Fatigue strength evaluation of root-failed welded joints based on one-millimeter stress

Zhi-Gang Xiao*, Kentaro Yamada**

*Dr. of Eng., Dept. of Civil Eng., Nagoya University, Furocho, Chikusa-ku, Nagoya 464-8603

**Ph.D., Professor, Department of Environmental Engineering and Architecture, Graduate School of Environmental Studies, Nagoya University, Furocho, Chikusa-ku, Nagoya 464-8603

A simple method of determining geometric stress was proposed for fatigue strength evaluation of weld toe-failed joints in previous studies. In this research, this method is extended to the fatigue strength evaluation of root-failed welded joints. The normal stress at one-millimeter in the expected crack path, which is not subjected to the effects of local geometries of weld root, is used to represent geometric effect. The fatigue test data of butt welds plotted with one-millimeter stress is taken as the reference of fatigue strength. The good agreement between the reference and the re-plot of root-failed cruciform joints in one-millimeter stress demonstrates the validity of the proposed method.

Key Words: fatigue strength, welded joints, geometric stress, root crack, FEA

1. Introduction

Weld root is a possible site of high stress concentration when it is transverse to the applied stress. Depending on the joint geometry and the extent of weld penetration, the weld root may be more severe than the weld toe and become the site for fatigue crack initiation¹⁾. Fig.1 shows schematically the root cracks in transverse butt welds containing lack of penetration (LOP) and load carrying cruciform joints. Generally, the fatigue strength of weld root-failed joints is lower than that of weld toe-failed ones. Weld root cracks are more difficult to inspect and more damaging than weld toe cracks, since in most cases, weld roots are "buried" within the welded joints, the fatigue crack starting from weld root cannot be seen on weld surface until it has penetrated through the weld throat and had an essential propagation, and at that time most of the fatigue life has already been consumed.

As with weld toe failures, classification method is most commonly used for fatigue strength evaluation of typical root-failed joints. Linear elastic fracture mechanics method (LEFM) is also applicable for evaluating root-failed welded joints provided that adequate information is available on stress and geometry of crack and the joints. However, the conventional hot spot stress (HSS) method, or geometric stress method, is not applicable to evaluation of root failures, since in most of existing HSS methods, extrapolation procedures are normally used to solve the geometric stress at weld toe from surface stresses by assuming linear or quadratic distribution of geometric stress near weld toe. It is obvious that the HSS thus obtained is not relevant

to the stress at weld root. It is specified in the fatigue design recommendations of International Institute of Welding (IIW) and other literatures that HSS in only applicable to the evaluation of weld toe failure²⁾³⁾.

For weld toe failures, by analyzing the local stress and geometric stress in weld toe region with a non-load-carrying cruciform joint and an in-plane gusset, the authors proposed a simple solution for geometric stress⁴). The normal stress at one-millimeter in the expected crack path, where the effects of local geometries of weld toe can not reach, is taken as the geometric stress, and the fatigue strength of several series of non-load-carrying cruciform joints whose one-millimeter stress factors equate unity is taken as the reference strength against the proposed geometric stress, i.e., one-millimeter stress. The applicability of the proposed method, i.e., one-millimeter stress method, to the fatigue strength evaluation of weld toe failure is shown by fatigue test results of typical welded joints⁴⁾ and

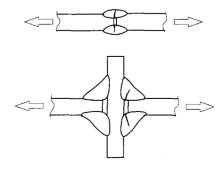


Fig.1 Root cracked butt weld and cruciform joint

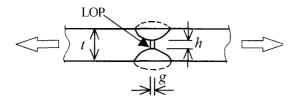


Fig.2 Reinforcement removed butt weld

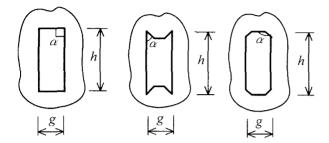


Fig.3 Geometries of LOP tip: right, sharp, and blunt angles

complex welded details⁵⁾⁶⁾.

In this research, the one-millimeter stress method is extended to the fatigue strength evaluation of weld root failures. The stress distribution along the expected crack path is investigated with double-sided butt welds containing LOP. It is found that the effects of local geometries of weld root are restrained within one-millimeter in the crack path. As with the case of weld toe failure, the normal stress at one-millimeter in crack path is used to represent geometric effect. The fatigue test data of butt welds plotted in one-millimeter stress is taken as the reference for fatigue strength evaluation of root failure. The good agreement between the reference and the re-plot of root-failed cruciform joints in one-millimeter stress shows the validity of the proposed method.

2. Stresses along the Expected Crack Path of Butt Welds Containing LOP

Transverse butt welds containing LOP may develop fatigue cracks at weld toe or weld root, depending on the size of weld and the extent of penetration. However, if reinforcement is removed by grinding, fatigue cracks will start from weld root and penetrate through the weld transverse to applied stress, provided that other defects such as porosity are not so severe as weld root. The stresses along the expected crack path in reinforcement-removed butt welds containing LOP will be studied through finite element analyses (FEA) in the following.

Affecting factors to stress along crack path to be investigated include the thickness of butting plate, *t*, the width of LOP, *g*, the depth of LOP, *h*, and local geometry of LOP tip, as shown in Figs. 2 and 3. The local geometry of a weld root is very difficult to control and therefore widely varied. Fig.3 shows schematically three cases of LOP tip, with a right, sharp and blunt angle, respectively, to represent an intermediate and two extreme cases of LOP tip geometry. Nine FE models are created

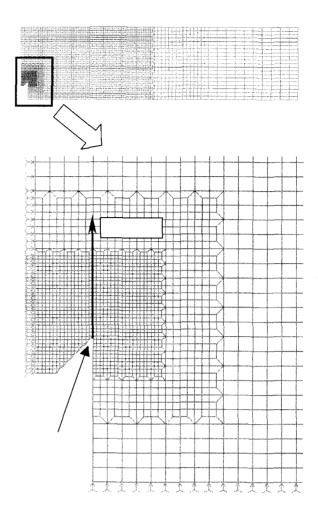


Fig. 4 Meshing of Model G1D4A45

using plane strain elements, as shown in Table 1. The software package used is COSMOS/M 2.6⁷⁾. As an example, the meshing is shown in Fig.4 for a FE model, G1D4A45, only one quarter of which is modeled by taking advantage of symmetry. Fine mesh is generated around LOP tip with the minimum elements of 0.05 mm by 0.05 mm in size.

2.1 Effects of local geometries of LOP tip

Concerning the geometries of LOP tip, five models are analyzed, i.e., G1D4A15, G1D4A30, G1D4A45, G1D4, and G1D4A135, covering sharp, right, and blunt angles of LOP tip. Except the angle values of LOP tip, other dimensions of these models are the same, as shown in Table 1. The normal stress along the expected crack path normalized by the average throat stress, which is defined as K_b is plotted in Fig.5 for each model. It could be seen from Fig.5 that the general trend is that the stress at LOP tip increases as the tip is sharpened. Another phenomenon is that, even though the stress at LOP tip is very sensitive to the change of tip geometry, the stress tends to be stabilized from one-millimeter onward in crack path. It indicates that the stress at one-millimeter in crack path includes little effect of local geometry of LOP tip and is mainly governed by the overall geometry of the welded joint, i.e. the size of LOP and the

thickness of the butting plates.

2.2 Effects of the width of LOP

The normalized stress results are plotted along crack path in Fig.6 for models G0D4, G1D4, and G3D4 to study the effect of LOP width. The with of LOP (gaps between two butting ends of main plates) is 0.2, 1.6, and 3.2 mm, respectively, and other dimensions of these models are the same, as shown in Table 1. It could be seen from Fig.6 that small LOP width results in high stress concentration at the tip of LOP, but the trend is reversed from one to two millimeter in crack path onward, with the stress in the wide LOP model being larger than that in the narrow one, due to the complement relationship between the stress in this part and that in the initial part of crack path. It could be seen that the difference in stress between different models is shown evidently in a large region far beyond one-millimeter in crack path, thus the effect of LOP width should be classified as a global one rather than a local one.

2.3 Effects of the thickness of butting plates

To study the effect of plate thickness, the stress results are compared in Fig.7 for three models, B12G1D4, G1D4, and B20G1D4, among which the only difference is the thickness of the butting plates, being 12, 16, and 20 mm, respectively, as shown in Table 1. It could be seen in this figure that the stress increases with the thickness of plates along the whole crack length. Thus it can be said that the thickness of plate has a global effect on stress along crack path.

Table 1. FE models of double-sided butt welds

Model	Thickness of plate, $t(mm)$	Width of LOP, g (mm)	Depth of LOP, h (mm)	Angle of LOP tip, α (degree)
G1D4A15	16	1.6	4	15
G1D4A30	16	1.6	4	30
G1D4A45	16	1.6	4	45
G1D4A135	16	1.6	4	135
G0D4	16	0.2	4	90
G1D4	16	1.6	4	90
G3D4	16	3.2	4	. 90
B12G1D4	12	1.6	4	90
B20G1D4	20	1.6	4	90

3. Determination of Referential Fatigue Strength with Test Results of Butt Welds Containing LOP

It is seen that, in butt welds containing LOP, the stress at one-millimeter in crack path includes little effect from local geometries of LOP tip, but mainly reflects the effect of the global features of the joint, such the width of LOP and the thickness of butting plates due to the short distance of

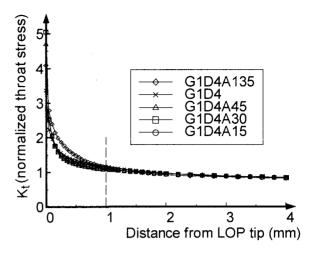


Fig. 5 Effects of geometries of LOP tip on stress along crack path

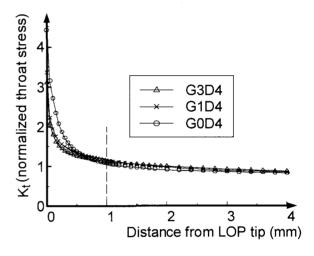


Fig. 6 Effects of LOP width on stress along crack path

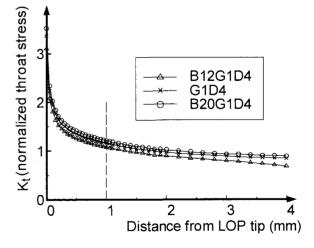


Fig. 7 Effects of plate thickness on stress along crack path

one-millimeter involved. As with the weld toe failure, the normal stress at one-millimeter in crack path is rationally taken as the stress for fatigue strength evaluation of root failures, and the reference fatigue strength can be determined with the fatigue test results of butt welds containing LOP.

3.1 Fatigue test results of butt welds

Kim carried out fatigue tests on double-sided butt-welds containing LOP^8). The butt welds were produced with the process of CO_2 shielded automatic gas metal arc welding. The weld reinforcement was removed to generate root cracks. The thickness and width of the SM490YA steel butting plates were 16 and 120 mm, respectively. Two series of specimens with different widths of LOP, g, were designed, with the intended zero wide LOP (face-to-face contact of butting plates) in one series, B0G0, and the intended 1.5 mm wide LOP in another, B0G1. The measurement of the fractured surfaces showed that the actual LOP width of B0G0 was below 0.05 mm and the LOP width in B0G1 was between 1.0 and 1.4 mm. The intended depth of LOP, h, is 4 mm for all specimens, and the measurement showed that the actual LOP depth varied between 2.5 and 7.0 mm.

In all specimens tested, fatigue cracks developed from the tip of LOP at multiple sites along the length of the welds. After some propagation, the cracks coalesced into a large flat crack and continued propagation until breaking the joint. Fatigue test results based on the measured area of throat section are plotted in Fig.8. It can be seen in Fig.8 that the fatigue strength of B0G1 is slight lower than that of B0G0, due to the fact that the height of LOP in the former is larger than that in the latter⁸. All fatigue test data are above the fatigue class FAT45 specified by IIW²) for transverse double butts containing LOP. Fatigue strength class is not specified for this detail in the Fatigue Design Recommendations of Japanese Society of Steel Construction (JSSC)⁹).

Nishi et al. carried out fatigue tests on single sided transverse butt welds containing LOP¹⁰⁾. The single side butt weld was produced with the TIG welding process in 8 passes. Weld reinforcement was also removed to generate fatigue cracks from weld root. The main dimensions of test specimens are shown in Fig.9. The thickness of the 316L stainless steel abutting plates is 40 mm, and the depth of LOP is about 3.8mm. The LOP was made without gap opening. The width of the specimen is 50 mm. Fatigue test results are plotted in Fig.8 in terms of average stress over the weld throat. The data are below those of double butt welds shown above, which is mainly due to the increase in stress concentration at weld root caused by the increase in plate thickness and the eccentricity of LOP compared with the double butt welds. Fatigue strength class is not specified by JSSC and IIW for single side butt welds of incomplete penetration²⁾⁹⁾, but IIW specifies the fatigue class FAT45 for full penetrated single side butt welds with weld root conditions not controlled by

means of nondestructive testing (NDT). It could be seen that the test results of single side butt welds satisfy this class.

3.2 Determination of reference fatigue strength against one-millimeter stress

Linear elastic finite element analyses are carried out for all series of specimens, as was done for the FE models in Section 2. The tip of LOP tip is modeled with a right angle. Taking into account the actual sectional area of weld throat, the one-millimeter stress factor K_b , i.e. the normal stress at one-millimeter in crack path (in throat section) normalized by the average stress over the throat section, is calculated for each specimen, and the fatigue test results in Fig.8 are re-plotted with one-millimeter stress range in Fig.10.

It could be seen that the separation between different series of data in Fig.10 becomes less evident than in Fig.8. However, there still exists a slight separation in the re-plots between single butt and double butt welds, which might be due to the difference in welding processes, since the use of TIG could produce satisfactory root bead shapes compared with the conventional gas metal arc welding¹⁾.

Assuming the inverse slope of S-N curve m=3, the regression analyses of the re-plotted data in Fig.10 are carried out by taking $\log N$ as dependent variable. The mean and mean $\pm 2s$ (s, standard deviation) lines are plotted in Fig.10. The two-million-cycle fatigue strength of the mean, mean-2s, and mean $\pm 2s$ is 85.0, 68.3, and 105.9 MPa, respectively. Compared with the references against one-millimeter stress for weld toe failure evaluation, these values are significantly lower.

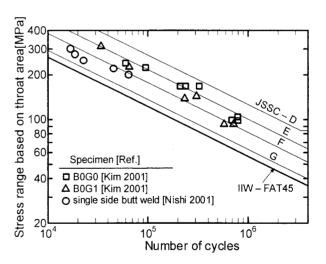


Fig.8 Fatigue test results of butt welds containing LOP

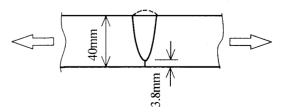


Fig.9 Test specimen of single side butt weld

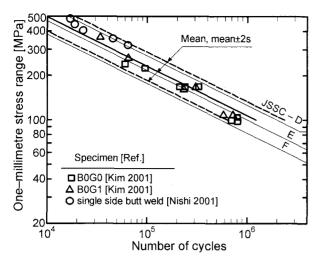


Fig.10 Fatigue test results of butt welds containing LOP plotted in one-millimeter stress range

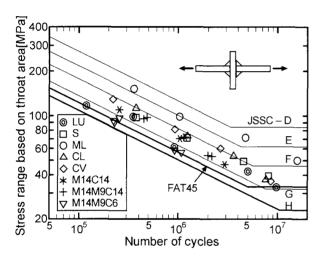


Fig.11 Fatigue test results of root-cracked load-carrying cruciform joints

4. Correlation of fatigue strength between root-failed cruciform joints and butt welds containing LOP

4.1 Fatigue test results of root-failed cruciform joints

It is found that cruciform joints are very likely to contain compressive residual stresses in weld root region³⁾¹¹⁾. Test results of root-failed cruciform joints may include the beneficial effect of compressive residual stress under some conditions such as low level of applied stress. Since no evidence is shown that the previous test results of root-cracked butt welds include effects of compressive residual stress, thus only the test results of root-failed cruciform joints not subjected to compressive residual stress should be collected for the purpose of fatigue strength correlation. Another criterion for data collection is that the actual area of the cracked throat section is known, since the actual weld size may differ significantly from the intended weld size, and fatigue strength of root failure is very sensitive to the actual size of weld throat.

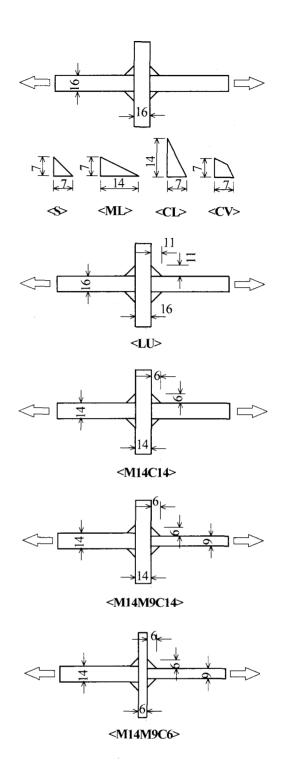


Fig.12 Test specimens of load-carrying fillet welded cruciform joints (dimensions in millimeter)

The test data of eight series of root-cracked load-carrying cruciform joints satisfying the abovementioned criteria are collected from the doctoral dissertation of Kainuma¹²⁾ and are plotted in Fig.11. The design dimensions of significance are plotted in Fig.12 for all specimens. The width of S-, ML-, CL-, and CV-specimens is 35 mm, and the other specimens are 25 mm wide. All the specimens failed in the weld throat. Fatigue cracks starting from weld root propagated in initial stage in the direction almost parallel to the central plate, being 10 to 15° to

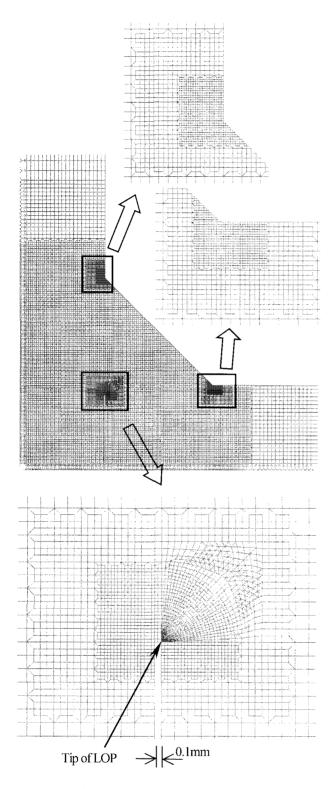


Fig.13 FEM model of LU-specimen

the axis of the central plate¹²⁾. Similar finding was also obtained by Frank et al¹³⁾. After some significant vertical propagation through the weld, usually one-third to one-half of the weld section, the remaining area of the weld throat section was insufficient to resist the maximum load, and unstable propagation of fatigue cracks occurred before the subsequent rupture.

The test results in Fig.11 are largely scattered, with the

fatigue strength ranging between JSSC-H and JSSC-D. All the fatigue test results satisfy the fatigue class JSSC-H specified for root-failed load-carrying cruciform joints by the Fatigue Design Recommendations of JSSC⁹. The fatigue class given by IIW is FAT45 for root cracked load-carrying cruciform joints produced with fillet welds or partial penetration K-butt welds², which is slight higher than JSSC-H (40MPa at two million cycles). It could be seen in Fig.11 that two data in the M14M9C6 series fall below the class FAT45, which indicates that the fatigue strength suggested by IIW for load-carrying cruciform joints might be unsafe in some cases, e.g. when the thickness of the central plate is small. Based on the review of fatigue test data, Maddox suggested the fatigue class FAT36 for the design of cruciform joints failing in the weld throat under axial loading³).

4.2 FEA and one-millimeter stress

(1) FE model

By taking advantage of symmetry, one-quarter plane strain models are created for specimens LU, S, ML, CL, CV, and M14C14, and one-half plane strain models for M14M9C14 and M14M9C6. Some modeling details are described in the following by taking the FE model of LU as an example.

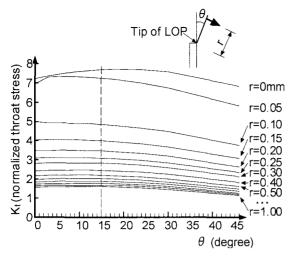
The LOP in LU-specimens is modeled with a gap of 0.1 mm wide, as shown in the FE model of LU in Fig.13. The depth of penetration of the fillet welds varied between 0.6 and 1.3 mm¹²), thus a penetration of 1.0 mm, which is close to the average value of the penetration depth, is modeled by setting the tip of LOP at 1.0 mm below the main plate surface. Fine mesh is generated around weld roots and weld toes, as shown in Fig.13. The tip of LOP is modeled with a right angle, and the geometry of weld toe is modeled with a zero-radius and a 45° angle.

The part of weld next to weld root is modeled with radiating mesh to facilitate the extraction of stress in different directions. Thirty radiating sectors are generated within the 90° region of weld, with each sector taking a radiating angle of 3°. The mesh size along the radius direction is 0.05mm, and the element size in the tangential direction decreases proportionally as the mesh approaches the LOP, until becomes zero when meeting the tip of LOP, where the surrounding radiating elements collapse from 4-node elements into 3-node triangular elements.

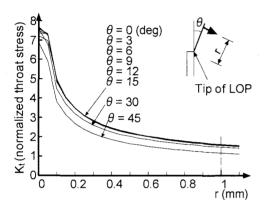
(2) Stress results and determination of one-millimeter stress

The stress distributions in weld root region are plotted in Fig.14. The stresses summarized are the normalized stress components normal to the radius in different directions (possible crack path), as shown in the insets of Fig.14. Hereafter, a normal stress at a node will be referred to by the location of the node, i.e. an angle to the vertical, θ , and a distance to the LOP tip, r. Hence the angle of a normal stress does not mean its real direction, but the direction that it is normal to.

In Fig.14a, the normal stress at a certain distance from the LOP tip, r, is plotted against the angle to the vertical. To give a whole visualized picture, the same stress results are rearranged



(a) Plot of stress with respect to radiating directions



(b) Plot of stress with respect to distance to weld root

Fig.14 Stress distribution in weld root region

in terms of radiating directions by plotting them against the distance to LOP tip in Fig.14b. It could be seen from these figures that stress gradient is high in regions close to weld root, and it becomes much lower at one-millimeter away from the root. Another phenomenon is that, when the angle is changed between 0 and 15°, the magnitude of the normal stress remains almost constant, except in the core region of 0.05 mm to the LOP tip, which corresponds to the first layer of mesh next to the LOP tip. Further deviating from the vertical, i.e. the angle to the vertical being increased, the normal stress component decreases gradually. This confirms the fact described earlier that the direction of initial fatigue crack propagation is limited within 15° to the vertical, mostly 10° to 15°. These phenomena also occur in other FEM models. In the direction of 15° to the vertical (θ =15°), the normal stress at 1 mm away from the LOP tip is therefore taken as the geometric stress for fatigue strength correlation, i.e. one-millimeter stress. From Fig.14a it could be seen that, in the curves in which r is close to 1 mm (including the curve of r=1 mm), when the angle increases from 0 to 15°, there is a slight increase in the normal stress magnitude. Thus, it can be thought that the normal stress at 1mm and 15° is the maximum or can approximate the maximum normal stress.

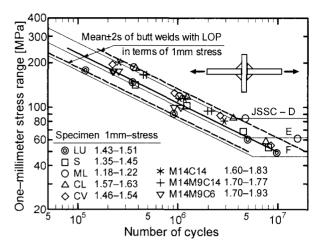


Fig.15 Fatigue test results of load-carrying cruciform joints in terms of one-millimeter stress range

4.3 Fatigue strength correlation

The one-millimeter stresses as determined above are extracted from FEA results for each model. For each series of specimen, the variation in one-millimeter stress resulting from the difference in weld size between the actual joint and the FE model is valuated by conducting FEA for two extreme cases, i.e. the maximum and minimum weld size. The one-millimeter stress for each specimen is then modified accounting to the actual weld size. The previous fatigue test results of root-failed cruciform joints are plotted in Fig.15 in terms of the modified one-millimeter stress. A point to note is that, the one-millimeter stress in the specimens of M14M9C14 and M14M9C6 is the value at the thick plate side, i.e. the side of 14 mm thick main plate, since the one-millimeter stress at the thick plate side is significantly larger than that at the thin plate side, and fatigue cracks actually developed at the thick plate side.

In Fig.15, also plotted are the data range of butt welds with LOP in terms of one-millimeter stress, which is obtained in Section 3. It could be seen that the re-plot of test data of load-carrying cruciform joints is in good agreement with that of transverse butt welds with LOP. Except two test data of ML-specimens, all the re-plot of cruciform joints fall within the range of re-plotted data of the butt welds with LOP. This indicates that the maximum normal stress at one-millimeter away from weld root and the re-plotted data of butt welds are suitable for fatigue strength evaluation of root-failed cruciform joints.

However, though not evident, the separation of test data can still be seen between some series. For example, the re-plot of LU data appears slight lower than the others. This may be due to the fact that a penetration of 1.0 mm is taken into account in the modeling of the fillet weld in LU-specimen. The information on penetration of fillet welds is not available for the other series¹²⁾, and penetration is therefore not modeled for those specimens.

However, penetration is reasonably believed to exist to some extent in those specimens, whether it is deeper or shallower. The modeling of penetration in LU model and the negligence of penetration in other models are the main reason that leads to the slight separation in the re-plot.

From FEA, it is found that the value of one-millimeter stress in root-failed joints changes with weld throat area. It is a case-by-case value depending on dimensions such as width of LOP and depth of penetration, though being not so dependent on the local geometries of weld root, which suggests the necessity of FEA on one hand, and the difficulty of correlating with the less varied nominal stress on the other hand. As part of their future efforts, the authors would like to investigate the relationships between one-millimeter stress and the nominal stress, but at present the one-millimeter stress method primarily bases its solution on the case-by-case FEA.

5. Summary and conclusions

In this study, the method of determining geometric stress with the normal stress at one-millimeter in crack path for fatigue strength evaluation of weld toe failure is extended to evaluate weld root failures.

Finite element analyses of butt welds containing LOP show that, though the local geometry of LOP tip has significant influence on stress concentration in the region surrounding LOP tip, but the affecting region is restrained within about one-millimeter in crack path. In comparison the width of LOP and the thickness of butting plates demonstrate their effects over much larger part of crack path.

Fatigue test results are analyzed for double and single side butt welds containing LOP. With the FEA in consideration with the actual size of weld throat, the normal stress at one-millimeter in crack path was taken to re-plot the test data. The scatter of the data is significantly reduced in the re-plot, and the separation between different series of test specimens becomes insignificant. The re-plot is taken as the reference for fatigue strength evaluation of root failed welded joints.

Test data of fillet welded load-carrying cruciform joints are collected based on the criteria of known size of throat area and being not subjected to the beneficial influence of compressive residual stress. FEA is conducted for each series of specimens, and the maximum normal stress at one-millimeter in the expected crack path, which is in between 0 to 15° to the LOP direction, is taken as the one-millimeter stress, and fatigue test results are subsequently re-plotted with the one-millimeter stresses thus determined. Good agreement is obtained between the re-plot and the data of butt-welds in one-millimeter stress, though slight separation still exists among some series of re-plotted data, which is attributed to the modeling of penetration in one series but the negligence of penetration in the other models due to the lack of penetration information.

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