Improving free vibration characteristics of horizontally curved twin I-girder bridges

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The use of horizontally curved steel multi I-girder bridges is dramatically increasing for highway bridges and interchanges during the past three decades. However, dynamic behavior of these bridges is more sophisticated and less understood than that of straight bridges. In addition, this bridge type has very small torsional stiffness so it can be easy to vibrate by external dynamic loadings. By these reasons, a series of horizontally curved steel twin I-girder bridges are carried out in detail by using FEM in this study not only to learn about the free vibration characteristics but also to improve these characteristics of this bridge type. Through these analyses, several stiffening structures are recommended.

Key Words: *Curved twin I-girder bridge, Natural frequency, Diaphragm, Lateral bracing, FEM*

1. Introduction

During the last three decades, horizontally curved bridges have become an important component in modern highway systems as a viable option at complicated interchanges or river crossings where geometric restrictions and constraints of limited site space make extremely complicated to adopt standard straight superstructures [1), 2)]. Curved alignments offer, in addition, the benefits of aesthetically pleasing, as well as economically competitive construction costs with regard to straight bridges. Besides, the advancement in fabrication, erection technology and the availability of digital computers to carry out the complex mathematical computations of the structural analysis and design of such girders are also primary reasons contributing to the development of these horizontally curved bridges[1)].

Among some superstructures utilized in horizontally curved bridges, the multi I-girder structures are commonly used because of the simplicity of their fabrication and construction, speed of erection as well as low cost for maintenance. However, curvature of the bridges induces the combinations of bending and torsion happened in the girders, makes the behaviors of these curved bridges complicated. These I-girders have very little torsional stiffness, so it is easy to vibrate both torsionally and vertically. In addition, the wide girder spacing and simplified lateral bracing system cause many problems related to vibration serviceability due to external dynamic loads[3)]. These undesired vibrations usually lead to fatigue damages in bridge members that are considered as one of big problems in maintenance and retrofit of bridges.

In spite of the complexities in design and construction, very limited documentations on the study of horizontally curved bridges have been made available contrary to straight bridges. Significant investigations into the design and analysis of horizontally curved steel I-girder bridges began only in the late 1960s when the Federal Highway Administration (FHWA) in United States formed the Consortium of University Research Teams (CURT) project, a large-scale research one funded by 25 states[2)]. The work eventually led to publication of the *Guide Specifications for Horizontally Curved Highway Bridges* in 1980 which was subsequently updated in 1993 and 2003. In Japan, researches involved a number of single girder and girder component experimental studies coupled with analytical works developed the *Guidelines for the Design of Horizontally Curved Girder Bridges* published by the Hanshin Expressway Public Corporation in 1988[4)]. Since then, the horizontally curved bridges have been studying theoretically and experimentally in many universities. Nevertheless, the studies on free vibration characteristic of horizontally curved bridges are still limited. H.Maneetes and D.G.Linzell [5] investigated the effects of cross-frame and lateral bracing parameters on the structure's free vibration response of a single-span, non-composite, curved I-girder bridge by both experiments and FEM analyses. These parametric studies provided influential parameters affecting dynamic response of the system. A beam finite element formulation for free vibration analysis of horizontally curved steel I-girder bridges based on Kang and Yoo's thin-walled curved beam theory was proposed by Ki-Young Yoon et al.[6]. Each node of this element possesses seven degrees of freedom including the warping one. This numerical formulation was extensively carried out the free vibration analyses of curved bridges considering the effects of curvature, boundary condition, modeling method, and degrees of freedom of cross-frame which provided invaluable information. Most of these papers used simple elements or not enough number of curvatures to have a clear look on dynamic response of curved bridges.

And although changing natural frequencies of a structure is an effective way to mitigate its vibrations, only a few papers dealing with this problem have been published in literature. Presented herein are a numerous investigations of a horizontally curved twin I-girder bridge during free vibration by using 3-D finite element method of MSC/Nastran. These investigations are not only to enhance the understanding about free vibration characteristics but also to find out the methods improving these characteristics of the bridge. Through this study, several stiffening structures are recommended for bridge designing or retrofitting.

2. Geometry of the studied bridges

The original bridge chosen in this study is a simply supported, horizontally curved, composite steel twin I-girder one whose span, which is the length of the centerline between two main girders, is fixed as 50m. Several radii of bridge measured from the origin of the circular arc to the centerline of the bridge deck are considered to take into account the effects of curvature. Thus, length of the two main girders varies in accordance with the chance of bridge's curvature; whereas, the total mass of the bridge remain unchanged. The two main I-girders are 3m deep and spaced transversely at 6m. These main structural members are tied together by a reinforced concrete slab which acts compositely with the girders and transverse steel members. The transverse members are radial cross-beams which are spaced equally along the span. Basic geometric properties and cross-section layout and of the studied bridges are presented in **Table 1** and **Fig. 1**, respectively.

Tab1e 1 Basic geometric properties of the original bridge

Span length [m]	50
Deck width x thickness [m]	10.2 x 0.3
Dimensions of the main girders [mm]	WEB 3000x24 Upper FLG 500x30 Lower FLG 800x50
Dimensions of the intermediate cross-beams [mm]	WEB 1000x16 FLG 300x25
Dimensions of the end cross-beams [mm]	WEB 2000x16 FLG 300x25

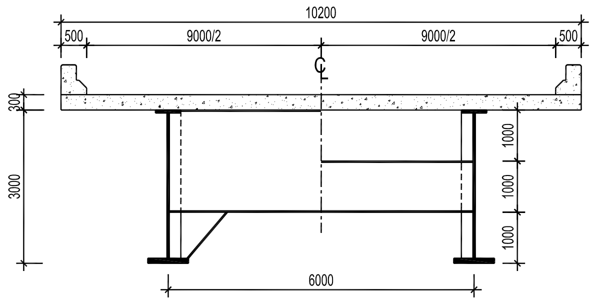


Fig. 1 Cross section of the original bridge (mm)

3. Finite element modeling

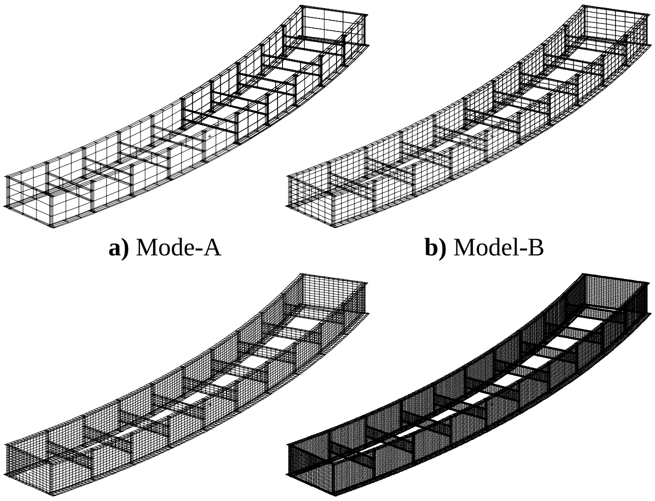
Proper modeling of finite elements is a key in finite element analyses, it is usually considered in the very first stage of any problems. In addition, bridge is a hybrid complex structure composed by several structural components with different material behaviors. So to get reliable results, numerical convergent tests of free vibration analyses are carried out in this section. The numerical free vibration results of the studied bridge with 100m radius of curvature are examined with the following categories: (1) mesh size; (2) element type and element order; and (3) mass formulation.

(1) Mesh size

Four different mesh sizes considered in these convergent tests are shown on **Fig. 2** along with their model names. It can be observed that the mesh size of the latter is two times smaller than that of the former one. And the latter mesh refinement is performed by subdividing the former mesh, so the former mesh is embedded in the latter mesh. So the mesh of Model-D is the finest one which is eight times smaller than that of Model-A – the coarsest.

(2) Element type and order

In bridge detailed analyses, the bridge deck is usually modeled by solid or shell elements[7]. Each type of elements has its own advantages and also disadvantages. In this study, according to the geometry of the bridge deck, hexagonal solid



a) Mode-A **b)** Model-B

c) Mode-C **d)** Model-D

Fig. 2 Four different mesh sizes (bridge deck is not shown)

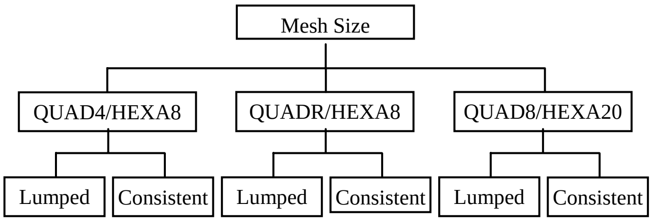


Fig. 3 Flow chart of convergent test procedures

Table 2 Number of elements/d.o.f of studied models

Model	QUAD4/HEXA8		QUADR/HEXA8		QUAD8/HEXA20	
	Element	D.o.f.	Element	D.o.f.	Element	D.o.f.
A	1,719	12,276	1,719	12,276	1,719	39,576
B	3,764	28,362	3,764	28,362	3,764	91,596
C	10,104	79,434	10,104	79,434	10,104	257,736
D					30,192	623,112

element is selected. There are two orders of the solid element considered in these analyses called HEXA8 with eight nodes and a higher order HEXA20 with twenty nodes.

There are many ways to model bridge steel I-girders. They can be modeled by beam, shell elements or both of them[7),8),9),10)]. In this study, shell element is chosen to idealize all of steel members. MSC/Nastran provides an extensive library of elements to model isoparametric shell elements. Among these elements, quadrilateral one is commonly used. In these analyses, three types of isoparametric quadrilateral element are adopted: (1) QUAD4 four-node element with optional coupling of bending and membrane stiffnesses, (2) QUAD8 eight-node element with optional coupling of bending and membrane stiffness, and (3) QUADR four-node element with no coupling of bending and membrane stiffnesses and the membrane stiffness formulation includes rotation about the normal to the plane of the element[11)]. Traditional shell elements, such as QUAD4, have five degrees-of-freedom per node: three translations and two bending rotations. The stiffness for the rotational degree-of-freedom normal to the element (the drilling degree-of- freedom) is zero. This creates modeling difficulties which may eventually lead to poor solutions. The QUADR elements are improved shell elements, which have six degrees-of-freedom, and are much less sensitive to high aspect ratios and values of Poisson's ratio near 0.5[11)]. In these elements, a rotational stiffness is computed about the normal to the element at the vertices and used in the formulation of the element stiffness.

(3) Mass matrix formulation

In mass matrix formulation, lumped mass is the simplest procedure for defining the mass properties of any structures. This method assumes that the entire mass is concentrated at the points at which the translation displacements are defined only. By doing so, the lumped mass matrix has a diagonal form, i.e. all of terms beyond the diagonal of the matrix (off-diagonal terms) are zero. On the other hand, consistent mass takes into account the effect of rotational inertias and leads to mass coupling between rotations and translations. This is more advanced method and, in many cases, gives more accurate results. However, the dynamic analysis of a consistent mass system requires considerably more computational effort than a lumped mass system does.

The flow chart demonstrating the convergent tests is shown on **Fig. 3**, and the number of elements and degrees-of-freedom of all the studied models are depicted in **Table 2**. In summarization, four mesh sizes are considered. Each of the mesh size, three groups of finite elements are carried out. And two mass formulations is analyzed in each FE group.

(4) Results and discussion

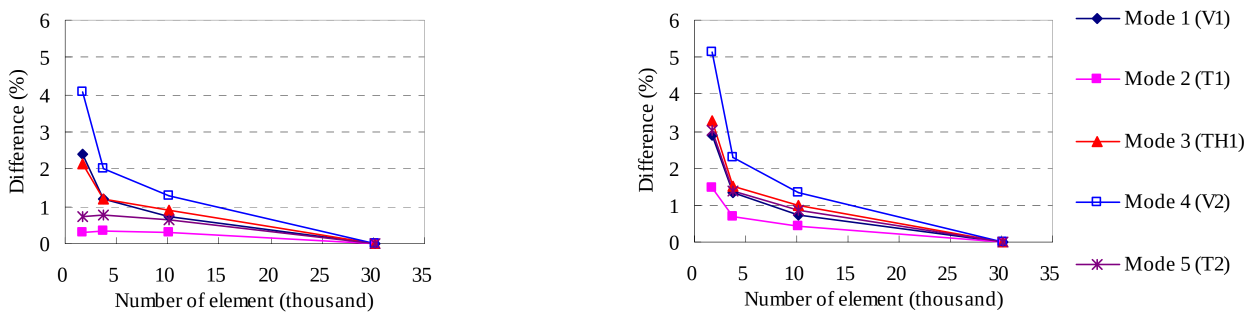
There is no formulation that can predict accurately the natural frequencies of such this complex structure, therefore, the calculated frequencies of the Model-D with QUAD8/HEXA20 element types are considered numerical benchmark values because of the fine mesh's density of the Model-D and the higher order of the finite elements.

The typical differences of natural frequencies between the "exact" values and those of remaining models are graphically presented in **Fig. 4**. Because the changing tendencies of the differences with number of elements of all the studied models are the same, only those of QUAD4/HEXA8 element group with both lumped and consistent mass are displayed in **Figs. 4a** and **4b**, respectively. According to the figures, regardless element type and mass formulation, the convergences of the results happen highly in models whose numbers of elements are less than five thousands. From five thousands upward, the differences of the frequencies compared with the "exact" solutions are small and negligible. It is easy to see that the convergences of all modes in the systems with consistent mass plotted in the **Fig. 4b** are smooth and have an understandable tendency toward the "exact" solutions. On the other hand, in the lumped mass systems shown in the **Fig. 4a**, several modes have unclear convergent trends such as the second (mode 2) and the fifth (mode 5) modes.

To verify the performances of the element groups in connection with the two mass matrix formulations, the same results are plotted in **Fig. 5**. It is known that the consistent mass systems usually perform better than lumped mass ones do. However, the figure seems to show the opposite results, the results of lumped mass systems appear slightly more accurate than those of consistent systems. It is known that the lumped mass formulation introduces an error that tends to predict lower frequencies. This error counteracts with the error in stiffness matrix formulation that tends to predict higher ones[12)]. It's largely serendipity, but for the problems, of which the effect of rotational inertias is very small, the two errors tend to balance each other and the lumped mass systems predict more accurate frequencies. In consistent mass systems, the frequencies always converge from above the exact values. Whereas, the frequencies often oscillate about the final values in lumped mass systems, but are closer than consistent. This also could explain for the abnormal tendencies happened in the second and fifth modes of lumped mass systems as aforementioned.

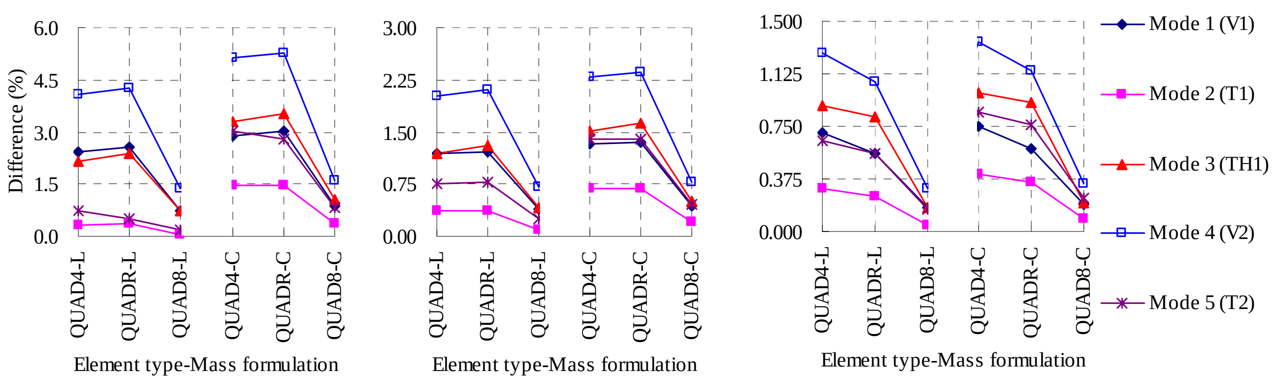
The **Fig. 5** also shows the superb performance of QUAD8/HEXA20 element group. It is easy to understand because of the high order element types. It is interesting to see the improved quadrilateral QUADR element with six degrees-of-freedom per node perform worse than the ordinary quadrilateral QUAD4 with five degrees-of-freedom per node does in the Model-A and Model-B shown on **Figs. 5a** and **5b**, respectively. Its performance is better only in Model-C presented in **Fig. 5c**. This could be addressed to none coupling of bending and membrane stiffness of QUADR element. Because of this none coupling effect, the improved QUADR element is not recommended for curved surface[11)]. When the mesh size is reduced, the angle between two adjacent elements is also decreased. That is the reason why the QUADR in Model-C performs better than in Model-A and Model-B. It can be said that in these curved system, the QUADR performs better than QUAD4 only if the mesh size is fine enough.

The smaller of mesh size or the higher of the element order, the better of results are obtained. That also means the more expensive computational cost is required. However, the differences of the results between these models are small. The maximum difference is only approximately 5%. In other



a) QUAD4/HEXA8+Lumped mass **b)** QUAD4/HEXA8+Consistent mass

Fig. 4 Differences of frequencies with varying of number of elements



a) Model-A **b)** Model-B **c)** Model-C

Fig. 5 Differences of frequencies with varying of number of elements words, the natural frequencies of the structure are not very sensitive with the mesh size, these studied element types. Based on the results of these studies, the mesh size of Model-C, the QUAD4/HEXA8 element group, and the lumped mass formulation are recommended for the next sections.

4. Free vibration characteristics

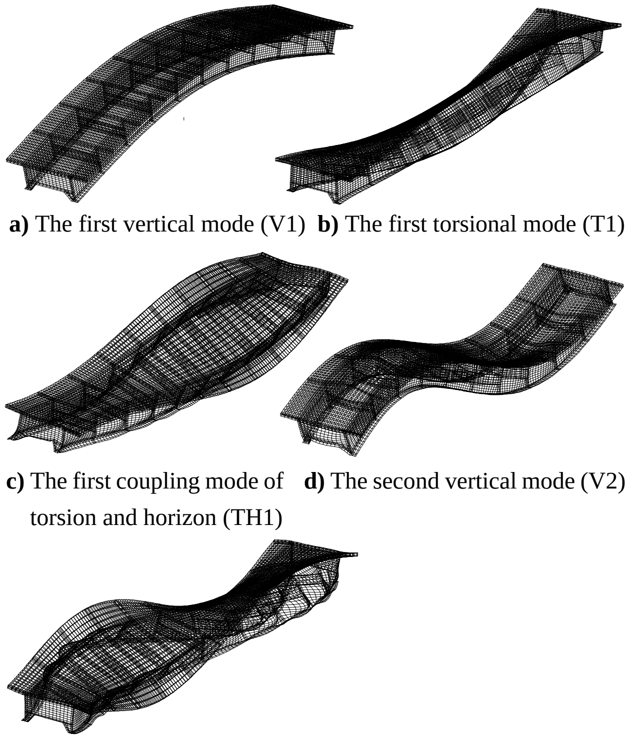
Although free vibration analysis does not relate to any types of loading, it is one of the most important steps in any dynamic analysis process. It is the usual first step in any dynamic analyses, and its results, which are natural frequencies and mode shapes, characterize the basic dynamic behavior and are an indication of how structures will respond to dynamic loadings[11]. An overall understanding of normal modes analysis as well as knowledge of the natural frequencies and mode shapes is important for all types of dynamic analysis. This section concentrates on the changes of natural frequencies and corresponding mode shapes with the varying of bridge's curvatures. Five different bridge's radii are investigated, namely R = 100m, 200m, 400m, 800m and ∞ (straight) to take into account the effect of initial curvatures.

Based on the results of the previous section, the mesh size of the Model-C, QUAD4/HEXA8 element group and lumped mass matrix formulation are used for these studied models. All of the finite elements are defined based on the cylindrical coordinate system located in the center of bridge's curvature. The boundary conditions at the ends of the main girders, which are also based on the cylindrical coordinate system, are hinged and movable-supported in tangential directions as presented in **Table 3**.

Table 3 Boundary conditions

Type	u_1	u_2	u_3	θ_1	θ_2	θ_3
Hinged	*Fix*	*Fix*	*Fix*	*Free*	*Free*	*Free*
Movable	*Fix*	*Free*	*Fix*	*Free*	*Free*	*Free*

u_1, u_2, u_3 are translations in the R, θ, Z directions.
θ_1, θ_2, θ_3 are rotations about the R, θ, Z directions.



a) The first vertical mode (V1) **b)** The first torsional mode (T1)

c) The first coupling mode of torsion and horizon (TH1) **d)** The second vertical mode (V2)

e) The second torsional mode (T2)

Fig. 6 The first five mode shapes of the straight model

The usual first step in performing a dynamic analysis of a structure is determining the natural frequencies and mode shapes of the structure with damping neglected. The number of natural frequencies and associated mode shapes is equal to the number of degrees-of-freedom that have mass or the number of dynamic degrees-of-freedom in the structure. However, amongst many natural frequencies and mode shapes, only some of the first ones are usually interested because of their influences in dynamic response of the structure. In this study, only the first five modes, which are shown on **Fig. 6** along with their names and abbreviations, are taken into account. These modes are chosen because they represent the behaviours of whole system vibrations, not local vibrations of only some members.

Unlike in straight system whose mode shapes are easily to recognize, all the mode shapes in curved models are coupled of bending and torsion vibrations. The higher of curvature, the larger of coupling effects can be seen. However, based on the primary difference in vibrations of the two main girders, these mode shapes can be classified. In the vertical-related modes, the two main girders vibrate in the same directions and the magnitude of the outside main girder's vibrations is always larger than that of the inside one and vice versa in the torsion-related modes.

Numerical results of natural frequencies and frequency ratios of the five studied models are graphically presented in **Fig. 7** with varying of curvatures. From the figure, it is easy to realize that the curvature has significant effects on the natural frequencies of the studied models especially in the models whose radii are smaller than 400m. It is noted that the boundary conditions, the length of centerline, the girder spacing, the number of cross frames, and the geometric properties of all models are unchanged. So the changes of the natural frequencies are mainly caused by different models' curvatures. The figure clearly show that while the frequencies of the vertical-related modes decrease, those of the torsion-related one tend to increase with the increase curvatures. Increasing of curvature means shortening the length of the inside main girder and lengthening that of the outside one. In connection with the primary difference in

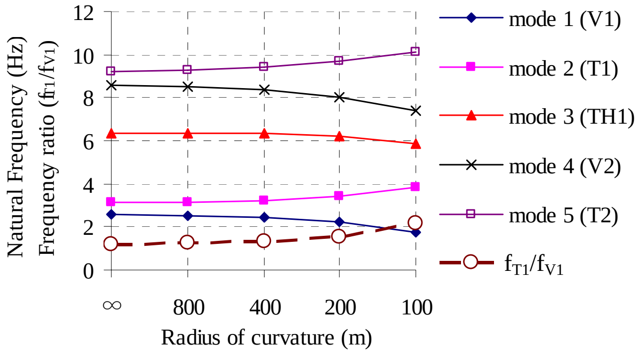


Fig. 7 Natural frequencies/frequency ratios with varying of curvatures

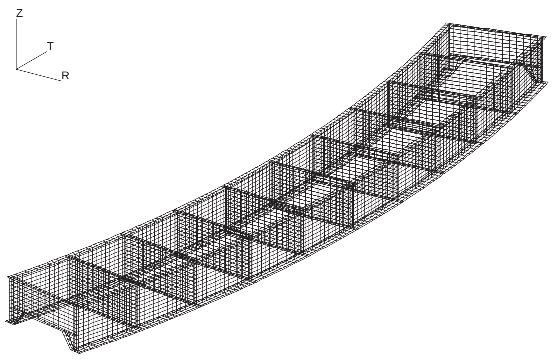


Fig. 8 Typical full-height diaphragm model (*R*=200)

vibration between torsional and vertical modes as aforementioned, the changes of frequencies could be understood.

Consequently, the frequency ratios, which are the ratios of the first torsional and vertical modes, change proportionally with the curvatures of bridges. It can be observed that the frequency ratios of almost all studied models are still small. So it is practical to increase this value by improving torsional rigidity of the studied bridge. And this is also the objective of the next section.

5. Improving Free vibration characteristics

When a bridge subjected to aerodynamic forces or an eccentrically running vehicle, it usually vibrates both vertically and torsionally. These unexpected vibrations cause fatigue damage in bridge members especially at connections due to stress concentration and sometimes lead to brittle fracture of the bridge. An effective method to mitigate these vibrations in bridge members is changing of its natural frequencies. In this section, the natural frequencies of the studied bridge are enhanced by several stiffening structures such as intermediate full-height diaphragms, lateral bracings and combinations of these structures.

(1) Effects of intermediate full-height diaphragms

Intermediate cross-frames or diaphragms of straight composite steel girder bridges can serve two distinct functions. They are designed to brace the girders' compression flanges and distribute loads among the girders. They act as secondary members to maintain the structural integrity. On the other hand, in horizontally curved and skewed bridges, the interaction of bending and torsion causes these components to become very important load-carry members[5)]. Therefore, the stiffnesses of diaphragms play an important role in the overall rigidity of curved bridges.

In order to assess the effect of intermediate diaphragms on free vibration characteristics of horizontally curved bridges, the same number of models which is analyzed in the previous section is carried out with the only difference in intermediate diaphragms as typically shown in **Fig. 8**. The original intermediate cross-beams, whose depth is 1.0m, are changed by new intermediate diaphragms of 3.0m deep, which is equal to that of the main girder, and called full-height diaphragms.

The differences of the first five modes' natural frequencies are displayed on **Fig. 9** with different radii of curvature. It is known that the higher stiffness of a member generally increases the natural frequencies of the system, on the contrary, its heavier structural mass cause decreasing. This could explain for the changes of the frequencies as displayed in the figure. It is clear to see the counteractions between the effects of mass and stiffness on these natural frequencies in vertical modes. The differences change from negative in large radius models to positive in small ones. In other words, the higher of curvatures are, the lagger positive effects can be achieved.

The overall effects of the studied diaphragms can only be seen in the models whose radii are smaller than 200m. However, these enhancements of full-height diaphragms are small and not worth changing from ordinary intermediate cross-beams.

(2) Effects of bottom-plate lateral bracings

The lateral bracing used in this study is the 20mm steel plates linking the bottom flanges of the two main girders and hereafter are called bottom-plates. There are total six different bottom-plate configurations investigated in this study as shown on **Fig. 10** along with their models' names (concrete deck slab is not shown). The simplest bottom-plate configuration is bp2a model which has two bottom-plates in outmost exterior bays as shown on **Fig. 10a**. The number in each model's name is the number of bays braced by bottom-plates in that model. Hence, the two bp4a and bp4b models, which are plotted in **Figs. 10b,** and **10c**, have the same number but different locations of braced bays. In the bp4a model, there are four bottom-plates that are arranged

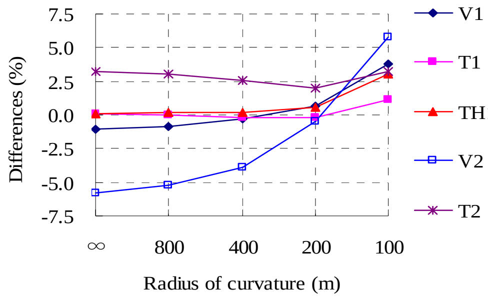
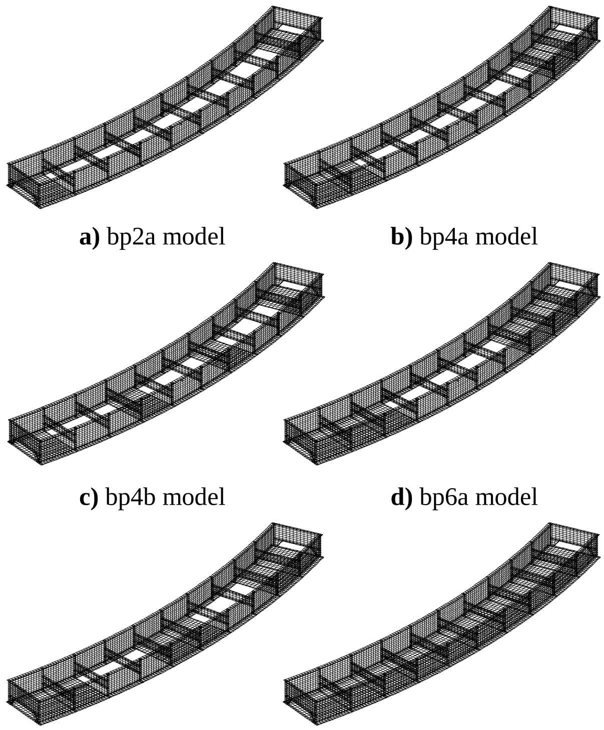


Fig. 9 Differences between full-height diaphragm and original models

symmetrically at both ends of the bridge. In bp4b model, beside the two bottom-plates arranged identically to those in bp2a model, there are two other bottom-plates positioned in the middle of the bridge span. This situation is the same in bp6a and bp6b models which are shown on **Figs. 10d** and **10e**. The purpose of these models is to determine the effects of not only the number but also the location of braced bays on free vibration characteristics of the bridge. The last model is bp10 one shown on **Fig. 10d**. In this model, all ten bays of the structure are braced and this makes this model somehow like a steel box girder bridge more than I-girder one. In practice, there is no bridge that is designed like this model; however, this model is intended to learn the ultimate enhancements of a structure by these bottom-plates. The same as previous sections, five models, whose radii are equal to 100m, 200m, 400m, 800m and infinitive, are considered.

Because the changing tendencies of the frequencies of all studied models are similar, only calculated results of typical models with $R=\infty$ and 100m are graphically demonstrated on **Figs. 11a** and **11b**, respectively. The results evidently reveal the greatly effects of bottom-plates on the natural frequencies of the torsion-related modes (mode 2, 5) and modestly effects on those of the vertical-related ones (mode 1, 4). In addition, frequencies of the studied systems are influenced considerably by both of the number and location of braced bays. With the same number of braced bays, the systems which are braced at exterior bays always achieve better results (bp4a, bp6a) than the others do (bp4b, bp6b). With all the bottom-plates at exterior bays, the natural frequencies of the systems increase proportionally with the number of braced bays.

To confirm the effect of different bottom plates on the performance of the system, the frequency ratios of these models, which are the ratios between the natural frequency of the first torsional and that of the first vertical modes, are depicted in **Fig. 12**. It can be assured the better performance of bottom plate configurations which have exterior stiffened bays such as bp2a, bp4a, bp6a, and bp10 models. The bp6a and bp10 models produce very good responses; however, using too many bottom-plates becomes impractical when



a) bp2a model **b)** bp4a model

c) bp4b model **d)** bp6a model

e) bp6b model **f)** bp10 model

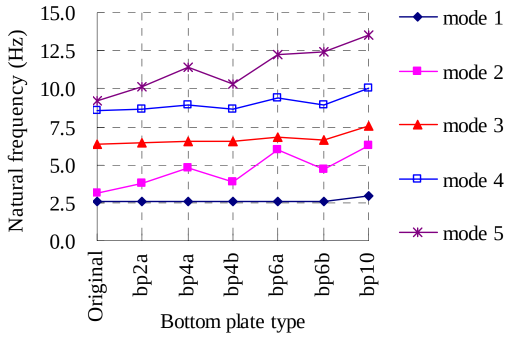
Fig.10 Bottom-plate configurations (R=100m)

considering the cost effectiveness. Consequently, the bp2a and bp4a configurations are considered the suitable ones for the last section of this study.

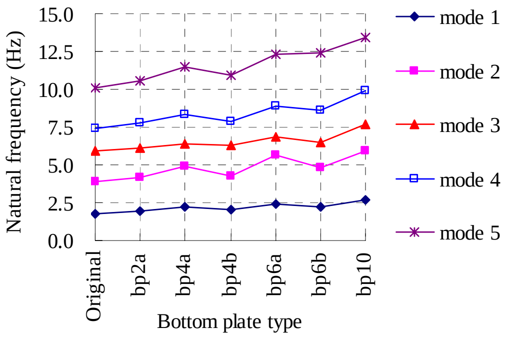
(3) Stiffening structures and their effects

In the two previous sections, the effects of full-height diaphragms and bottom plates are carried out separately. From the calculated results, it is learned that the full-height intermediate diaphragms have a very little effect on the free vibration characteristics, whereas the bottom plates in exterior bays greatly enhance these characteristics of the studied bridge by increasing its torsional stiffness. It is also noted that the full-height diaphragms are in the vertical planes that are perpendicular to the main girders, whereas the bottom-plates are in the horizontal one. The combinations of these structural members could increase the spatial rigidity of the studied bridge and then greatly improve its free vibration response. From this deduction, this section investigates the effects of combinations of the full- height intermediate diaphragms and bottom-plates hereafter called end stiffening structures.

Details of these end stiffening structures and their model names are shown in **Fig. 13**. In these models, beside the use of bottom plates, there are some replacements of ordinary intermediate cross-beams by full-height diaphragms. bp2a_2, bp4a_2 and bp6a_2 models are the combinations of the bp2a, bp4a, and bp6a models, respectively, with 2 full-height diaphragms at both interior edges of the bottom-plates. These



a) *R*= ∞



b) *R*=100m

Fig. 11 Natural frequencies of bottom-plate models

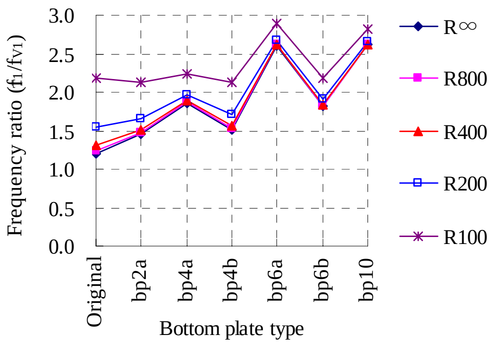
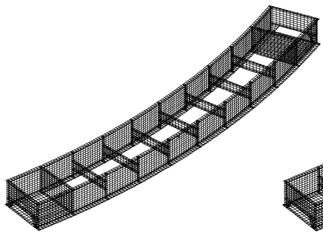
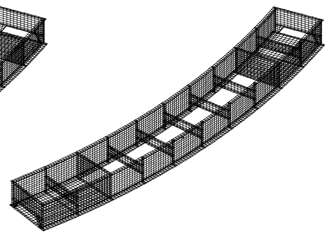


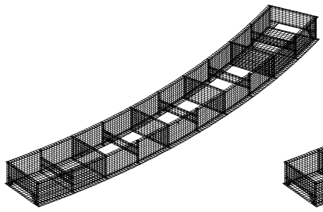
Fig. 12 Frequency ratios of bottom plate models



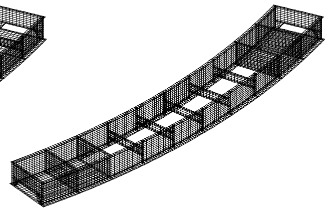
a) bp2a_2 model



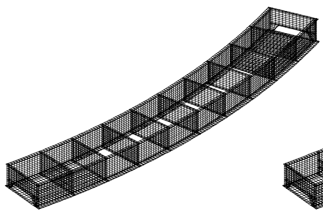
b) bp4a_2 model



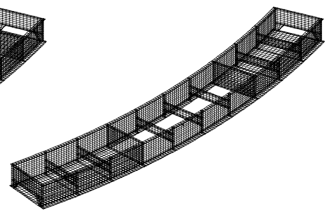
c) bp4a_4a model



d) bp4a_4b model



e) bp4a_9 model



f) bp6a_2 model

Fig. 13 Typical end stiffening configurations (*R*=100m)

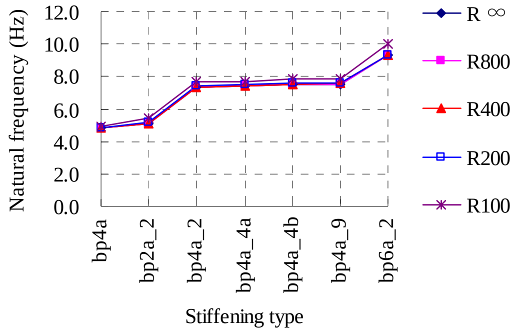
diaphragms, end diaphragms, main girders, bottom-plates, and concrete slab all together form two closed boxes at both ends of the studied bridge. As displayed in each model's name, the first group of words represents for the bottom-plate configuration and the second is the number of intermediate cross-beams replaced by full-height diaphragms. Therefore, in other models beside the two diaphragms as described in above models, there are other ones which are placed inside, outside or both of the closed boxes. Five different curvatures *R*= ∞, 800, 400, 200, and 100 (m) are investigated in each end stiffening structure.

From the results of previous section, it is known that the stiffer at both ends of bridge structure, the better of its dynamic response is achieved. Therefore, the formation of two close boxes in these combined configurations could enhance the natural frequencies of the system. As expected, these combinations result in significant influences on frequencies especially of the torsion-related modes as presented in **Fig. 14**. Even the simplest model bp2a_2 can produce favorable results that are comparable to those of the bottom-plate bp4a model. It can also be observed from the figure that the torsional frequencies of the bp4a_2, bp4a_4a, bp4a_4b, and bp4a_9 models are similar in spite of numbers of their diaphragms are totally different. This finding also correlates well with the results in previous section that the diaphragms achieve nearly no better results with regard to the original cross-beams. Therefore, the minimum replacement by two intermediate full-height diaphragms at both interior edges of bottom-plates is necessary in these models.

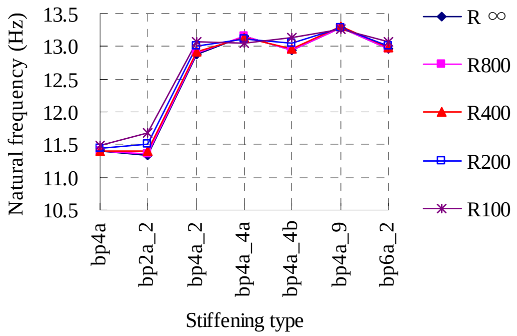
To have an overview about the improving effects of the structures studied in this paper, the frequency ratios of the most reasonably studied structures in each section are displayed in **Fig. 15**. It is easy to see the performance of the bp4a_2 model is superb, whereas, that of full-height diaphragm is similar to that of original model one. In addition, the frequency ratios of the higher curvature models are always larger than those of the others.

6. Conclusions

The present study has been investigated the free vibration characteristics of horizontally curved twin I-girder bridges by using 3-D finite element method of MSC Nastran. The results of many detailed FEM models provide sufficient evidents for the following remarkable conclusions:



a) The first torsional modes (T1)



b) The second torsional modes (T2)

Fig. 14 Torsion-related frequencies with stiffening structures

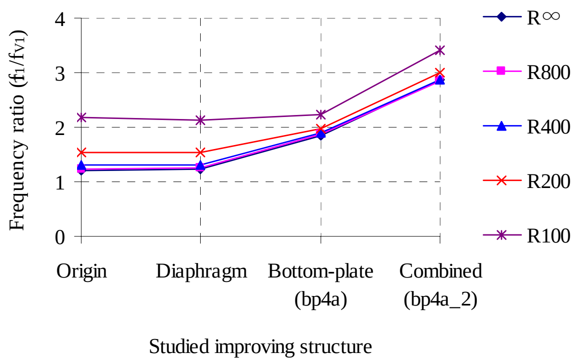


Fig. 15 Frequency ratios of studied improving structures

(1) In this studied bridge with a reasonable mesh size and given finite elements, the free vibration analyses are not very sensitive with the mesh size and element types. The performance of quadric quadrilateral shell elements is superb; that of 24-node shell element is better than 20-node shell element only if the mesh size is fine enough. Lumped mass system is slightly better than consistent one.

(2) In the first five modes, frequencies of vertical-related modes tend to decrease; whereas those of torsion-related modes, on contrary, tend to increase with the increase of curvature.

(3) The use alone of full-height diaphragms in this study does not gain desired enhancement with regard to ordinary cross-beams in the original models. In other words, the ordinary intermediate cross-beams have adequate stiffness in this studied bridge.

(4) The bottom-plates considerably enhance the natural frequencies of torsion-related modes. These frequencies are greatly affected by both number and location of bays braced by bottom-plates. With the same number of braced bays, the systems which are braced at exterior bays always achieve better results. With all the bottom-plates at exterior bays, the natural frequencies of the systems increase proportionally with the number of braced bays.

(5) Finally, the combinations of bottom-plates and diaphragms exhibit a significant enhancement of the natural frequencies. However, only minimum use of 2 intermediate diaphragms is necessary.

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