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Vibration Simulation of a Rail-Sleeper-Ballast System on Layered-Ground under Train Loads

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A computation approach has been developed to investigate the dynamics of a rail-sleeper-ballast-ground system under moving train loads. The interacting train and track are both modelled as the dynamic systems. The total system is divided into two substructures for the analysis: The track system and the ground system. The dynamics formulations are solved using the Fourier transform. The present solution is differed from the total system analysis, which has been developed previously by assuming a continuous interface all along the track direction. Illustrative example case studies are presented, based on the available track components properties and ground profile.

Key Words: Ground motion, rail Vibration, substructure Method, Fourier transform, layered ground, computer simulation

1. Introduction

In high-speed railway transportation, in order to guarantee the safety of train operation and to reduce train induced nearby ground vibration, it is important to predict the track vibrations in view of the critical speed that are characterized by resonance velocities at which the surface wave energy accumulates under the train wheels either in rails or in supporting ground. Most of the related works are theoretically based on the continuous contact assumption between wheels and rails and ground. Even simplification of these elelemets gives first engineering insight into the related mechanics. But it should be further elaborated for thorough understanding of the dynamics of track and ground system.

Train track dynamics have been dealt with practically by a beam model on the Winkler foundation. In the elaborate theory, however the ground has been idealized either by a half space[1)] or a layered system[2),3)] or a stack of layers[4), 5), 6), 7), 8)] for the wave propagation in subgrade subsoil media. In case of a half space assumption, the wave field is predominantly governed by the Raleigh wave, so that when the moving load speed approaches its Raleigh wave velocity, an extraordinary response occurs. In case of a layer/layers assumption, on the other hand, the dispersive nature appears and the wave field is governed by the modal waves[2)]. The modal waves can be characterized by different wave speeds for different frequencies. The energy transmission is carried out by the modes that are most concerned with the situation. There exists a situation in which higher modes come into a major contribution and the response feature changes drastically from that for a half space.

In the previous works, the wave number-frequency solution method has been utilized. For a moving load of a constant speed c, the wave number in the moving direction is related to the frequency decisively by $\xi_x = \omega / c$. For a system with discrete sleeper supports, the forces from sleepers act spatially in discrete on the ground according to those positions[9)]. Therefore, the solution method should better be developed in the space-frequency domain in order to take into account of the coupled motions through sleepers.

In what follows, the author presents the substructure formulation for a rail-sleeper-ballast-ground system. In the track model, the rail is treated as a Euler beam discretely supported, via rail pads, by rigid sleepers. Below each sleeper, layers of soil including embankment are accounted for to model the ballast and the ground. The mass, stiffness, damping values and the sleeper spacing of the track system can be arbitrarily varied, so that the properties of track components and ground profiles can be taken into account. This solution may be differed from the previously developed total system analysis by the first author and others by assuming a continuous interface all along the track direction.

2. Track Modeling and Formulation

In order to facilitate the analysis of train tracks of discrete supports by sleepers, we apply the substructure method by dividing the whole system into a Euler beam analysis for rails and the elastodynamics analysis for the supporting ground. **Fig.1** illustrates the model.

2.1 Train Loading

Consider a moving train comprising N numbers of cars. The successive axle loading due to the train passage along the x-axis with a constant velocity c is described by

$$f_N(x-ct)=\sum_{n=0}^{N-1} f_n(x-ct-\sum_{e=0}^{n} L_{e,t}) \qquad (1)$$

in which $f_n(x-ct-\sum_{e=0}^{n} L_{e,t})$ indicates a single n-th car contribution by the mass movement whose detail expression is

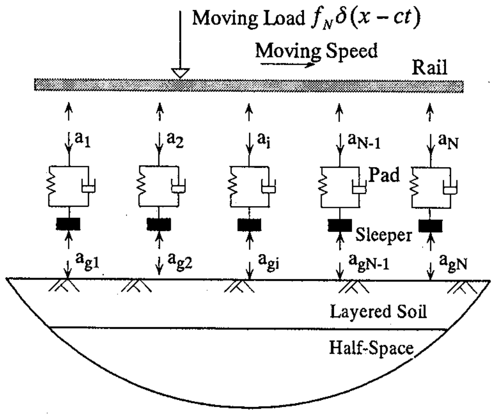

Fig. 1 The Compound Track/Ground Model

$$\begin{aligned} f_n(x-ct,y,z,t) = \\ & f_{n1}\delta(x-ct+L_0)+f_{n1}\delta(x-ct+A_n+L_0) \\ & +f_{n2}\delta(x-ct+A_n+B_n+L_0) \\ & +f_{n2}\delta(x-ct+A_n+B_n+L_0) \end{aligned} \qquad (2)$$

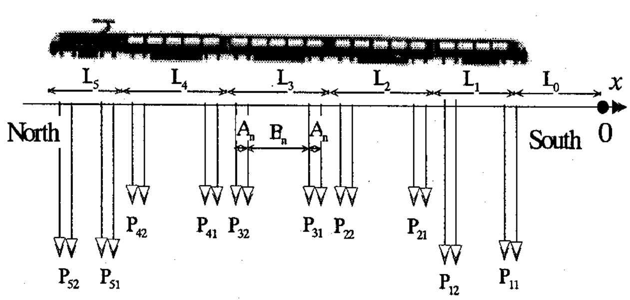

Fig. 2 Profile of train wheel loads

with f_{n1} and f_{n2} defining respectively the axle loads from the front and rear bogies, $L_{e,t}$ is the respective car length and L_0 is the reference position ahead of the first axle load position. A_n, B_n are the distances between axles (see **Fig.2**). The $\delta(\bullet)$ denotes the Dirac's delta function for an impulse loading. The frequency domain representation after the Fourier Transform is then

$$f^{xt}_{N_t}(\xi_x,\omega)=\frac{2\pi}{c}\delta(\xi_x-\frac{\omega}{c})f_x(\xi_x) \qquad (3)$$

where

$$f_x(\xi_x)= \sum_{n=0}^{N_t-1}\{f_{n1}(1+e^{-iA_n\xi_x})+f_{n2}(e^{-i(A_n+B_n)\xi_x}+e^{-i(2A_n+B_n)\xi_x})\} \qquad (4)$$

2.2 Track modeling

A pair of rails is modeled by a Euler beam (see **Fig.1**). The governing equation for a moving load $f(t)$ therefore can be expressed by

$$EI\frac{\partial^4 u}{\partial x^4}+m\frac{\partial^2 u}{\partial t^2}+C_d\frac{\partial u}{\partial t}=f(t)\delta(x-ct)+\sum_{m=1}^{N}a_m(t)\delta(x-x_m) \qquad (5)$$

where EI denotes the bending rigidity of a pair of rails, m for the mass per unit length, and C_d for the material damping factor. The second term defines the reactions from the ground. N is the number of the sleeper supports involved. is the moving load on the rails, a_m is the reaction forces at sleeper supports, x_m (m=1, 2, ... N) is the location of sleepers along the track.

Applying the Fourier Transform to Eq.(5) with respect to the coordination x and time t, we get

$$(EI\xi^4-\omega^2 m+i\omega C_d)u^{xt}(\xi,\omega)=f^t(\xi c-\omega)+\sum_{m=1}^{N}a^t_m(\omega)e^{i\xi x_m} \qquad (6)$$

When Eq.(6) is solved for $u^{xt}(\xi,\omega)$

$$u^{xt}(\xi,\omega)=G^{xt}(\xi,\omega)[f^t(\xi c-\omega)+\sum_{m=1}^{N}a^t_m(\omega)e^{i\xi x_m}] \qquad (7)$$

where $$G^{xt}(\xi,\omega)=\frac{1}{(EI\xi^4-\omega^2 m+i\omega C_d)} \qquad (8)$$

The superscript 't' indicates the value in the Fourier transform domain with respect to time and the superscript 'x' the corresponding value with respect to space coordinate x.

First we get the Green function in the transformed domain of the Euler beam from Eq.(7) for unit moving load $f_0\delta(x-ct)$. To simplify the formulation of Green function, the internal damping effect of rails has been included into elasticity modulus E^*. The inverse transformation back into the space domain is performed by applying the counter integration with the residue theory. The explicit form is derived as

$$\begin{aligned} G^t(x,\omega) &= \int_{-\infty}^{\infty} G^{xt}(\xi,\omega)e^{-i\xi x}d\xi \\ &= \frac{1}{4E^*I\Omega^3}\left[-e^{-\Omega|x|}+ie^{-i\Omega|x|}\right] \end{aligned} \qquad (9)$$

where $\Omega=(m\omega^2/E^*I)^{\frac{1}{4}}$, $E^*=E(1+i2\zeta)$ with ζ being the damping ratio of rails.

With use of the Green function of Eq.(9), the solution for the moving load and the support reactions are formulated by the convolution integral form. Then the response due to axle loads with a given moving speed c is obtained straightforwardly from Eq.(7). Taking the inverse Fourier transform of Eq.(7) with respect to the wavenumber

$$u^t(x,\omega)=F(x,\omega)-\sum_{m=1}^{N}a_m(\omega)G^t(x-x_m,\omega) \qquad (10)$$

where

$$F(x,\omega)=\frac{f_0}{4EIc\Omega^3}\times\begin{cases}F1(x_1,\omega)-F1(x_0,\omega) & x\le x_{-\infty}\\ F2(x,\omega)-F2(x_0,\omega) & x_{-\infty}\le x\le x_{+\infty}\\ \quad +F1(x_1,\omega)-F1(x,\omega) & \end{cases}\tag{11}$$

where $x_{-\infty}$ and $x_{+\infty}$, the distance range of the train moves, are introduced for the sake of the numerical computation to limit the negative and positive infinities in x, and

$$F1(x,\omega)=\frac{e^{(x-y)\Omega-i\frac{\omega y}{c}}}{-\Omega-i\frac{\omega}{c}}+\frac{e^{i(x\Omega-y\Omega-\frac{\omega}{c}y)}}{-\Omega-\frac{\omega}{c}}\tag{12}$$

$$F2(x,\omega)=\frac{e^{(-x+y)\Omega-i\frac{\omega y}{c}}}{\Omega-i\frac{\omega}{c}}+\frac{e^{i(-x\Omega+y\Omega-\frac{\omega}{c}y)}}{\Omega-\frac{\omega}{c}}\tag{13}$$

For the discretely supported rails, the displacement at the sleepers' positions of Eq.(10) is extended into

$$u^t(x_i,\omega)=F(x_i,\omega)-\sum_{m=1}^{N}a_m(\omega)G^t(x_i-x_m,\omega)\tag{14}$$

The matrix $G^t(x-x_m,\omega)$ defines the Green function matrix, and the force vector $F(x_i,\omega)$ is obtained for the train axle loads in Eq.(4). The Eq.(14) is now given in a matrix form

$$U=GA+F\tag{15}$$

where, G is the matrix of $G^t(x_l-x_m,\omega)$, and

$$U=[u^t(x_1,\omega),u^t(x_2,\omega)\ \dots\ u^t(x_N,\omega)]^T\tag{16}$$

$$F=[F(x_1,\omega),F(x_2,\omega),\dots,F(x_N,\omega)]\tag{17}$$

$$A=[a_1(\omega),a_1(\omega),\dots a_N(\omega)]\tag{18}$$

The response due to the ground reactions through sleepers is obtained after solving the interaction system with the ground as in what follows.

3. Ground Modeling

In order to make the formulation easy to understand, first the formulation is introduced with Winkler spring, then the layered effect of ground are implemented into the formulation.

3.1 Elastic Winkler foundation

Generally, the ground is considered as the Winkler foundation, and we can get the support stiffness at the location of sleepers as the equivalent value for a sleeper span area. Then the equivalent ground stiffness becomes a constant, $K_{const}=\alpha d w$, where α is the Winkler spring coefficient, d is the distance of adjacent sleepers along the track direction, w is the width of track. Then, the ground deformations at sleeper's positions are provided by

$$u^t_{ground}(x_i,\omega)=a_i/K_{const}\quad i=1,2,\dots N\tag{19}$$

Now the displacement compatibility is applied for all sleepers in consideration.

$$u^t(x_i,\omega)=F(x_i,\omega)-\sum_{m=1}^{N}a_m(\omega)G^t(x_i-x_m,\omega)=a_i/K_{const}\quad i=1,2,\dots N\tag{20}$$

After rearranging, Eq.(20) is rewritten as

$$\begin{bmatrix}G(x_1-x_1,\omega)+\frac{1}{K_{const}}\cdots & G(x_1-x_m,\omega) & \cdots & G(x_1-x_N,\omega)\\ \vdots\ \ddots & \vdots & & \\ G(x_m-x_1,\omega)\cdots & G(x_m-x_m,\omega)+\frac{1}{K_{const}}\cdots & & G(x_m-x_N,\omega)\\ \vdots\ \ddots & \vdots & & \\ G(x_N-x_1,\omega)\cdots & G(x_N-x_m,\omega)\cdots & & G(x_N-x_N,\omega)+\frac{1}{K_{const}}\end{bmatrix}\times\begin{Bmatrix}a_1^t(\omega)\\ \vdots\\ a_m^t(\omega)\\ \vdots\\ a_N^t(\omega)\end{Bmatrix}=\begin{Bmatrix}F(x_1,\omega)\\ \vdots\\ F(x_m,\omega)\\ \vdots\\ F(x_N,\omega)\end{Bmatrix}\tag{21}$$

Using the Gauss linear elimination method, from the equations above, we can solve for the sleeper reaction: $a_i\ (i=1,2,\dots N)$ in frequency domain.

$$u(x,\omega)=F(x,\omega)-\sum_{m=1}^{N}a_m(\omega)G(x-x_m,\omega)\quad (m=1,2,\dots,N)\tag{22}$$

To get the solution in time domain, we applying Fourier Transform method with respect to frequency, for which the FFT algorithm is actually used.

3.2 Ground Modeling by Layered Soils

The ground is modelled by layered soil with individual properties. The values within each layer in this paper, see **Fig.3**.

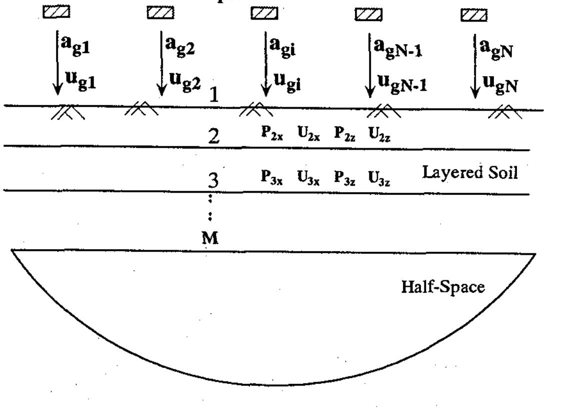

Fig. 3 Layered Ground Model

The formulation starts with the Navior's equations. The original three-dimensional wave field is first decoupled into the P-SV wave field and the SH wave field. These wave fields are formulated by the layer stiffness or flexibility technique through the concerned layer propagator matrix.[4),10)] The nodes are placed at the interface of layers. The total layer stiffness matrices are associated with the degrees of freedom at those appropriate layer matrix order. They are assembled and transformed into the original space coordinates as,

$$P_{layers}(\xi,\omega) = K_{layers}(\xi,\omega)U_{layers}(\xi,\omega) \tag{23}$$

The global stiffness matrix of ground is assembled from the individual layer stiffness.

$$K_{layer}(\xi,\omega) = \begin{bmatrix} K^1_{11} & K^1_{12} & & & & \\ K^1_{21} & K^1_{22}+K^2_{11} & K^2_{12} & & & \\ & K^2_{12} & K^2_{12}+K^3_{11} & \cdots & & \\ & & \vdots & \ddots & & \\ & & & & K^M_{22}+K^M_{11} & K^M_{12} \\ & & & & K^M_{21} & K^M_{22}+K^{half} \end{bmatrix} \tag{24}$$

We denote u^{xt}_{ix} , u^{xt}_{iz} for the displacement of x direction(horizontal) and z direction(vertical) at the ith layer surface wavenumber and frequency domain, p^{xt}_{ix} , p^{xt}_{iz} for the applied load at the ith layer surface, and U^{xt}_{layer}, P^{xt}_{layer} for the displacement vector and load vector of each soil layer whose definitions are

$$U^{xt}_{layer} = [u^{xt}_{1x}, u^{xt}_{1z}, u^{xt}_{2x}, u^{xt}_{2z} \ \dots \ u^{xt}_{Mx}, u^{xt}_{Mz}]^T \tag{25}$$

$$P^{xt}_{layer} = [p^{xt}_{1x}, p^{xt}_{1z}, p^{xt}_{2x}, p^{xt}_{2z} \ \dots \ p^{xt}_{Mx}, p^{xt}_{Mz}]^T \tag{26}$$

In this paper, we are only interested in the ground vertical (z direction) displacement u_{1z} . We get the displacement solution from Eq(23).

$$U^{xt}_{layer} = K^{-1}_{layer}(\xi,\omega)P^{xt}_{layer} \tag{27}$$

For computing the displacement on the location of sleepers $x_i(i=1,2,\dots,N)$, the forces in frequency domain can be specified as:

$$P^{xt}_{layer} = [0, \ \sum_{1}^{N} a_i e^{jkx_i}, 0,\dots, 0]^T \tag{28}$$

Specially the vertical displacement of ground surface, u^{xt}_{1z} :

$$u^{xt}_{1z} = K^{-1}_{layer22}(\xi,\omega)p_{1z} = K^{-1}_{layer22}(\xi,\omega)\sum_{1}^{N} a_i e^{jkx_i} \tag{29}$$

This solution in terms of wavenumber and frequency is transformed into the frequency domain solution. We therefore apply the discrete Fourier Transform with respect to the wavenumber k .

$$u^t_{zx} = \int_{-\infty}^{\infty} K^{-1}_{layer}(\xi,\omega)\sum_{1}^{N} a_{gi} e^{jk(x_i-x)}dk \tag{30}$$

The compatibility of the displacement of the rails and ground is given by

$$u^t_{z,x_m} = u^t(x_m,\omega) \qquad m=1,2,\dots,N \tag{31}$$

so that we can get another relationship of u_i and a_i

$$\begin{aligned} u_{x_m} &= \frac{1}{2\pi}\int_{-\infty}^{\infty} K^{-1}_{layer}(\xi,\omega)\sum_{i=1}^{N} a_{gi} e^{jk(x_i-x_m)}dk \\ &= \sum_{i=1}^{N} a_{gi}\int_{-\infty}^{\infty} K^{-1}_{layer}(\xi,\omega)e^{jk(x_i-x_m)}dk \end{aligned}$$

$$m = 1,2,\dots,N \tag{32}$$

After applying the discrete inverse Fourier transform, we can get

$$u_{x_m} = \sum_{i=1}^{N} \frac{1}{N}\left(\sum_{l=1}^{N} K^{-1}_{layer}(k_l)e^{jk_l(x_i-x_m)}\right)a_{gi} \tag{33}$$

in which, k_l $(l=1,2,\dots,N)$ is the discrete wave number. If we denote

$$H_{gmi} = \frac{1}{N}\sum_{l=1}^{N} K^{-1}_{layer}(k_l)e^{jk_l(x_i-x_m)} \tag{34}$$

then Eq(33) is expressed as

$$u_{x_m} = \sum_{i=1}^{N} H_{gmi}a_{gi}, \quad m=1,2,\dots,N \tag{35}$$

So, It can be assembled into a matrix form

$$\begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_j \\ \vdots \\ u_N \end{bmatrix} = \begin{bmatrix} H_{g11} & H_{g12} & \cdots & & H_{g1N} \\ H_{g21} & H_{g22} & & & \\ \vdots & \vdots & & & \\ & & & H_{gij} & \\ & & & & \\ H_{gN1} & & & & H_{gNN} \end{bmatrix} \begin{bmatrix} a_{g1} \\ a_{g2} \\ \vdots \\ a_{gi} \\ \vdots \\ a_{gN} \end{bmatrix} \tag{36}$$

For the convenience for expression, the sleepers are assumed to be placed in equi-distance, d .

$$k_l = \frac{l-1}{Nd}, \ x_i = (i-1)d, \ x_l = (l-1)x \tag{37}$$

The computation of H_{gmi} is then

$$\begin{aligned} H_{gmi} &= \frac{1}{N}\sum_{l=1}^{N} K^{-1}_{layer}(k_l)e^{jk_l(x_i-x_m)} \\ &= \frac{1}{N}\sum_{l=1}^{N} K^{-1}_{layer22}\left(\frac{l-1}{Nd}\right)e^{j(l-1)(i-m)/N} \end{aligned} \tag{38}$$

We abbreviate the preceding matrix as

$$U_g = H_g A_g \tag{39}$$

where U_g is the ground displacement vector and A_g is the ground reaction force vector

$$A_g = [a_{g1}, a_{g2}, \ldots a_{gN}]$$

4. Integrated System of Ground and Track including Effect of Rail-pad

The railpad is considered to have a spring constant K_p and a damper ratio D_p, and the sleeper has a mass M_s. **Fig.4** gives the mechanical model.

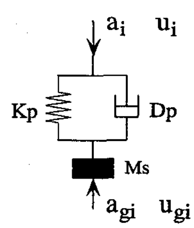

Fig. 4. Railpad model

$$-a_i = (u_i - u_{gi})K_p + (\dot{u}_i - \dot{u}_{gi})D_p \tag{40}$$

$$-a_i - a_{gi} = M_s \ddot{u}_s \tag{41}$$

The stiffness matrix of the ground is to be integrated with the stiffness of railpad to lead the displacement compatibility. The governing equation is then in the matrix forms as

$$U = HA \tag{42}$$

where

$$H = (M_s \omega^2 H_g - I)^{-1}((1 - \frac{M_s \omega^2}{K_p + i\omega D_p})H_g + \frac{1}{K_p + i\omega D_p} - I) \tag{43}$$

Combining the statements of Eq.(15) and Eq.(42) and eliminating the displacement vector U, we get the equation for the reaction forces A at the sleepers locations.

$$(G - H)A = -F \tag{44}$$

We utilize the linear Gauss elimination method to get ground reaction forces at sleepers' positions A. After the interaction forces between sleepers, rail-pads and ground have been obtained, and substituting these into the rail displacement statement, then we can get the transformed solution from Eq.(22). Further, the ground response under an array of load at sleeper's positions can be obtained by additional computations.

5. Numerical Computation and Results

The foregoing formulation is implemented into a computer code. For an illustrative computation, an actual geometry and properties of the track and ground of Swedish X-2000 on the West Coast line are used to simulate the vibration induced by train loading. The train geometry is described in **Fig.1** The total number of 61 sleepers are considered beneath the rails with the rail-pads between rails and ballast. The geometrical origin is located on the (31^{st}) sleeper. The motion of the load is supposed to take place from left to right and start at the origin point, at t=0. The sleepers are equally spaced with 0.7m distance. Two moving speeds of the load, 70km/h and 200km/h, is considered, which represent the low speed and the high speed, respectively. We modelled the embankment as a layer of comparatively stiff soil. Beneath the given soil layer we assume a half-space. The Soil profile used here is described in **Table 1**, the profile of multi-wheel axels loads in **Table 2,** and the geometric parameters of the track in **Table 3**

Table 1 Soil Profile in Computation

Soil Layer	Thickness (m)	Mass Density (g/m^3)	Shear Velocity V_S (m/s)		Poison Ratio	Damping ratio	
			C=70km/h	C=200km/h		C=70km/h	C=200km/h
Embankment	1.4	1,800	250	150	0.49	0.04	0.04
Surface Crust	1.1	1,500	72	65	0.49	0.04	0.063
Organic clay	3.0	1,260	41	33	0.49	0.02	0.058
Clay	4.5	1,475	65	60	0.49	0.05	0.098
Clay	6.0	1,475	87	85	0.49	0.05	0.064
Half-space	-	1,475	100	100	0.49	0.05	0.060

Table 2 Profile of Multi-wheel Axels Loads

Car NO.(Left to right)	P1(KN)	P2(KN)	A(m)	B(m)	L(m)
1	122.5	122.5	2.9	11.6	0.0
2	122.5	122.5	2.9	14.8	22.2
3	122.5	122.5	2.9	14.8	24.4
4	122.5	122.5	2.9	14.8	24.4
5	180	181.5	2.9	6.6	24.4

Table 3 Rail Parameter in Computation

Mass of unit rail per meter (kg)	Flexural rigidity of rail ($MN.m^2$)	Rail-pad Spring constant(MN/m)	Rail-pad Damping const.(KN.s/m)	Mass of sleeper (kg)
56.0	4.86	110	650	205

The analysis with substructure method has been implemented in the numerical computation and there are some sample results below. For the application of FFT algorithm for the inverse Fourier Transform, the frequency increment $\Delta f = 0.0977 Hz$ is taken and the Fourier point number is 1024. The Nyguist frequency $f_{nyg} = N\Delta f / 2$, therefore is 50Hz. The corresponding time increment is $\Delta t = 0.01$ sec.

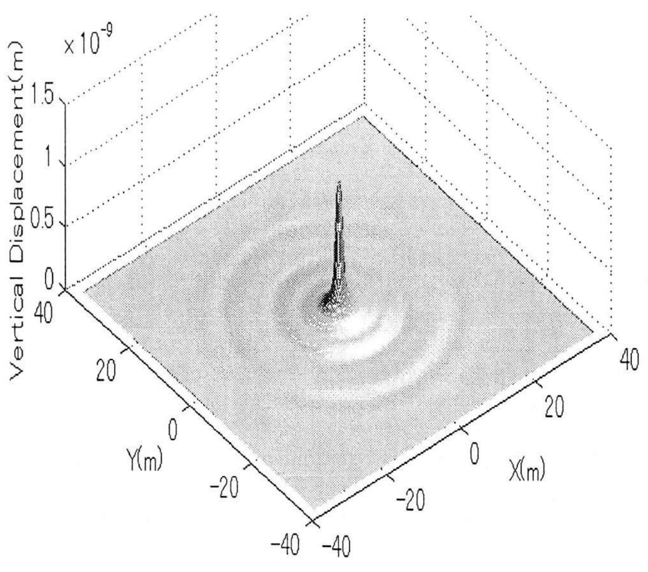

Fig. 6 Vertical displacement of ground directly under 1m×1m size unit harmonic load of 60HZ

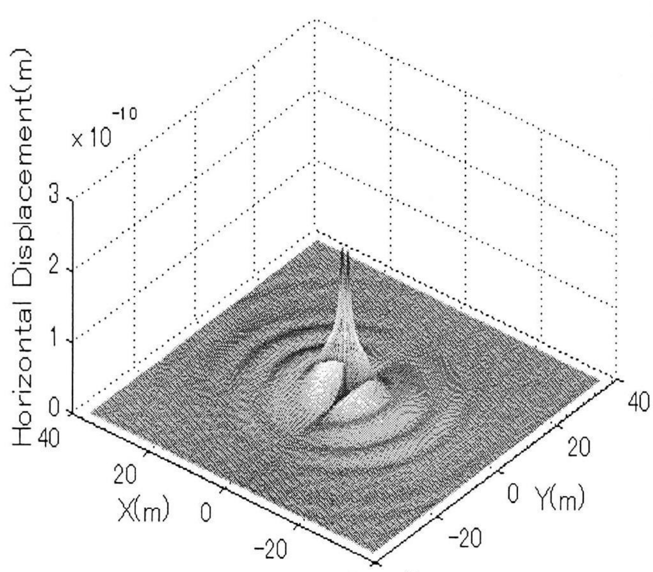

Fig. 7 Horizontal displacement of ground directly under 1m×1m size unit harmonic load of 60Hz

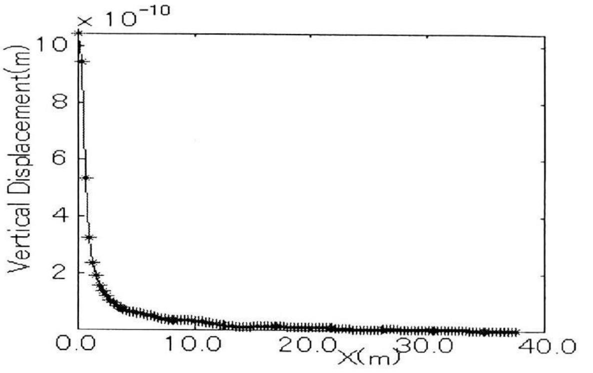

Fig. 8 Vertical cut at Y=0 in **Fig. 6**

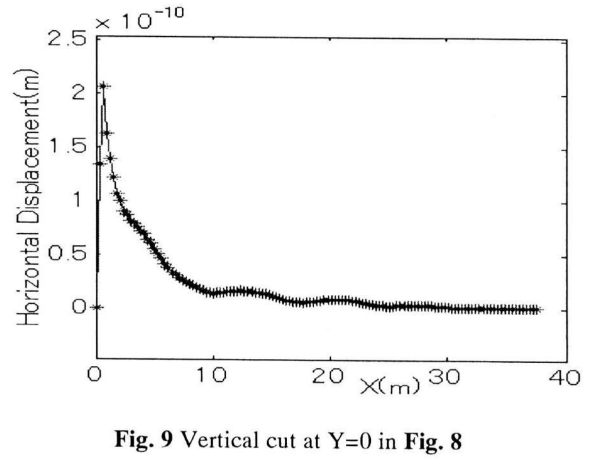

Fig. 9 Vertical cut at Y=0 in **Fig. 8**

In **Fig.6,** the 3D aspect of ground vibrations of vertical motion caused by the unit harmonic load uniformly distributed over 1m×1m area of 60Hz frequency are depicted. The horizontal component displacements are shown in **Fig.7**. **Fig.8** and **Fig.9 are** the central cut section at Y=0 of 3D graph of **Fig.6** and **Fig. 7.**

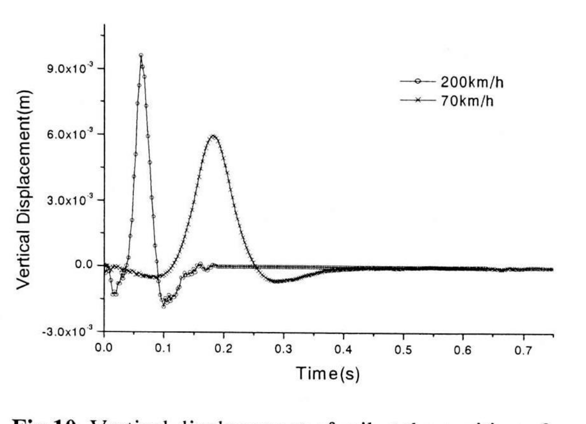

Fig.10 Vertical displacement of rail at the position of a sleeper when the point load moves by.

In **Fig10** shown is the displacement time history of rails at the sleeper's position under moving point load with 70km/h and 200km/h.

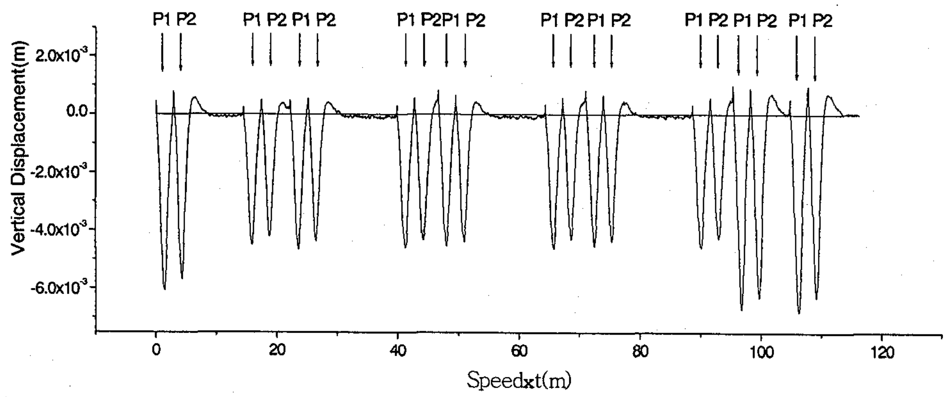

Fig. 11 Vertical displacement of rail under multi-wheel axels loads(70km/s)

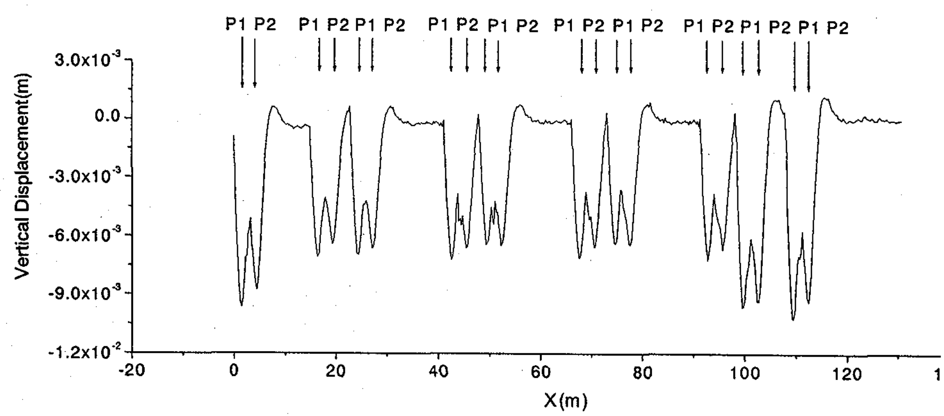

Fig. 12 Vertical displacement of rail under multi-wheel axels loads (200km/s)

The vibrations of rails under multi wheel loads (**Table 2**) with speed of 70km/h and 200km/h are obtained from the superposition of those corresponding with the appropriate magnitudes and time lags and showed in **Fig11** and **Fig.12** respectively for different speed, in which the axis has been converted to the space distribution from time.

We introduced a concept of 'effective distance' to define the extent in which a number of neighbouring sleepers are brought into the response by a moving load passing by a focused sleeper's position.

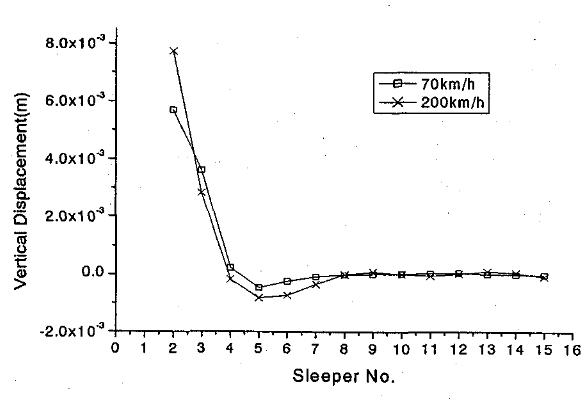

Fig. 13 Effect length of sleepers when the load moves by the 32nd sleeper

In **Fig.13**, shown are the influenced sleepers which are effect by the load passing by the 32nd sleeper with speed of 70km/h and 200km/h. The effective length increases as the moving speed grows. From the numerical experiments, we find that the effective distance is highly dependent on the stiffness of the rails and slightly on the moving speed of the load within the range of parameters here.

The load transmitted from the rail to the sleepers also will be available from the results we get, which can be used in the ground motion simulation. In **Fig14**, the reaction force on the 8th sleeper when the load passes by it is presented.

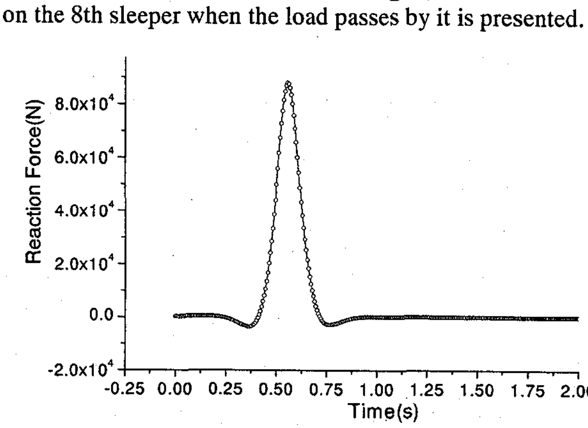

Fig. 14 The reaction force of 8th sleeper when load passes with speed of 70km/h

When the sleeper reactions are obtained from the stiffness matrix equation, they are substitute as the vibration-generating source forces to evaluate the ground motion. The displacement time history at each observation point can be obtained from the superposition of those caused by the concerned sleepers. For a single moving load case, the problem can be solved by this way ; while for a series of train load one more time of superposition should be needed in view of the geometry of train wheels. In **Fig.15** and **Fig.16,** the ground motions at place 10m away from the track center line are shown for a series of train wheels loads with speed of 70km/h and 200km/h respectively. The geometry of train wheel loads is shown in **Table2.**

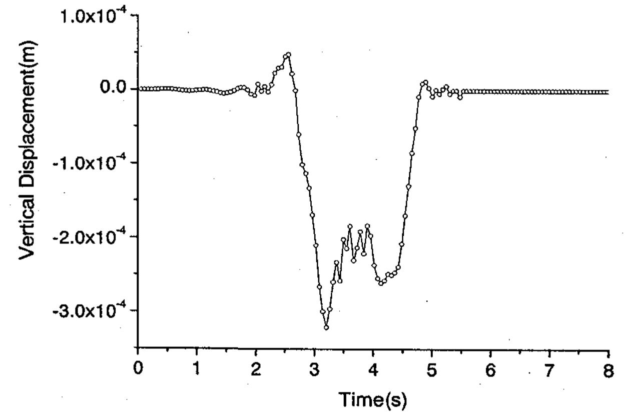

Fig. 15 Field vibration (10m off) under train load with speed of 70km/h

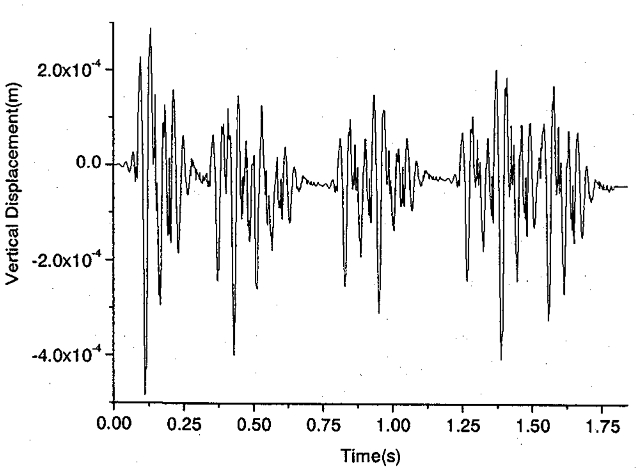

Fig. 16 Field vibration (10m off) under a train load with speed of 200km/h.

6. Conclusions

In this paper, a train track and ground dynamic interaction has been developed by the substructuring technique, with an emphasis on the discretely positioned sleepers that become the source of vibration generation in the ground. The vertical displacements of rails and the nearby ground are the main focus. The dynamic stiffness method of layered ground is utilized in order to model better the reality conditions, in contrast to the traditional continuous Winkler's foundation assumption. The analytical results are obtained in frequency domain, and the time histories of rail vibration are computed from the inverse FFT both for single and multi-axels loads. When the load moves passing by the sleeper position, the effective distance in the neighbouring sleepers is clarified. This effect distance is not clearly studies in the former efforts with the assumption of continuous spring-supports of Winkler foundation.

The computation results show that the discrete properties of supports affect the behaviour of rail vibration more in low frequency range than in high frequency range; Then the discretely distributed sleeper forces become the load in turn to simulate the nearby ground motion. In the example case, the ground vibrations also are presented both in transformed domain and the time histories.

The present substructure formulation may be applied to advantage also to the extended viaduct track analysis with substantially spaced piers.

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