

Fig.12 Relations between maximum displacement response and wave period



Fig.13 Relations between maximum bending stress response and wave period

Fig.14 and Fig.15 show relations between maximum bending stress responses and wave height, and relations between maximum displacement responses and wave height, respectively. From the comparison of the wave force interaction and the Morison, there is a slightly difference for the wave periods over *13sec*, but the difference gradually increases as the wave period under T=13sec. It is understood that the wave force of spar 4 has a similar value of the Morison over T=13sec, and it has the difference for other wave periods as shown in Fig.7. It is noted that the wave force interaction effects to the dynamic response of truss spar structure. If the location and distance of spar is changed, the wave force interaction effects would take some contributions on the dynamic

response evaluation of truss spar structure. From the analytical results, it is expected to evaluate efficiently the wave force by the Morison equation for the truss spar structure used in this study. However, in case the effect of wave force interaction is increasing as becomes shorter the distance between spar and spar and larger the diameter of spar, Morison equation can not express exactly the wave force acting on spar. So that, in order to perform a reliable design of truss spar structure, it is important to clarify the wave force interaction effects with respect to the wave force elevation due to the Morison equation.



Fig.14 Relations between maximum bending stress responses and wave height



Fig.15 Relations between maximum displacement responses and wave height

### Conclusion

The dynamic response characteristics of truss spar structure under the wave conditions are examined. The results are summarized as follows:

 The wave force interaction due to the diffraction theory is examined by the eigenfunction method. It is suggest that since the wave force interaction has important effects on the response evaluation for the nondimensional wave number under 0.5, it is important to examine the effects of the location, the distance of spar and the incident direction of wave for the truss spar structure.

(2) It is suggest that the dynamic response of the present truss spar structure can be evaluated availably with the Morison equation, if the situation can be negligible to the interaction effects with structure. However, in order to carry out the reliable design of truss spar structure, it is important to examine the wave force interaction effects.

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