Application of Radiation Modes in Active Control of Sound Radiated from a Highway Bridge

Theeraphong CHANPHENG*, Hitoshi YAMADA**, Toshio MIYATA**, Hiroshi KATSUCHI***

* Grad. Student, Dept. of Civil Engineering, Yokohama National University, 79-5, Tokiwadai, Yokohama 240-8501
** Dr. of Eng., Professor. Dept. of Civil Engineering, Yokohama National University
*** Dr. of Eng., Assoc. Professor. Dept. of Civil Engineering, Yokohama National University

Low frequency sound radiated from highway bridges is considered as the result from vibration of bridge. In order to reduce such sound, attempts have been used in minimizing bridge vibration. However, mechanisms of vibration and sound radiation are different so that minimizing of structural vibration modes may not guarantee the reduction of sound radiation. The concept of radiation modes, which is appropriate to describe mechanisms of sound radiation, is reviewed. The radiation modes of a highway bridge are investigated. Active control of sound radiation using radiation modes is performed. It is found that sound radiation can be significantly suppressed by using this approach.

Key Words: radiation mode, highway bridge, sound reduction

1. Introduction

Low frequency sound radiated from highway bridges is a well-known environmental problem in Japan. It is caused by vibration of bridge due to moving vehicles. In order to reduce sound radiation, methods of minimizing bridge vibration were proposed such as modification of joint connections, and installation of Tuned Mass Damper. Most of the proposed methods were based on minimization of structural vibration modes. And it was considered that reduction of significant structural vibration modes that dominate overall vibration characteristics will result in reduction of sound radiation. Although sound reduction could be achieved, this may not be optimal because mechanisms of structural vibration and sound radiation are different.

Differences between structural vibration and sound radiation are explained as follows. If we consider an elastic vibrating body, the behavior of *structural vibration* is dominated by some fundamental structural vibration modes. That is, in order to minimize structural vibration, it can be done by reducing such significant structural vibration modes. However, if we consider the behavior of *sound radiation*, each vibration mode radiates sound with differing radiation efficiency and some vibration modes are coupled each other. That is, in order to reduce sound radiation, it cannot be done in the same manner as in structural vibration control. Due to the fact that each vibration mode couples with each other when it radiates sound, formulations that can be used to describe the sound radiation in terms of a set of independent velocity distributions on the structure and can eliminate coupling effects between structural modes are necessary. These formulations are found in many literatures in acoustic engineering. They are termed *radiation modes* [1].

Radiation modes are considered more appropriate for control of sound radiation than structural vibration modes. Although applications of radiation modes are confined to simply structures such as baffled panels and baffled beams, they are considered to be applicable to large scale structures such as highway bridges. The objective of this research is to use the benefits of radiation modes to perform numerical study on the active control of sound radiation from a highway bridge and to investigate the validity of this approach.

The formulation of radiation modes is developed from the expression of acoustic radiated power. The acoustic radiated power is explained in the following sections.

2. Acoustic Radiated Power

Although sound pressure level (unit in decibel) is one of the quantities generally used as representation of sound radiation, it varies with distance, depends on propagation path and pertains to diffraction, reflection, etc. which make it difficult to be interpreted. In contrast, acoustic radiated power, which is independent of propagation path, etc., is more useful for understanding characteristics of sound radiation.

Consider an arbitrary shape of elastic structure in free space surrounded by air in Fig. 1. Assume that it vibrates at single frequency ω ; and corresponding sound pressure p propagates to spherical surface at far field. The acoustic radiated power W can be described by integral of sound pressure p over the spherical surface at far field as

$$W(\omega) = \frac{1}{2\rho c} \iint |p(\omega)|^2 dA \quad (1)$$

where ρ :air density, c: sound velocity, and A: area of spherical surface at far field. In the discretized form of pressure at far field $\mathbf{p}$, Eq.(1) can be written as

$$W(\omega) = \frac{1}{2\rho c} \mathbf{p}^H \mathbf{p} \Delta A \quad (2)$$

where the superscript H refers to Hermitian (transpose conjugate) operation. This equation can be used as an approximation of acoustic radiated power from the measured sound pressure. However measuring of sound pressure is undesirable in case of large scale structures, expression of acoustic radiated power in terms of structural vibration is required.

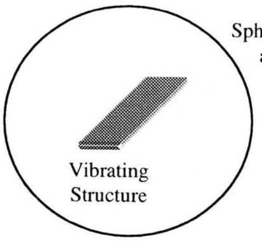


Fig.1 Vibrating structures in free space

2.1 Acoustic Radiated Power - Vibration Modes

In Eq.(2), sound pressure at far field $\mathbf{p}$ relates to normal surface velocity vector $\mathbf{v}$ by

$$\mathbf{p} = \mathbf{Z}\mathbf{v} \quad (3)$$

where $\mathbf{Z}$ is acoustic transfer matrix. The matrix $\mathbf{Z}$ can be computed by Rayleigh integral for a planar baffled structure or by Boundary Element Method (BEM) for complicate structures. From modal analysis of structure, it is possible to relate velocity vector $\mathbf{v}$ and modal displacement $\mathbf{q}$ by modal shape matrix $\mathbf{\Phi}$, then Eq.(3) can be written in terms of modal displacement $\mathbf{q}$ as

$$\mathbf{p} = \mathbf{Z}\mathbf{\Phi}\mathbf{q} = \mathbf{Z}_\mathbf{m}\mathbf{q} \quad (4)$$

where $\mathbf{Z}_\mathbf{m}$ is modal acoustic transfer matrix. After substitute Eq.(4) into Eq.(2) , acoustic radiated power can be expressed in terms of modal displacement $\mathbf{q}$ as

$$W(\omega) = \mathbf{q}^H \mathbf{Z}_\mathbf{m}^H \mathbf{Z}\,\mathbf{q} = \mathbf{q}^H \mathbf{P}_\mathbf{m}(\omega)\mathbf{q} \quad (5)$$

In general, matrix $\mathbf{P}_\mathbf{m}$ is not diagonal. It is Hermitian and positive definite. That means there are some coupling effects among modal vibrations.

Modal displacement $\mathbf{q}$ can be solved from surface velocity $\mathbf{v}$ by least mean squares techniques as

$$\mathbf{q} = [\mathbf{\Phi}^T\mathbf{\Phi}]^{-1}\mathbf{\Phi}^T\mathbf{v} = \mathbf{\Omega}\,\mathbf{v} \quad (6)$$

Substitute Eq.(6) into Eq.(5), acoustic radiated power can be written as

$$W(\omega) = \mathbf{v}^H\mathbf{\Omega}^H\mathbf{P}_\mathbf{m}\mathbf{\Omega}\,\mathbf{v} = \mathbf{v}^H\mathbf{P}(\omega)\,\mathbf{v} \quad (7)$$

Eq (5) is transformed to Eq.(7) because the radiation modes are better interpreted in the form of velocity distribution on structure not in the form of modal displacement. This idea is explained in the following section.

2.2 Acoustic Radiated Power - Radiation Modes

From Eq.(7), matrix $\mathbf{P}$ at frequency ω can be factorized by singular value decomposition (SVD) as follows:

$$\mathbf{P}(\omega) = \mathbf{Q}^T(\omega)\mathbf{\Sigma}(\omega)\mathbf{Q}(\omega) \quad (8)$$

where $\mathbf{Q}$ is the orthonormal transformation matrix; $\mathbf{\Sigma}$ is diagonal matrix whose elements are singular values, decreasing monotonically along the diagonal. This diagonal matrix is the key point to eliminate coupling effects between structural vibrations. Substitute Eq.(8) into Eq.(7), the acoustic radiated power can be expressed as

$$W(\omega) = \mathbf{v}^H\mathbf{Q}^T(\omega)\mathbf{\Sigma}(\omega)\mathbf{Q}(\omega)\mathbf{v} \quad (9)$$

Define vector $\mathbf{r}$ as

$$\mathbf{r} = \mathbf{Q}(\omega)\mathbf{v} \quad (10)$$

which is the transformation of normal surface velocity $\mathbf{v}$. $\mathbf{r}$ is termed *radiation mode amplitude* at frequency ω and $\mathbf{Q}$ is termed *radiation mode shape*. The row vectors in radiation mode shape matrix $\mathbf{Q}$ are interpreted as a set of independent patterns of velocities distribution on the vibrating surface that radiate sound independently. Acoustic radiated power in Eq.(9) can be written by

$$W(\omega) = \mathbf{r}^H\mathbf{\Sigma}\mathbf{r} = \sum_j \sigma_j |r_j|^2 \quad (11)$$

This relationship indicates that acoustic radiated power can be decomposed into a combination of radiation mode amplitudes multiplying with their associated singular values. The magnitude of the jth diagonal element σ_j of $\mathbf{\Sigma}$ determines the relative importance of the jth radiation mode compared to other radiation modes. Summation of radiation modes in Eq.(11) can be truncated to remain only significant radiation modes which have high contribution to acoustic radiated power.

3. Active Control of Sound Radiation by Radiation Modes

In active control of sound radiation, the objective is to reduce acoustic radiated power to a certain level. As shown in Eq.(11), acoustic radiated power is the summation of significant radiation modes. The reduction of acoustic power can be achieved if some significant radiation modes are suppressed.

In case a structure vibrating at single frequency ω, radiation mode shapes $\mathbf{Q}(\omega)$ and their associated singular value matrix $\mathbf{\Sigma}$ can be computed; and the significant radiation modes can be identified. Then, suppression of significant radiation modes can be done directly. Unfortunately, when consider a wide band of frequency of vibration; it is not convenient to do since radiation mode shape $\mathbf{Q}$ and their associated singular value matrix $\mathbf{\Sigma}$ are dependent on frequency. This makes concept of radiation modes difficult to be applied.

To overcome such difficulties, Borgiotti and Jones [2] describe the nesting property of radiation mode shapes (or radiation spatial filters). It is stated that "The space S of the radiation mode at any frequency ω below ω_{max} is a subspace of S_{max} of the radiation mode at ω_{max}." By using this property, it is proved that a feedback control system driving the radiation mode amplitudes $\mathbf{r}$ computed from radiation spatial filter $\mathbf{Q}(\omega_{max})$ to zero will suppress the radiation mode amplitudes $\mathbf{r}$ for all ω below ω_{max}.

According to concept mentioned above, in this research, radiation mode shapes at ω_{max} or $\mathbf{Q}(\omega_{max})$ are used as spatial filters. Then the radiation mode amplitudes $\mathbf{r}$ will be suppressed by an active control system.

Block diagram of the active control system using such concept is illustrated in Fig.2. The objective of control is to suppress the radiation mode amplitudes $\mathbf{r}$ due to disturbance force $\mathbf{d}$. In other words, it is to minimize the transfer function from $\mathbf{d}$ to $\mathbf{r}$ as small as possible. In this research, robust controller is designed by μ–synthesis [3]. This design method is selected because it can incorporate the lack of model fidelity directly into the design procedure, and simultaneously optimize for performance and robustness. Moreover, frequency characteristics of model uncertainties and performance weight functions can be expressed explicitly. Review of μ–synthesis design method in acoustic radiation can be found in [4].

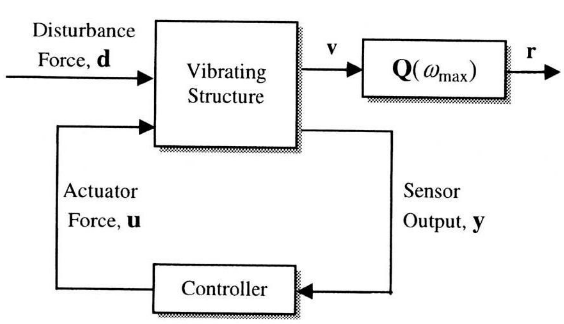


Fig.2 Block diagram of active control system

4. Application of the Concept of Radiation Modes to a Highway Bridge

In this section, a highway bridge which radiated low frequency sound is studied. The parameters of the bridge are obtained from the steel girder bridge B in reference [5]. Vibration characteristics of the bridge are first computed, and then its radiation characteristics which can be described by radiation mode shapes $\mathbf{Q}$ and their associated singular value $\mathbf{\Sigma}$ are computed. Finally, active control of sound radiation is numerically performed.

4.1 Structural Model of a Highway Bridge

Model of the highway bridge is shown in Fig. 3. This highway bridge has 4 girders, span length 36.48 m, deck width 12.80 m, and is simply supported. Bridge deck is modeled by solid elements and girders are modeled by plate elements.

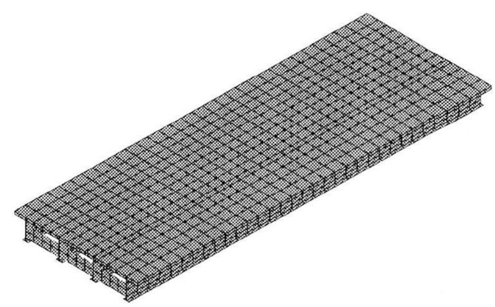


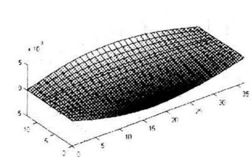
Fig.3 FEM model of Highway Bridge

4.2 Vibration Characteristics

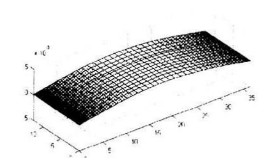
Vibration mode shapes and their corresponding natural frequencies from eigenvalue analysis are shown in Table 1. Letter B in the table denotes bending mode, T: torsion mode, and L: local deck vibration mode. Structural vibration mode shapes (consider only deck vibration) are shown in Fig. 4.

Table 1. Natural frequencies and mode shapes

No.	Freq. (Hz)	Shape
1	2.79	T1
2	2.91	B1
3	9.83	B2
4	11.90	T2
5	12.02	L1
6	17.05	L2
7	21.29	T3
8	24.72	L3
9	24.82	L4
10	26.84	L5



Mode 1



Mode 2

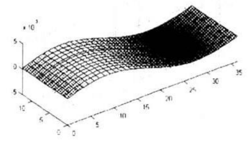
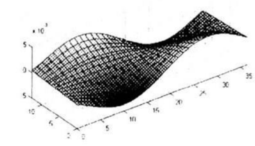
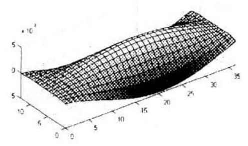
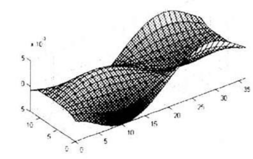
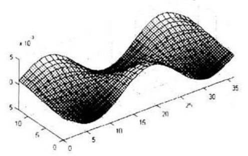
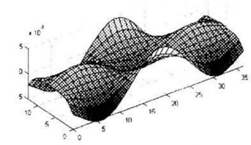
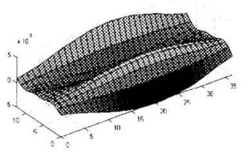
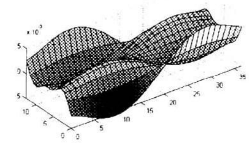

Mode 3

Mode 4

Mode 5

Mode 6

Mode 7

Mode 8

Mode 9

Mode 10

Fig.4 Structural vibration mode shapes

4.3 Radiation Characteristics

Using the vibration mode shapes and BEM method, radiation mode shapes and their associated singular values are computed. The first four radiation mode shapes at some frequencies are presented in Fig. 5 to Fig.9. From these figures, it is clear that radiation mode shapes are frequency dependent. A plot of the first four singular values, σ_1 through σ_4 as a function of frequency is presented in Fig. 10. It shows that at low frequency range (0 to 17 Hz), the 1st and 2nd radiation modes are significant radiation modes. But at higher frequency range (more than 17 Hz), the higher radiation modes become significant.

From data in reference [5], this bridge radiates low frequency sound as shown in Table 2. Frequencies of radiated sound agree well with the natural frequencies of vibration mode 2, 3, 5 and 6. It is observed that frequencies of radiated sound match the natural frequencies of all vibration modes except those of torsion modes. As shown in Fig.5 to 9, the 1st and 2nd radiation mode shapes which are significant radiation modes are similar to bending or local deck vibration mode shapes. This is the reason why torsion modes have no effects on sound radiation.

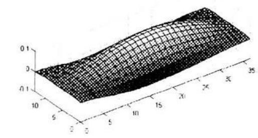
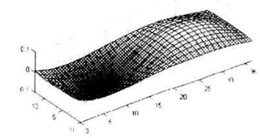
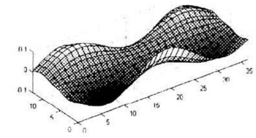
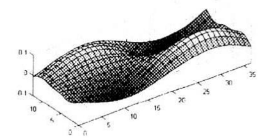

Mode 1

Mode 2

Mode 3

Mode 4

Fig.5 Radiation mode shapes at 2.91 Hz

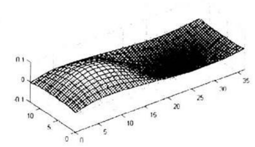
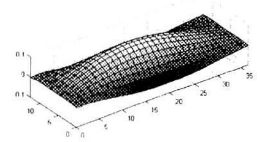
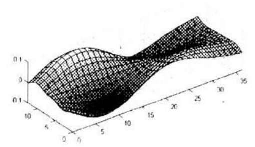
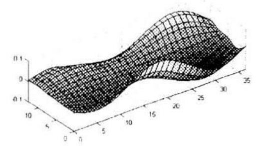

Mode 1

Mode 2

Mode 3

Mode 4

Fig.6 Radiation mode shapes at 9.83 Hz

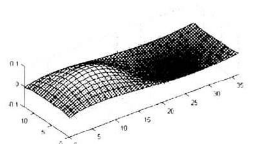
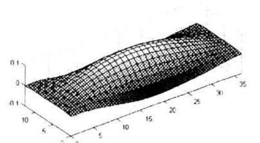
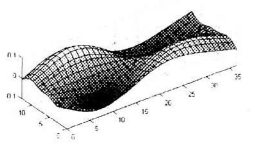
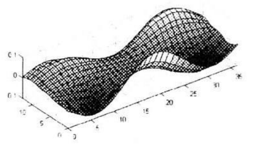

Mode 1

Mode 2

Mode 3

Mode 4

Fig.7 Radiation mode shapes at 12.02 Hz

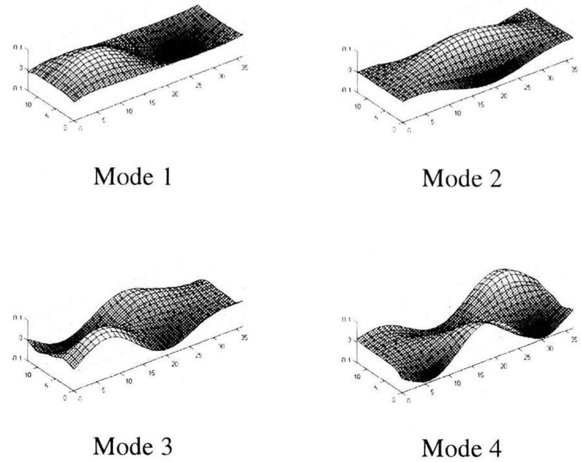


Fig.8 Radiation mode shapes at 17.05 Hz

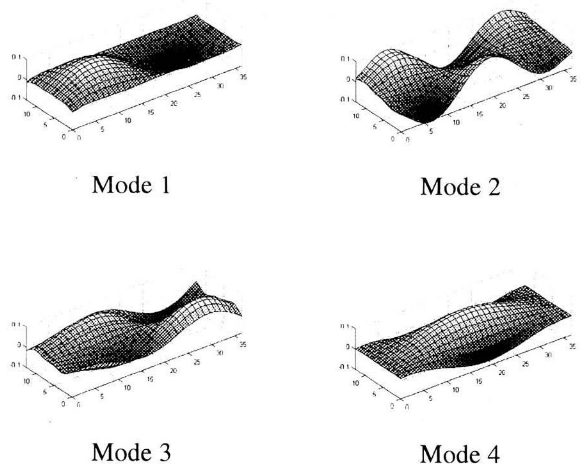


Fig.9 Radiation mode shapes at 20 Hz

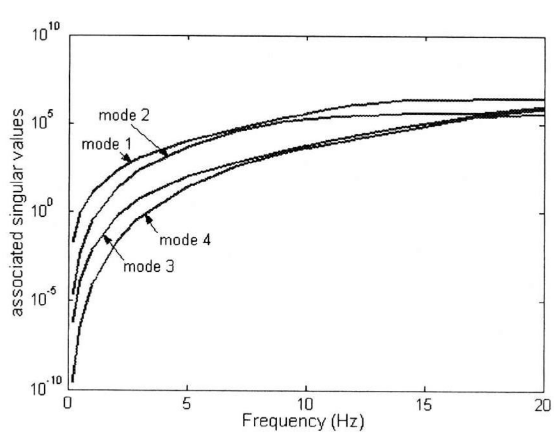


Fig.10 Associated singular values σ_j of 1st to 4th radiation modes

Table 2. List of measured radiated sound pressure level

No.	Freq. (Hz)	SPL (dB)
1	3	100
2	10	80
3	12	80
4	16	75

4.4 Numerical Simulation of Sound Radiation Control

Active control of sound radiation can be achieved by suppression of radiation modal amplitudes **r**. Radiation mode shapes **Q** are fixed at the maximum frequency of control 20 Hz and consider only first four radiation modes. The radiation mode shapes used in controller design are shown in Fig.9. These shapes also have influence on selection of positions of actuators and sensors. In this study, nine actuators and nine velocity sensors are placed near the anti-nodes of radiation mode shapes.

Consider a block diagram for μ–synthesis design of controller shown Fig. 11. Disturbance, **d** on the system is a harmonic concentrated force applied at mid span. Uncertainty which presents in the system is unmodelled dynamics resulting from truncation of higher structural vibration modes. This is represented by additive weight function $\mathbf{W}_a$. $\mathbf{\Delta}_a$ is uncertainty block in framework of μ–synthesis. Performance weight function of controller force is constant matrix $\mathbf{W}_u$. Performance weight function of quantity to be controlled (radiation mode) **r** is also fixed as constant matrix $\mathbf{W}_p$. In design controller **K**, only performance weight $\mathbf{W}_p$ is trial parameter. After several trials, controller **K** can be designed.

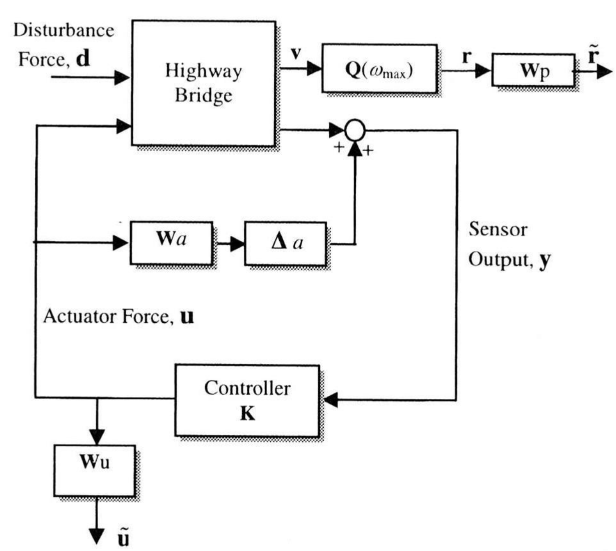
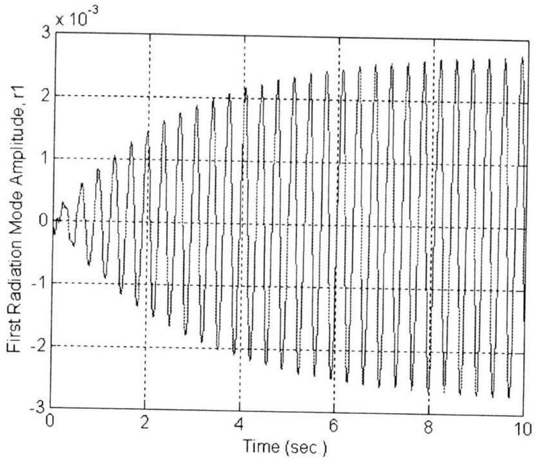


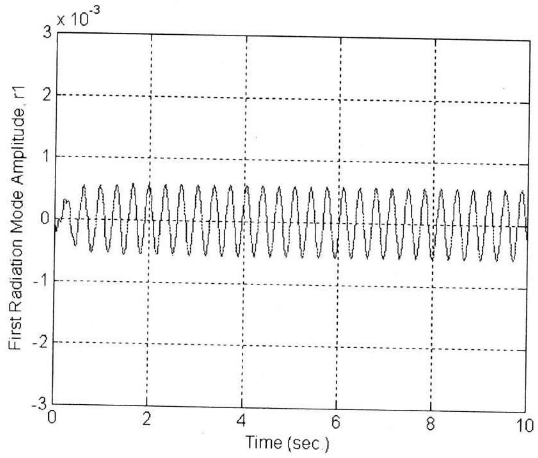
Fig.11 Interconnection for μ–synthesis design

In Fig.12 and 13, time histories of 1st radiation mode amplitude are plotted. The bridge is excited by a concentrated force at frequency 2.91 Hz and 12.02 Hz respectively. With controller, the 1st radiation mode amplitude is significantly suppressed as shown in

Fig.12(b) and 13(b). From these simulation results, it is considered that using radiation modes in active control of sound radiation is effective.

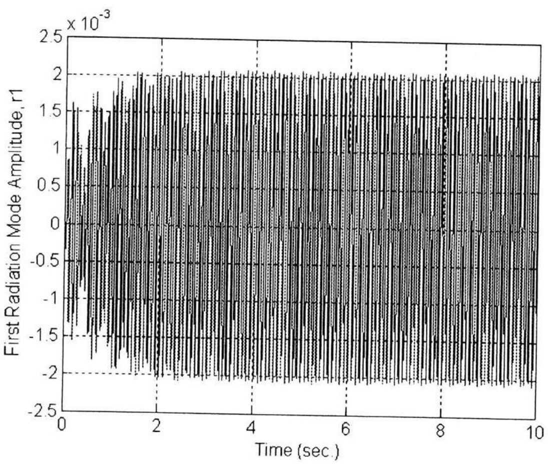


(a) Uncontrolled 1st radiation mode

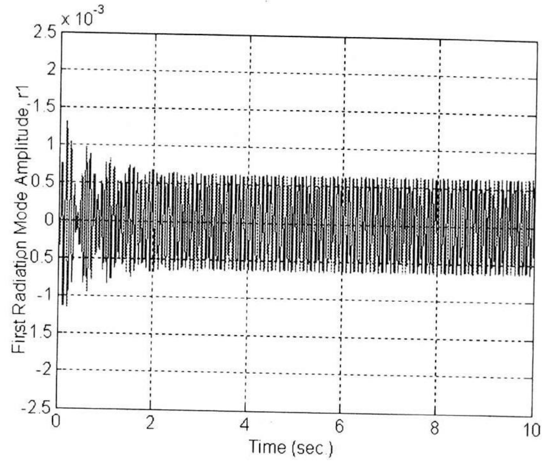


(b) Controlled 1st radiation mode

Fig. 12 1st Radiation mode amplitude excited at 2.91 Hz



(a) Uncontrolled 1st radiation mode



(b) Controlled 1st radiation mode

Fig. 13 1st Radiation mode amplitude excited at 12.02 Hz

5. Conclusions

In this work, we can summarize two important concepts as follows:

(1) Using the radiation modes provide some physical insight into radiation mechanisms.

(2) Using the radiation modes in active control of sound are considered to be effective.

References

1) S.J. Elliott and M.E. Johnson, Radiation modes and the active control of sound power, Journal of the Acoustic Society of America, 94(4), pp.2194-2204, 1993
2) G.V. Borgiotti and K.E. Jones, Frequency independence property of radiation spatial filters, Journal of the Acoustic Society of America, 96(6), pp.3516-3524, 1994
3) G.J. Balas, J.D. Doyle, K. Glover, A.K. Packard, and R. Smith, μ–Analysis and Synthesis Toolbox, MUSYN Inc., 2001
4) D.E. Cox, G.P. Gibbs, R.L. Clark, and J.S. Vipperman, Experimental robust control of structural acoustic radiation, Journal of Vibration and Acoustics, 121, pp.433-439, 1999
5) 足立義男,橋梁の振動性状が低周波域の空気振動に及ぼす影響に関する研究,土木研究所報告,第 164 号-1,1984年 11 月

(Received September 13,2002)