Static and Free Vibration Analysis of Cylindrical Shells on Elastic Foundation

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Static and free vibration analyses of cylindrical shells partially buried in elastic foundation were conducted by means of finite element method. Foundation is modeled by continuous elastic springs and distributed circumferentially on limited arc by expansion of Fourier series. Formulation of foundation stiffness matrix was done by observation of the relationships between coupled harmonic terms. Applicability of present method to analyze considered problem subjected to any arbitrary loads and end conditions is verified. Free vibration analysis of chosen simply supported cylindrical shells was carried out. Variation of natural frequencies with certain geometry of the shells is presented systematically for axisymmetric structure.

Key Words: Cylindrical Shells, Elastic Foundation, Finite Element Method

1. Introduction

Cylindrical shells laid on the soil as a foundation are commonly used in many engineering fields in the form of structural components because of their strength and effectiveness. With such important roles, cylindrical shells should be analyzed adequately and reliably, especially for safety issues related to their dynamic behavior. Simple and wide applicable method is needed to perform such analysis. Many papers have been published for beam type structures. Free vibration of cylindrical shells without the existance of foundation can be found in text book written by Seide[1)] while many publications have been made for dynamic problems of cylndrical shells in the air. Yang[2)] has investigated the completely buried pipeline subjected to sinusoidal seismic load using shell element to model the pipe. Free vibration of whole buried simply supported cylindrical shells in Winkler and Pasternak foundation has been studied extensively by Paliwal[3,4)] using direct solution to the governing equation of motion for shells. In practical applications, cylindrical shells are embedded partially in the elastic foundation. This yields a more complicated problem. The free vibration of simply supported cylindrical shells with non-uniform elastic bed and mass has been investigated by Amabili[5,6)] based on Rayleigh-Ritz method. In his paper, both mass and the elastic bed have to be assumed distributed uniformly on the whole cylinder length in longitudinal direction. In this study, application of finite element method for cylindrical shells partially buried in elastic foundation is studied and turns out to be a useful method for static and free vibration analysis. Present method is applicable for any arbitrary loads and end conditions. Effect of relative stiffness between shells and foundation to the natural frequency distribution is presented here. Characteristic of vibration in term of alteration of sectional mode has been investigated.

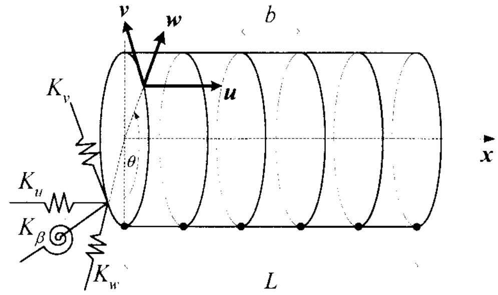
2. Analytical Model and Formulation

The structure is modeled by an isotropic thin elastic cylindrical shell with modulus elasticity E, Poisson's ratio υ and geometry of R, t and L for radius, thickness and length of the shell, respectively, which is assumed to be free from local instability such as buckling.

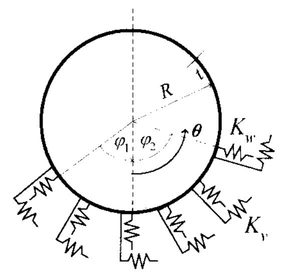
Soil as a foundation is modeled by continuous elastic spring attached on a limited arc surrounding the periphery of the shell in axial, circumferential, radial and radial slope direction as shown in Fig.1(a) for generalized model.

K_u, K_v, K_w and K_β are axial, circumferential, radial and rotation spring stiffness, respectively. φ_1 and φ_2 are angle corresponding to left and right of the enclosed arc. The geometry of model for the problem is shown in Fig.1(b).

The proposed method is based on finite element method. The displacement functions are defined by summation of product of



(a) Generalized model and reference direction



(b) Geometry of the structure

Fig.1. Generalized model and geometry of structure

simple polynomial in x with sinusoidal function in θ for a general problem as given in Eq.(1) for axial, circumferential, radial displacement and radial slope, respectively. Radial slope, β is defined as the first derivative of w with respect to x. Axisymmetric and non-axisymmetric corresponding functions are labeled by superscript S and U, respectively.

$$
\begin{aligned}
u(x,\theta) &= \sum_{m=0}^{M}\left\{\mathcal{U}_m^{S}(x)\cos(m\theta)+\mathcal{U}_m^{U}(x)\sin(m\theta)\right\}\\
v(x,\theta) &= \sum_{m=0}^{M}\left\{\mathcal{V}_m^{S}(x)\sin(m\theta)+\mathcal{V}_m^{U}(x)\cos(m\theta)\right\}\\
w(x,\theta) &= \sum_{m=0}^{M}\left\{\mathcal{W}_m^{S}(x)\cos(m\theta)+\mathcal{W}_m^{U}(x)\sin(m\theta)\right\}\\
\beta(x,\theta) &= \sum_{m=0}^{M}\left\{\mathcal{B}_m^{S}(x)\cos(m\theta)+\mathcal{B}_m^{U}(x)\sin(m\theta)\right\}
\end{aligned}
\tag{1}
$$

Axial and circumferential displacements are defined by linear polynomial in x, while radial displacement is expressed by third order polynomial.

Eq.(1) can be written as

$$
\{\boldsymbol{\delta}\} = \sum_{m=0}^{M}\left\{\left[\mathbf{N}^{S}\right]_m\left\{\boldsymbol{\delta}\mathbf{e}^{S}\right\}_m+\left[\mathbf{N}^{U}\right]_m\left\{\boldsymbol{\delta}\mathbf{e}^{U}\right\}_m\right\} \tag{2}
$$

in which $[\mathbf{N}^S]_m$ and $[\mathbf{N}^U]_m$ are axisymmetric and non-axisymmetric shape function matrix for typical term m.

2.1. Cylindrical Shells

The strain displacement relationship[7] is given by

$$
\{\boldsymbol{\varepsilon}\} = \begin{Bmatrix}\varepsilon_x\\ \varepsilon_\theta\\ \varepsilon_{x\theta}\\ \chi_x\\ \chi_\theta\\ \chi_{x\theta}\end{Bmatrix} = \begin{Bmatrix}\dfrac{\partial u}{\partial x}\\ \dfrac{1}{R}\dfrac{\partial v}{\partial\theta}+\dfrac{w}{R}\\ \dfrac{1}{R}\dfrac{\partial u}{\partial\theta}+\dfrac{\partial v}{\partial x}\\ -\dfrac{\partial^2 w}{\partial x^2}\\ -\dfrac{1}{R^2}\dfrac{\partial^2 w}{\partial\theta^2}+\dfrac{1}{R^2}\dfrac{\partial v}{\partial\theta}\\ 2\left(-\dfrac{1}{R}\dfrac{\partial^2 w}{\partial x\partial\theta}+\dfrac{1}{R}\dfrac{\partial v}{\partial x}\right)\end{Bmatrix} \tag{3}
$$

which includes three strain components ε_x, ε_θ, $\varepsilon_{x\theta}$ of the middle surface and three curvature changes χ_x, χ_θ, $\chi_{x\theta}$. By using Eq.(2), Eq.(3) can be written as

$$
\{\boldsymbol{\varepsilon}\} = \sum_{m=0}^{M}\left\{\left[\mathbf{B}^{S}\right]_m\left\{\boldsymbol{\delta}\mathbf{e}^{S}\right\}_m+\left[\mathbf{B}^{U}\right]_m\left\{\boldsymbol{\delta}\mathbf{e}^{U}\right\}_m\right\} = [\mathbf{B}]\{\boldsymbol{\delta}_{\mathbf{e}}\} \tag{4}
$$

Stiffness matrix for cylindrical shells can be derived by using

$$
[\mathbf{K_S}] = \int_V [\mathbf{B}]^{T}[\mathbf{D}][\mathbf{B}]\,\mathrm{d}V \tag{5}
$$

in which $[\mathbf{D}]$ is a 6×6 elasticity matrix.

$$
[\mathbf{D}] = \frac{E\,t}{1-\upsilon^2}\begin{bmatrix}1 & \upsilon & 0 & 0 & 0 & 0\\ \upsilon & 1 & 0 & 0 & 0 & 0\\ 0 & 0 & \dfrac{(1-\upsilon)}{2} & 0 & 0 & 0\\ 0 & 0 & 0 & \dfrac{t^2}{12} & \dfrac{\upsilon t^2}{12} & 0\\ 0 & 0 & 0 & \dfrac{\upsilon t^2}{12} & \dfrac{t^2}{12} & 0\\ 0 & 0 & 0 & 0 & 0 & \dfrac{(1-\upsilon)t^2}{24}\end{bmatrix} \tag{6}
$$

One can arrange the sub-stiffness matrix $[\mathbf{K_S}]_{m,n}$ for typical term m and n as given by

$$
[\mathbf{K_S}]_{m,n} = \int_V \begin{bmatrix}\left[\mathbf{B}^{S}\right]_m^{T}[\mathbf{D}]\left[\mathbf{B}^{S}\right]_n & \left[\mathbf{B}^{S}\right]_m^{T}[\mathbf{D}]\left[\mathbf{B}^{U}\right]_n\\ \left[\mathbf{B}^{U}\right]_m^{T}[\mathbf{D}]\left[\mathbf{B}^{S}\right]_n & \left[\mathbf{B}^{U}\right]_m^{T}[\mathbf{D}]\left[\mathbf{B}^{U}\right]_n\end{bmatrix}\mathrm{d}V \tag{7}
$$

Due to the well-known orthogonality property, uncoupled system between axisymmetric and non-axisymmetric can be expected, so that the upper right and lower left sub-matrix in Eq.(7) are completely vanished under any combination of m and n. By the same reason, harmonic term m and n are uncoupled, in the sense that only elements corresponding to $m = n$ do have a value under integrals. Final form of cylindrical shells stiffness matrix for typical harmonic term m and n is given by

$$[\mathbf{K_S}]_{m,n} = \begin{bmatrix} [\mathbf{K_S^{SS}}]_{m,n} & [\mathbf{0}] \\ [\mathbf{0}] & [\mathbf{K_S^{UU}}]_{m,n} \end{bmatrix} \tag{8}$$

For the free vibration analysis, mass matrix of cylindrical shells can be defined by

$$[\mathbf{M_S}] = \rho \int_V [\mathbf{N}]^{\mathrm{T}} [\mathbf{N}] \mathrm{d}V \tag{9}$$

where ρ is mass per unit volume of the shells and $[\mathbf{N}] = \sum_{m=0}^{M} \left[[\mathbf{N^S}]_m \quad [\mathbf{N^U}]_m \right]$.

Sub-mass matrix of cylindrical shell for given term m and n can be written as

$$[\mathbf{M_S}]_{m,n} = \rho \int_V \begin{bmatrix} [\mathbf{N^S}]_m^{\mathrm{T}} [\mathbf{N^S}]_n & [\mathbf{N^S}]_m^{\mathrm{T}} [\mathbf{N^U}]_n \\ [\mathbf{N^U}]_m^{\mathrm{T}} [\mathbf{N^S}]_n & [\mathbf{N^U}]_m^{\mathrm{T}} [\mathbf{N^U}]_n \end{bmatrix} \mathrm{d}V \tag{10}$$

By similar procedure as for stiffness matrix, the mass matrix of the cylindrical shells can be reduced to

$$[\mathbf{M_S}]_{m,n} = \begin{bmatrix} [\mathbf{M_S^{SS}}]_{m,n} & [\mathbf{0}] \\ [\mathbf{0}] & [\mathbf{M_S^{SS}}]_{m,n} \end{bmatrix} \tag{11}$$

in which the integrals of the remaining element in the matrix give value only when $m = n$. Eq.(11) again shows uncoupling axisymmetric and non-axisymmetric system for mass matrix of the shells.

2.2. Foundation

Soil as a foundation can be modeled by axial, circumferential, radial and rotation spring attached to the surface of the shells on a limited arc. Spring stiffness value is assumed to be constant along the limited arc.

In present method, generalization of element in circumferential direction is definitely necessary, but unfortunately the distribution of spring is not uniform over the circumference. Method described here can overcome this problem by defining the foundation distribution function which is valid for any points in circumferential direction and at the same time describing the existence of springs at considered point.

For this purpose, Fourier series is used to define the foundation distribution function along the circumference because of its capability of defining any kind of functions. The foundation distribution function is given by

$$\kappa_\partial(\theta) = \frac{K_\partial}{\pi} \left[a_0 + \sum_{\ell=1}^{\infty} \left\{ a_\ell \cos(\ell\theta) + b_\ell \sin(\ell\theta) \right\} \right] \tag{12}$$

where $a_0 = \dfrac{\varphi_1 + \varphi_2}{2}$, $a_\ell = \dfrac{\sin(\ell\varphi_1) + \sin(\ell\varphi_2)}{\ell}$

$$b_\ell = \frac{\cos(\ell\varphi_1) - \cos(\ell\varphi_2)}{\ell}$$

in which ℓ stands for the Fourier harmonic term in describing the foundation distribution and ∂ (= u, v, w, β). Eq.(12) took place in the $[\mathbf{k_f}]$ matrix as shown in Eq.(13) for corresponding direction.

$$[\mathbf{k_f}] = \begin{bmatrix} \kappa_u(\theta) & 0 & 0 & 0 \\ 0 & \kappa_v(\theta) & 0 & 0 \\ 0 & 0 & \kappa_w(\theta) & 0 \\ 0 & 0 & 0 & \kappa_\beta(\theta) \end{bmatrix} \tag{13}$$

Repeating the similar procedure which is used to obtain Eq.(7) and Eq.(10), the foundation stiffness matrix can be written by

$$[\mathbf{K_F}] = \int_A [\mathbf{N}]^{\mathrm{T}} [\mathbf{k_f}] [\mathbf{N}] \mathrm{d}A \tag{14}$$

Since $[\mathbf{k_f}]$ matrix also contains trigonometric term in ℓ, again more elaborate observation to the nature of integrals has to be performed in order to derive the foundation stiffness matrix. Final form of the matrix for given term m and n can be written as

$$[\mathbf{K_F}]_{m,n} = \begin{bmatrix} [\mathbf{K_F^{SS}}]_{m,n} & [\mathbf{K_F^{SU}}]_{m,n} \\ [\mathbf{K_F^{US}}]_{m,n} & [\mathbf{K_F^{UU}}]_{m,n} \end{bmatrix}$$
$$= \begin{bmatrix} \int_A [\mathbf{N^S}]_m^{\mathrm{T}} [\mathbf{k_f}] [\mathbf{N^S}]_n \mathrm{d}A & \int_A [\mathbf{N^S}]_m^{\mathrm{T}} [\mathbf{k_f}] [\mathbf{N^U}]_n \mathrm{d}A \\ \int_A [\mathbf{N^U}]_m^{\mathrm{T}} [\mathbf{k_f}] [\mathbf{N^S}]_n \mathrm{d}A & \int_A [\mathbf{N^U}]_m^{\mathrm{T}} [\mathbf{k_f}] [\mathbf{N^U}]_n \mathrm{d}A \end{bmatrix} \tag{15}$$

For an axisymmetric problem ($\varphi_1 = \varphi_2$), in this case only considering the structure's axisymmetric, upper right and lower left sub-matrix in Eq.(15) are reduced to zero matrix for any possible combinations of term m, n and ℓ, in other words, the foundation stiffness matrix of axisymmetric and non-axisymmetric are uncoupled each other. This can be verified by assuming symmetric distribution of foundation so that $\varphi_1 = \varphi_2 = \varphi$, then the sine term in Eq.(12) will be completely vanished because $b_\ell = 0$ and observation can be focused on the constant term and cosine term only.

For non-axisymmetric structures ($\varphi_1 \neq \varphi_2$), the whole system of foundation stiffness matrix can be expected as previously given in Eq.(15). As been realized that the constant and cosine term in Eq.(12) play an important role in determining each element in upper left and lower right of foundation stiffness matrix, the sine term in the same equation gives contribution in determining the value of each element in upper right and lower left matrix of foundation stiffness matrix. The coupled system is taken part for a non-axisymmetric structure.

Different from stiffness matrix of shell, foundation stiffness matrix forms couple system either between deformation system or between harmonic term m and n. Only for symmetric structure ($\varphi_1 = \varphi_2$), the foundation stiffness matrix is reduced to uncoupled system, but still shows the coupling phenomenon of harmonic terms.

In the calculation of Eq.(12), only a finite number of terms say NL, in the truncated series are taken into account. $NL = 20$, $M = N = 10$ and total number of element $NS = 40$ are used for numerical analysis in the next sections, based on the convergence studies which are not shown here. Analyses were carried out by consideration to radial spring stiffness only ($K_u = 0$, $K_v = 0$, $K_w \neq 0$, $K_\beta = 0$).

3. Static Analysis

Numerical examples for static analysis were carried out for gravity load case as shown in Fig.2. In all the calculations, the following parameters were used: $t/L = 0.001$, $\upsilon = 0.30$, $\varphi_1 = \varphi_2 = \varphi = 60°$, and $q^g/E = 5 \times 10^{-8}$, where q^g is gravity load per unit area.

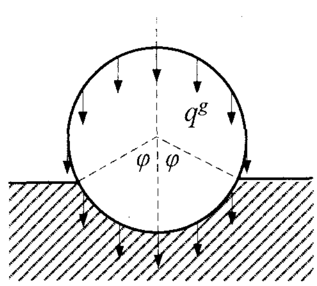


Fig.2. Gravity load cases

Fig.3 and Fig.4 show the overall deformation of shell for both clamped and simply supported edges with $R/L = 0.05$.

The effect of end conditions to the displacement on the top edge of the shell can be seen in Fig.5, which is plotted against ratio of L/R. For the cylindrical shell with small value of L/R, the effect of end conditions can not be neglected.

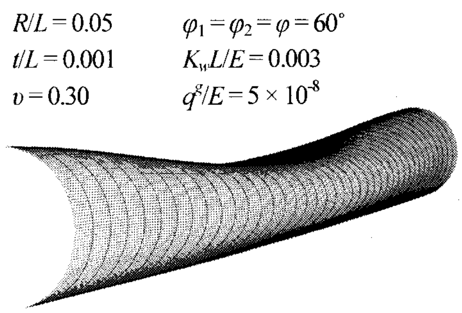


Fig.3. Clamped cylindrical shell on elastic foundation

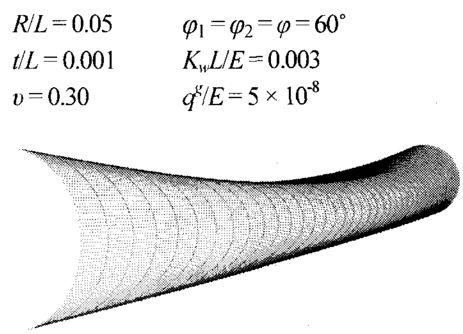


Fig.4. Simply supported cylindrical shell on elastic foundation

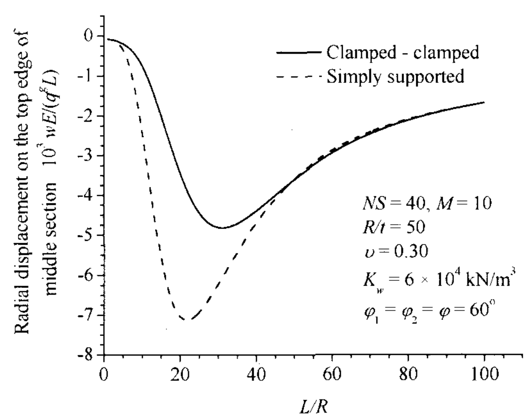


Fig.5. Effect of end conditions

The radial displacement on bottom, side, top and circumferential displacement on side edge at middle section are plotted against dimensionless value R/L with different foundation stiffness (only considering radial spring) for both clamped end conditions in Fig.6 to Fig.9. For cylindrical shells with small value of R/L, shell is penetrated into foundation without much deformation occurred in the shell as shown in Fig.6 and this result is supported by Fig.7 and Fig.9.

Radial displacement on top edge shows relatively fluctuated curve which is caused by alteration in sectional mode of deformation as can be seen in Fig.8.

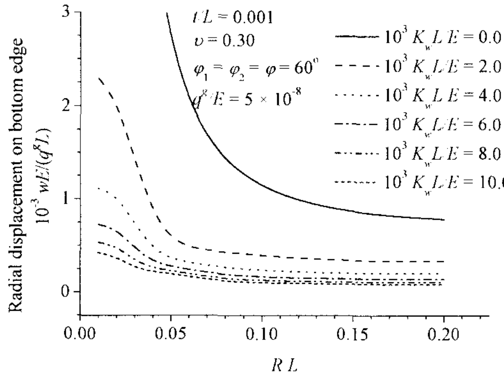


Fig.6. Variation of radial displacement on bottom edge with R/L for different K_wL/E

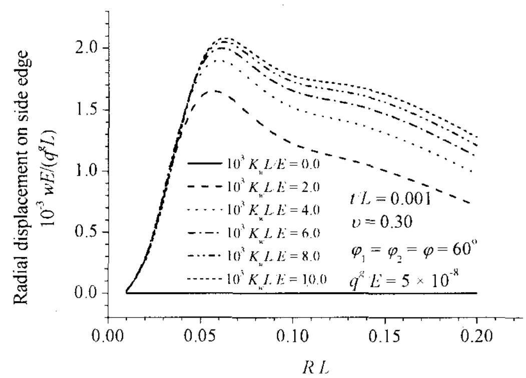


Fig.7. Variation of radial displacement on side edge with R/L for different K_wL/E

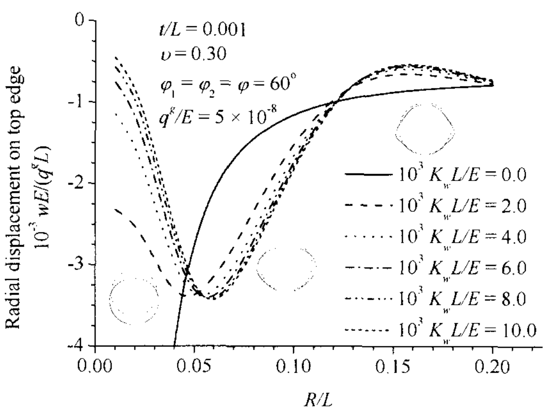


Fig.8. Variation of radial displacement on top edge with R/L for different K_wL/E

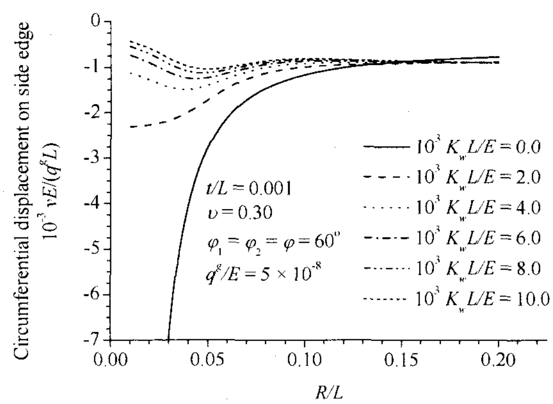


Fig.9. Variation of circumferential displacement on side edge with R/L for different K_wL/E

4. Free Vibration Analysis

For free vibration analysis, eigensystem can be written as follows:

$$\sum_{i=1}^{NS}\sum_{n=0}^{N}\left[\begin{bmatrix}\left[\mathbf{K}_S^{SS}\right]_{m,n}^{i} & [0] \\ [0] & \left[\mathbf{K}_S^{UU}\right]_{m,n}^{i}\end{bmatrix}+\begin{bmatrix}\left[\mathbf{K}_F^{SS}\right]_{m,n}^{i} & \left[\mathbf{K}_F^{SU}\right]_{m,n}^{i} \\ \left[\mathbf{K}_F^{US}\right]_{m,n}^{i} & \left[\mathbf{K}_F^{UU}\right]_{m,n}^{i}\end{bmatrix}\right]\begin{Bmatrix}\left\{\delta e^{S}\right\}_n \\ \left\{\delta e^{U}\right\}_n\end{Bmatrix}=$$

$$\omega^2\sum_{i=1}^{NS}\sum_{n=0}^{N}\begin{bmatrix}\left[\mathbf{M}_S^{SS}\right]_{m,n}^{i} & [0] \\ [0] & \left[\mathbf{M}_S^{UU}\right]_{m,n}^{i}\end{bmatrix}\begin{Bmatrix}\left\{\delta e^{S}\right\}_n \\ \left\{\delta e^{U}\right\}_n\end{Bmatrix},\text{for } m=0,1,2,\dots N \qquad (16)$$

where ω is the natural frequency.

Proceeding section covers the axisymmetric vibration of cylindrical shells on elastic foundation. The symmetric distribution of foundation is able to separate the whole system in Eq.(16), and then an axisymmetric system can be reduced to

$$\sum_{i=1}^{NS}\sum_{n=0}^{N}\left[\left[\mathbf{K}_S^{SS}\right]_{m,n}^{i}+\left[\mathbf{K}_F^{SS}\right]_{m,n}^{i}\right]\{\delta_e\}_n=\omega^2\sum_{i=1}^{NS}\sum_{n=0}^{N}\left[\mathbf{M}_S^{SS}\right]_{m,n}^{i}\{\delta_e\}_n$$

$$\text{, for } m=0,1,2,\dots N \qquad (17)$$

The above two equations can be solved for the natural frequencies and modes by any available algorithms. The resulting vector is corresponding to the amplitude of displacements at specified node for typical harmonic term.

Natural frequency, ω for shell with different thickness parameters shows some fluctuations as can be seen in Fig.10 to Fig.14 for the first five modes.

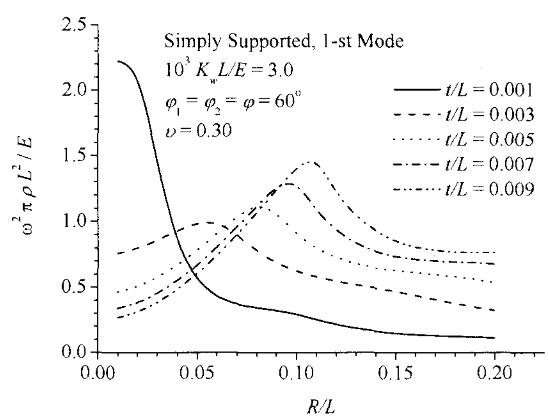


Fig.10. Variation of first natural frequency with R/L for different t/L

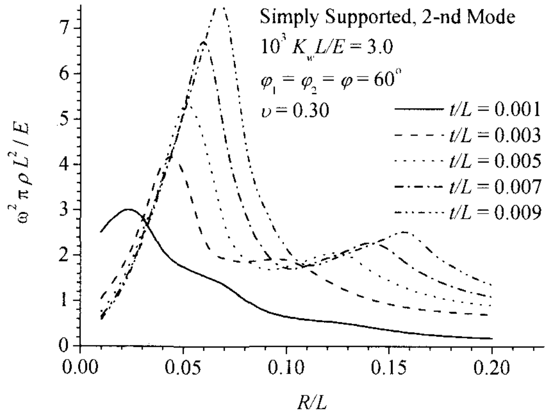


Fig.11. Variation of second natural frequency with *R/L* for different *t/L*

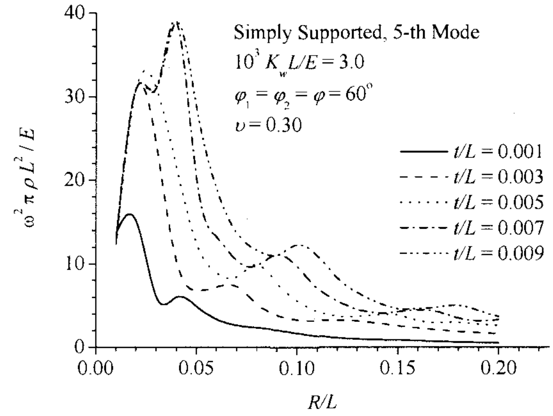


Fig.14. Variation of fifth natural frequency with *R/L* for different *t/L*

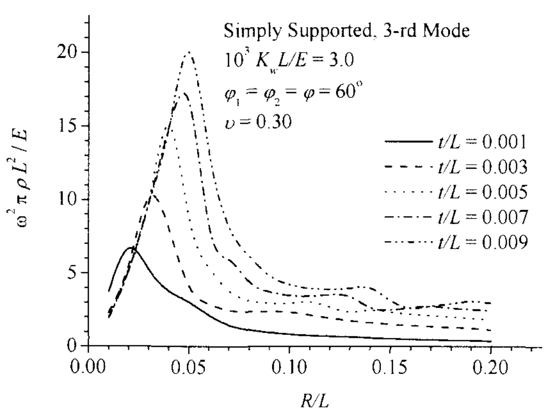


Fig.12. Variation of third natural frequency with *R/L* for different *t/L*

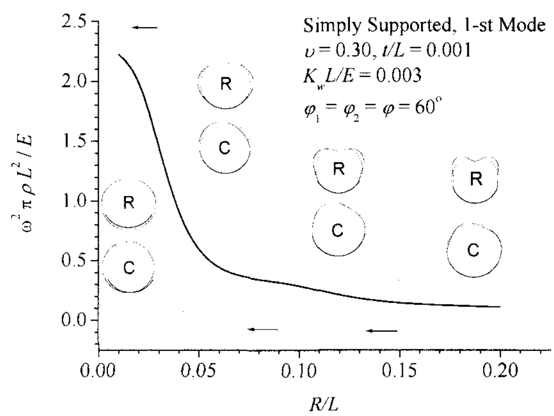


Fig.15. Changes in sectional mode shape for first frequency ($t/L = 0.001$)

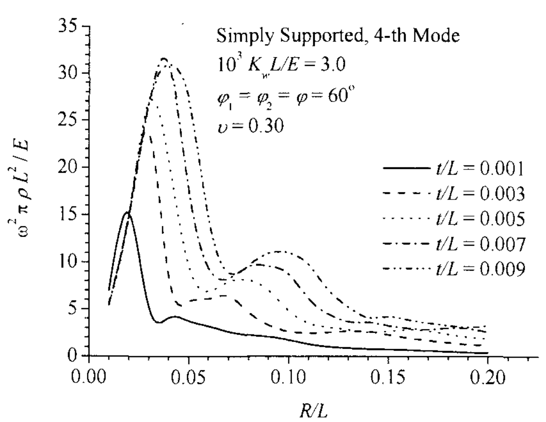


Fig.13. Variation of fourth natural frequency with *R/L* for different *t/L*

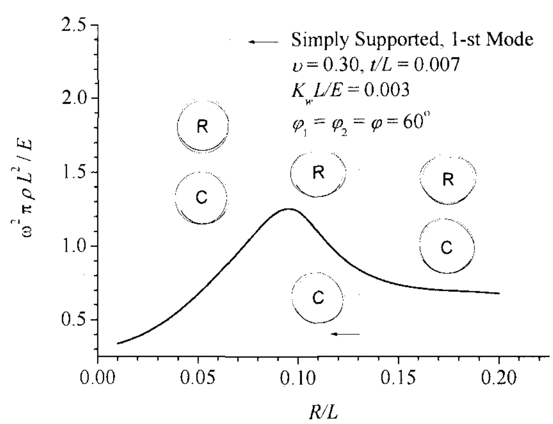


Fig.16. Changes in sectional mode shape for first frequency ($t/L = 0.007$)

These fluctuations are caused by changes in significant sectional mode shapes as can be seen in Fig.15 for $t/L = 0.001$ and first frequency. In the figures, sectional modes with notation R and C show the radial and circumferential displacements of the middle section. Only magnitudes of displacement are plotted in sub-figure for convenient, without any physical meaning especially for circumferential displacement. This argument holds for different thickness parameter (Fig.16) and also true for higher modes, as given in Fig.17 for second frequency.

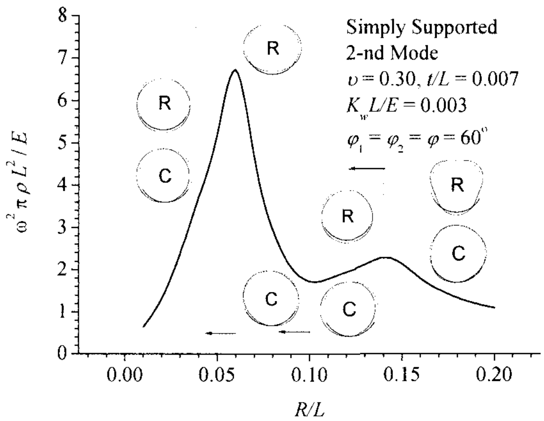


Fig.17. Changes in sectional mode shape for second frequency ($t/L = 0.007$)

For the first five frequencies, overall mode shapes of simply supported cylindrical shell on elastic foundation are given in Fig.18 for $R/L = 0.05$, $t/L = 0.001$, $\upsilon = 0.30$, $K_wL/E = 0.003$ and $\varphi_1 = \varphi_2 = \varphi = 60^\circ$. Present method is applicable to observe transversal and sectional vibration modes simultaneously.

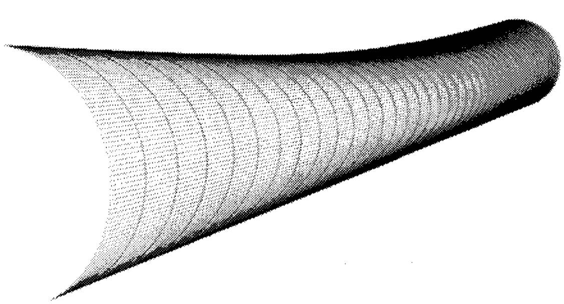


Fig.18(a). First axisymmetric vibration mode ($\omega^2\pi\rho L^2/E = 0.573$)

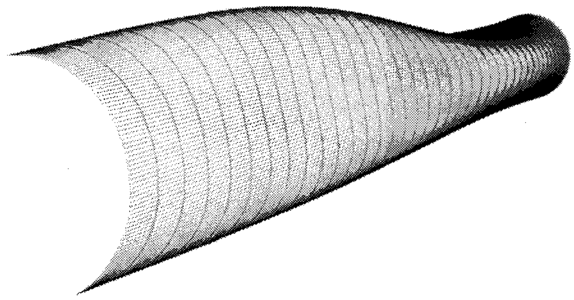


Fig.18(b). Second axisymmetric vibration mode ($\omega^2\pi\rho L^2/E = 1.754$)

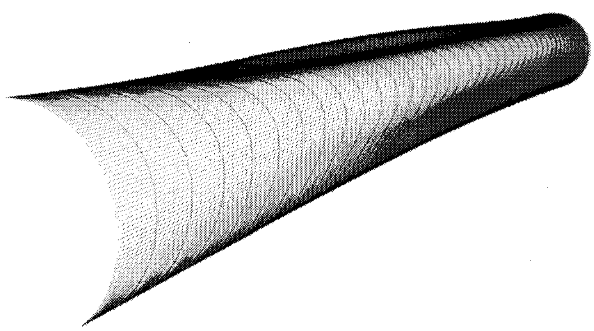


Fig.18(c). Third axisymmetric vibration mode ($\omega^2\pi\rho L^2/E = 3.038$)

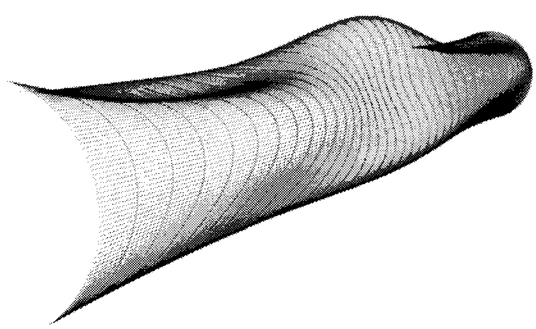


Fig.18(d). Fourth axisymmetric vibration mode ($\omega^2\pi\rho L^2/E = 3.791$)

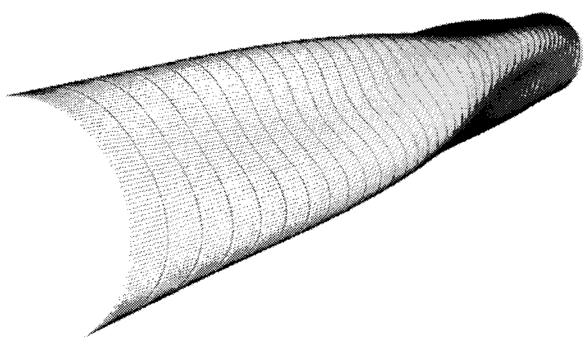


Fig.18(e). Fifth axisymmetric vibration mode ($\omega^2 \pi \rho L^2 / E = 4.865$)

Variation in the first and second natural frequencies with foundation parameters such as K_w and φ (= $\varphi_1 = \varphi_2$) are shown in Fig.19 and Fig.20 for given parameter of $R/t = 200$ and $R/t = 10$, respectively. For cylindrical shells with large value of R/t, linear distribution of ω^2 can be expected at small value of $K_w L/E$ but in the same curve, for relatively larger value of $K_w L/E$, changes in $K_w L/E$ value are become less sensitive to the square of natural frequency as shown in Fig.19 for first and second frequency. In contrary, cylindrical shells with small value of R/t show linear distribution of square natural frequencies which can be seen within the considered range of value $K_w L/E$ as shown in Fig.20. For vibration system under consideration, relationship between natural frequency and relative stiffness can be written as

$$\omega^2 = \frac{K_S^* + K_F^*}{M_S^*} = \frac{K_S^*}{M_S^*}(1+\alpha) \tag{19}$$

The numerator in Eq.(19) consists of generalized stiffness of shell (K_S^*) and generalized stiffness of foundation (K_F^*), while the denominator is generalized mass of shell (M_S^*). α appearing in Eq.(19) is the relative stiffness ratio ($\alpha = K_F^* / K_S^*$).

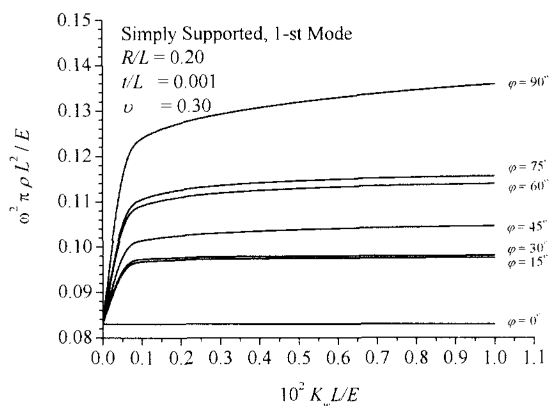


Fig.19(a). Variation of first natural frequency with foundation parameters ($R/t = 200$)

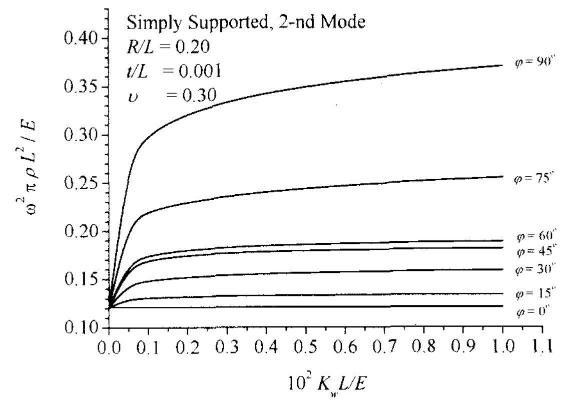


Fig.19(b). Variation of second natural frequency with foundation parameters ($R/t = 200$)

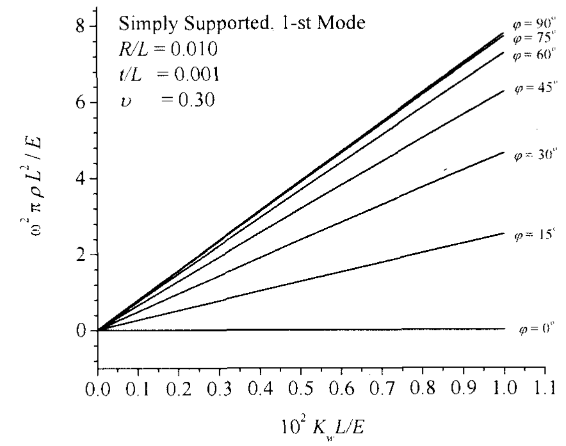


Fig.20(a). Variation of first natural frequency with foundation parameters for ($R/t = 10$)

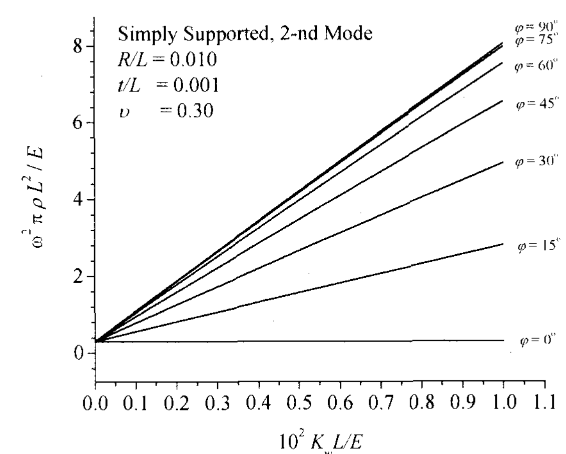


Fig.20(b). Variation of second natural frequency with foundation parameters ($R/t = 10$)

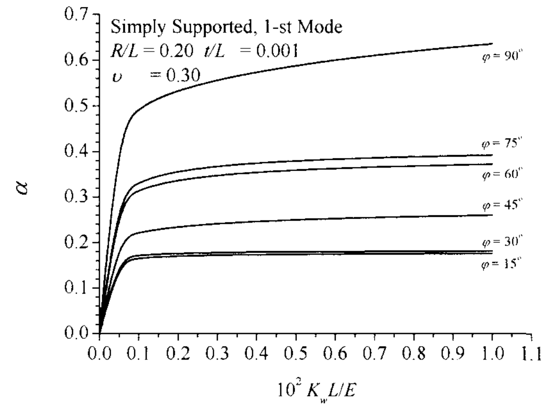


Fig.21. $\alpha - K_w$ relationship for first frequency ($R/t = 200$)

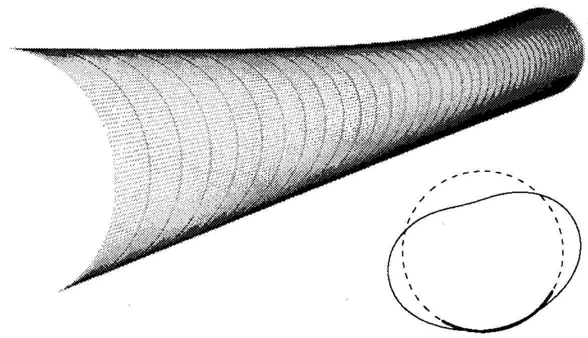


Fig. 23(b). Second non-axisymmetric mode
($\omega^2 \pi \rho L^2 / E = 0.437$)

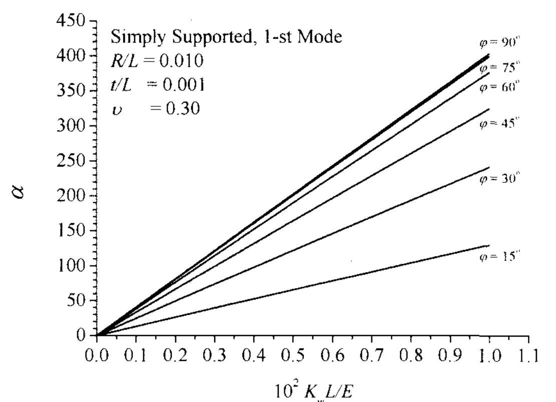


Fig.22. $\alpha - K_w$ relationship for first frequency ($R/t = 10$)

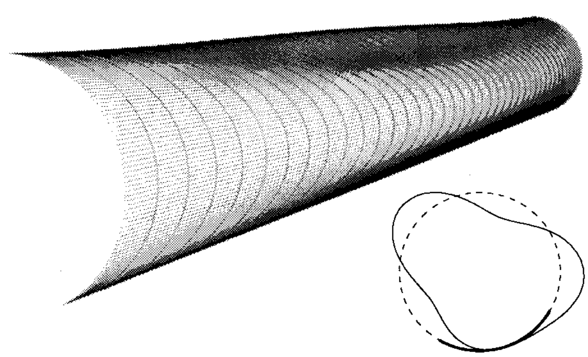


Fig.23(c). Third non-axisymmetric mode
($\omega^2 \pi \rho L^2 / E = 1.224$)

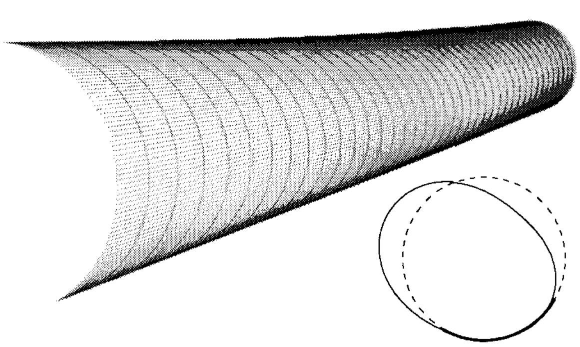


Fig.23(a). First non-axisymmetric mode
($\omega^2 \pi \rho L^2 / E = 0.366$)

The relationship between α and K_w is similar to that one between ω^2 and K_w. For the first mode, the relationship between relative stiffness (α) and K_w for different φ is given in Fig.21 and Fig.22. $\alpha - K_w$ relationship can be expressed by a single straight line for cylindrical shells with small value of R/t (Fig.22), while for large value of R/t, this relationship can be approximated by a piecewise linear distribution.

Finally, to show the applicability of present method to assess the non-axisymmetric vibrations, simply supported cylindrical shell with $R/L = 0.05$, $t/L = 0.001$, $\upsilon = 0.30$, $K_w L/E = 0.003$, $\varphi_1 = 30^\circ$ and $\varphi_2 = 60^\circ$ is analyzed. Overall mode shapes of the first three frequencies with their middle sectional deformation are given in Fig.23 in which, thick line on the periphery of the shells as shown in each sub-figure refers to the part of the shell which is covered by foundation.

5. Conclusions

The present method has been shown to be useful for the static and free vibration analysis of cylindrical shells partially buried in elastic foundation. Numerical studies indicate the advantage of proposed method. Boundary conditions can be applied in the calculation in straightforward manner. By using proposed method, the fluctuation of the ground surface in the longitudinal direction can be modeled by simple element meshing strategy.

The principal conclusions of this study, especially for free vibration analysis are summarized as follows:

(1) For fixed value of t/L, parameter R/L gives significant influence to the variation of natural frequency.

(2) Changes in sectional mode shape can be expected for increasing value of R/L.

(3) For cylindrical shells with large value of R/t, $\omega^2 - K_w$ relationship can be expressed by a piecewise linear distribution.

(4) The relationship between ω^2 and K_w can be expressed by a single straight line for cylindrical shells with small value of R/t.

The existence of inner and/or outer fluid in some practical applications, especially in the pipelining field, leads to the interactive problem between fluids and structures which had been neglected in this basic study. The coupled fluid-structure analysis will be the main task for further studies.

References

1) Seide, P., *Small Elastic Deformation of Thin Shells*, Aspen Publishers Inc, 1975.
2) Yang, R., Kameda, H. and Takada, S., Shell Model FEM Analysis of Buried Pipelines under Seismic Loading., Bull. Disas. Prev. Res. Inst., Kyoto University, 38(3), pp.115-146, 1988.
3) Paliwal, D.N., Pandey, R.K. and Nath, T., Free Vibrations of Circular Cylindrical Shell on Winkler and Pasternak Foundation., Int. J. Pressure Vessels and Piping, 69(1), pp.79-89, 1996.
4) Paliwal, D.N., Kanagasabapathy, H. and Gupta, K.M., The Large Deflection of an orthotropic Cylindrical Shell on a Pasternak Foundation, Comp. and Struc., 31(1), pp.31-37, 1995.
5) Amabili, M. and Dalpiaz G., Free Vibration of Cylindrical Shells with Non-Axisymmetric Mass Distribution on Elastic Bed, Meccanica, 32, pp.71-84, 1997.
6) Amabili, M. and Garziera, R., Vibrations of Circular Cylindrical Shells with Non-uniform Constraints, Elastic Bed and Added Mass : Part I : Empty and Fluid-Filled Shells, J. of Fluids and Structures, 14, pp.669-690, 2000.
7) Timoshenko, S.P. and Krieger, S.W., *Theory of Plates and Shells*, McGraw-Hill Kogakusha, Tokyo, Japan, 1959.

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