# Assessment on Fill Material Properties for Filled Steel Stub Columns

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In view of the mounting cost of maintaining or rehabilitating deteriorating infrastructure or reconstructing the collapsed ones, the application of advanced composite structural systems as well as new materials in the construction industry is now considered a necessity for sustainable economy as mankind heads towards the next millenium. Polymers or polymer-based materials are now considered to be both complementary and supplementary to cement concrete that include high durability, fatigue resistance, strength, ductility and energy absorption. Particularly, the versatile design of material properties is attracting. In this study, attempts are being made to assess the properties of the fill materials, such as polymer and polymer based materials on the mechanical behavior of filled steel tubular members (FST) in conjunction with the finite element analysis. This study is a continuation of previous experimental works of compression test, bending test and interface slip test. Numerical results confirm the finite element analytical modeling of fill materials and the behavior of FSTs under compression is assessed for the different material properties. It is concluded that the poisson's ratio and bulk modulus can affect the strength and ductility, and that the interface bond can affect the local buckling mode of FSTs.

Keywords: Filled steel tube, Compression, Finite element modeling, buckling mode, Fill materials

# 1. Introduction

Experience gained over the last few decades in Europe, Japan and the USA reveals that existing construction technology has not delivered the reliability needed, as is evidenced by the severely deteriorated infrastructure and inability to guarantee safety against natural hazards. In consequence, governments especially the USA are now investing heavily in harness with private industry to develop novel high performance construction materials and systems, with special interest being directed towards advanced composite materials1). On the other hand, in Japan, the devastating effects of the 1995 Hyogo-ken Nanbu earthquake has arisen further interest in composite members <sup>2-3)</sup>, the most practical so far being the concrete filled tubular members (CFT) 4-6) for columns. In addition, as an alternative cement-based fill material, the low-strength and light mortar filled in steel tubes has been investigated for beams, which are more critical to its dead weight<sup>7</sup>).

Numerous research works are also exploring the use of alternative polymer or polymer based materials to replace cement concrete in composite construction to alleviate several drawbacks inherent to ordinary cement concrete e.g. high shrinkage and creep, and low tensile strength, fatigue resistance and durability<sup>8-18)</sup>. A most unique feature of polymers realized was the wide array of properties presented by chemical chains, making them amenable or versatile to meet specific requirements of any desired construction, as well as for the ordinary material improvement, particularly concrete. Recently, the authors also conducted a series of experiments on steel tubular members filled with polymers and polymer-based materials in addition to ordinary mortar/concrete(FST). It is concluded that stub columns and beams reveal colossal increase in strength and ductility of the epoxy polymer-filled steel tubular members, highlighting their untapped potential for seismic resistance 19-22). It is clearly notified that the very high adhesive or shear strengths of epoxy resin has been demonstrated by means of steel-epoxy-steel interface, resulting in increased stiffness and strength of FSTs.

Numerous attempts have been made to enhance the superiority of CFTs in terms of confinement of fill materials, interfacial shear transfer, its structural modeling and applicability to earthquake-resistant structures<sup>23-29</sup>. However, none of them can conclude the optimum fill material properties for each structural component. Therefore, this study is motivated by the observed vast potential of polymer based materials in FST, and focused is the search of desirable fill material properties for the wide range of needs in conjunction with the finite element analysis, highlighting the superior composite interaction between fill materials and steel.

# 2. Finite element modeling of composite stub columns by ABAQUS

Previous experimental studies by the authors involved numerous fill materials with different fundamental properties. These properties had a combined effect on the response of the stub column, presenting a difficulty in clearly distinguishing which property is predominant at any particular circumstance, and hence relevant for design consideration. In order to assess the effect of each of the important fill material properties with the intention of seeking ideal fill material properties for optimum performance of filled steel composite members, finite element analytical model using ABAQUS<sup>30)</sup>, general purpose program was evolved in conjunction with experimental results. The model was then used to conduct parametric studies on the fill material properties affecting strength and ductility of composite stub columns.

### 2.1 Geometric and structural modeling

An elasto-plastic three-dimensional nonlinear finite element modeling and analysis of filled steel stub columns subjected to compressive load was conducted by employing the commercial finite element code ABAQUS, in conjunction with experimental results. The model was initially evolved from the dimensions in Table 1, consistent with experimental studies, where L is the length, D is the diameter and t is the thickness of the steel column.

The steel tube was modeled as 8-node doubly curved thin shell elements of reduced integration, with five degrees of freedom per node (S8R5), while the fill material core was modeled as 20-node quadratic brick (hexahedral) elements of reduced integration (C3D20R) considering the non-axisymmetric buckling deformation, all included in the ABAQUS software. To account for the interaction between

the steel tube and the fill material, bi-directional nonlinear spring elements oriented in the radial and axial directions were deployed at each of the interface nodes, but the spring constant in the normal of the interface was assumed to be large enough such as a rigid.

Load application was achieved by imposing incremental axial displacements through a rigid plate at the top, while maintaining a rigid support plate at the bottom. The modeling is visualized in Fig. 1.

Table 1 Dimensions of filled steel stub columns

L	D	t	D/2t	Area of	Area of	$A_{\text{fill}}$
(mm)	(mm)	(mm)		steel	fill	/As
				As	$A_{\mathrm{fill}}$	
				(mm <sup>2</sup> )	(mm <sup>2</sup> )	
192	96	1.6	30	474.5	6764	14.3
200	100	1.0	50	311.0	7543	24.3

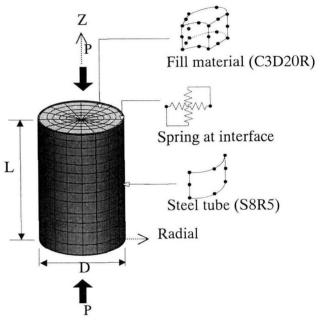


Fig. 1 Finite element modeling

#### 2.2 Material models

The properties of fill materials and the steel tube, as well as the interface characteristics were same as the ones in experimental studies<sup>19-22)</sup>. Fill material labels used in the following, LCM, E1A, E2A and CN indicate latex mortar with ordinary age (25days), hard polymer concrete(high stiffness epoxy mixed with aggregates), soft polymer concrete(low stiffness epoxy mixed with aggregates) and ordinary mortar(normal concrete) respectively. Specific material properties are tabulated in Table 2 and more detailed information can be referred to References 19 and 22.

In modeling the steel material behavior, an elastic-plastic constitutive model incorporating von Mises yield surface, an associated flow rule and isotropic strain hardening was adopted. ABAQUS programming requires the input of material true stress versus plastic strain data to be used in modeling the steel plasticity, and this was obtained from the coupon tensile test on steel strips, e.g. bi-linear like. Specific properties are also given in Table 2.

Table 2 Properties of Steel and Fill Materials

Fill	Young's	Poisson's	Yield	Ultimate
Material	Modulus	Ratio	Stress	Strength
	(KN/mm <sup>2</sup> )		$(N/mm^2)$	$(N/mm^2)$
Steel	214	0.342	228*	333
LCM	15.2	0.222	-	21.1
E1A	12.9	0.316	-	52.0
E2A	3.0	0.480	_	19.0
CN	29.6	0.171	-	26.0

(note) \*determined by 0.2% offset strain

On the other hand, for the fill materials, extended Drucker-Prager model with an associated flow rule and an exponential yield surface relating equivalent pressure stress  $p = (\sigma_z + 2\sigma_l)/3$  to Mises equivalent stress  $q = \sigma_z - \sigma_l$ was used, where  $\sigma_z$  and  $\sigma_l$  are the axial and lateral stresses on the encased fill material, respectively. The extended Drucker-Prager model is usually used to model frictional materials that exhibit pressure-dependent yield in that the material becomes stronger as the pressure increases, and also materials in which the tensile and compressive yield strengths are significantly different. The model allows for the material to harden as well as to soften isotropically. The general exponent form of yield criteria was adopted in this study as it provides the most flexibility in matching triaxial test data. The yield function of the general exponent form shown in Fig. 2 is written as(e.g., details can be referred to Ref.31);

$$F = a \cdot q^b - p - p_t = 0 \tag{1}$$

where,  $p_t = a \cdot \sigma_c - \sigma_c/3$  and  $\sigma_c$  is the yield stress. The material parameters a, b, and  $p_t$  can be calculated by the least square method in ABAQUS's data preparation procedure from the input triaxial experimental data of lateral confining stress versus axial confined strength. In this study, triaxial experimental data was approximated from the stub column tests based on the measured surface strains on the encasing steel tube and assuming elasto-plastic stress-strain relation for the steel. In this way, the axial and lateral stresses on the fill materials were indirectly approximated by satisfying equilibrium conditions. Fig. 3 gives the deduced triaxial data in the form of confined axial strength versus the

confining lateral stress. It is to be said that the triaxial data are really approximate values since their accuracy is dependent on the extent of buckling of the steel tube. However, in the absence of any other plausible data, they are deemed satisfactory for this modeling.

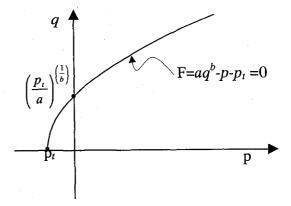


Fig. 2 Drucker-Prager general exponent yield surface

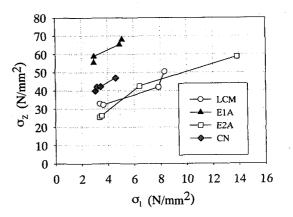


Fig. 3 Triaxial data

Finally, it was required to define the characteristic of the spring at the interface connecting the fill material and the steel tube. The bond or spring stiffness characteristics were obtained through experimental flexural tests on open sandwich beams complemented with finite element analysis. Details of nonlinear spring modeling at interface can be referred to Ref.21.

# 3. Verification of finite element model by experimental results

# 3.1 Load-strain characteristics

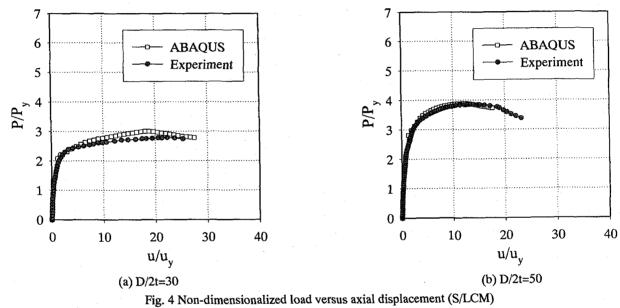
Figs. 4 to 7 relate the ABAQUS analyzed non-dimensional force to non-dimensional displacement of the filled composite vis-à-vis experimental results. The axial displacement has been normalized by shortening at yield of steel tube( $u_y$ ) while the axial force has been normalized by axial yielding force of steel tube( $P_y$ ).

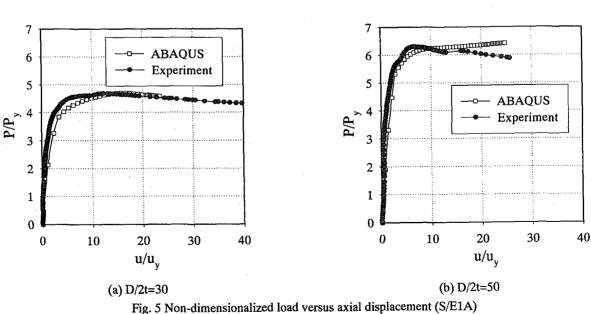
Generally, a close agreement between analytical and

experimental results is observed, especially for S/LCM2-50, S/E2A-30, S/E2A-50, S/CN-30 and S/CN-50, in which the last number indicates D/2t ratio. Difficulties were encountered in modeling S/E1A stub columns due unreliable experimental strain readings as a result of premature buckling of the steel in the middle region. Experimental strains measured on the steel surface show a decrease at some point implying buckling with consequent introduction of tensile bending strains, and this leads to unreliable deduction and over-estimation of triaxial data for the epoxy E1A material. It was found that E1A had to be modeled as having no hardening in an attempt to approximate the curve of S/E1A columns to some reasonable level.

All in all, the modeling duplicated the experimental results quite satisfactorily despite limited data of the fill material and the interface bond characteristics. When the

data-base on fill material properties is expanded in the future, this model will serve as significant stepping stone for the analysis of filled steel stub columns filled with various types of materials in addition to concrete. Since there is not yet a universally accepted test method for the determination of interface interaction or bond slip characteristics of composite filled steel members, researchers have used a variety of test methods. These include push-out tests on CFT, pure flexure tests and tests on I-girders framing into CFTs using simple shear-tab connectors. In this study, the interface interaction in filled steel members was simulated through flexural tests on open sandwich beams, so as to have direct access on both the fill and steel materials in the measurement of sectional strains and interface slip. Since there is no normal stress acting on the interface, this evaluation may be under estimation.





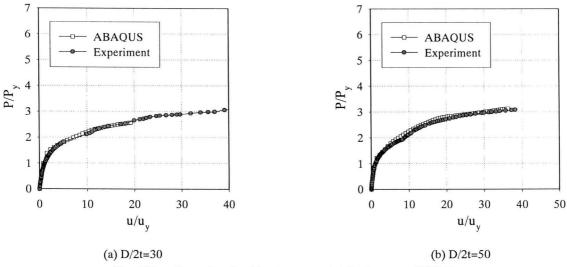


Fig. 6 Non-dimensionalized load versus axial displacement (S/E2A)

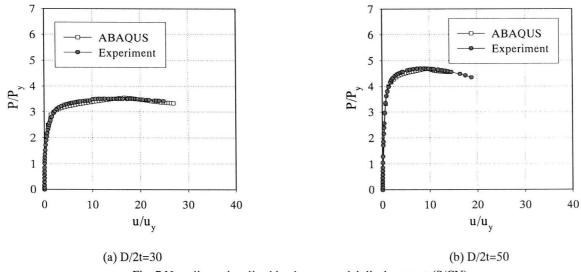
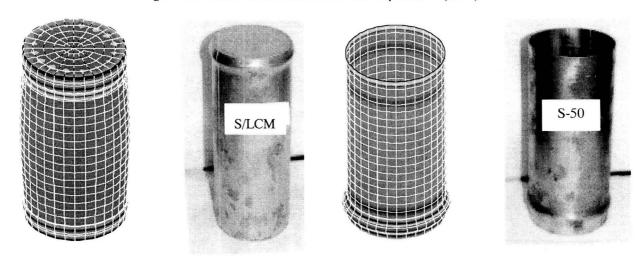


Fig. 7 Non-dimensionalized load versus axial displacement (S/CN)



(a) Filled steel stub column

Photo 1. Typical buckling of the stub columns with D/2t=50 (ABAQUS versus experiment)

#### 3.2 Buckling modes

The buckling modes of the specimens are shown in Photo 1, where the ABAQUS model simulates the experimental modes. Two types of buckling shapes are noted; localized buckling at the stub column ends, and global-like buckling in the middle region. From ABAQUS modeling, it was evident that the type of buckling was affected by the modulus of elasticity of the fill material and the bond characteristics. A high modulus of elasticity of the fill material improved the load-sharing ratio between the steel and the fill material, thus relieving the steel tube and eliminating localized buckling wave at the ends. Instead the highly stressed fill material of high modulus of elasticity exerted lateral pressure on the steel tube inducing hoop stresses, and resulting in bulging in the middle region.

#### 4. Parametric study

The analytical model, developed in conjunction with experimental results, was then utilized to study the effects of various fill material properties on the behavior of filled stub columns. Although the experimental study dealt with a variety of fill materials with different properties, it was difficult to distinguish the extent or contribution of any material property since the materials presented the combined effects of the properties. Hence parametric studies herein varied only the material property of interest while keeping all the other parameters constant. The variables investigated included fill material triaxial strength data, modulus of elasticity, and Poisson's ratio. In addition the effect of fill material-steel bond characteristic was also investigated. L/D ratio was maintained constant at 2.

For each type of composite column analyzed, loading was applied through a rigid plate by imposing incremental axial deformations monotonically until required deformation level had been attained or solution convergence was not possible. Results obtained were interpreted in the form of load-deformation behavior, with emphasis on stiffness, strength, ductility and softening gradient.

# 4.1 Effect of triaxial strength data of the fill material

Triaxial strength data in ABAQUS relates the lateral confining stress to the confined axial strength of the fill material. This data is used by ABAQUS to define a failure surface for the fill material, provided supplementally by compressive load test on the FSTs. The variation of triaxial strength data included four cases; very high strength, high strength, moderate strength and low strength, corresponding to triaxial strengths of epoxy concrete E1A, normal concrete CN, latex cement mortar LCM and epoxy concrete E2A, but

poisson's ratio is kept to be 0.25 which is similar to normal concrete. On the other hand, bond characteristics are also assumed to be the bond of LCM to steel.

Fig. 8 shows enormous differences resulting from different triaxial strength data. The highest strength is for the highest triaxial strength data, which was for fill material epoxy concrete E1A. On the other hand, the lowest strength is for the lowest triaxial strength data, which was for epoxy concrete E2A. No significant difference in ductility is seen for the different strengths, suggesting that other parameters influence ductility.

# 4.2 Effect of fill material modulus of elasticity

Modulus of elasticity of the fill material was varied from 1.0 KN/mm<sup>2</sup> to 30 KN/mm<sup>2</sup>, a range covering the low elastic modulus of polymers and the high elastic modulus of cement concrete, but the strength is kept to be equivalent to normal concrete. From Fig. 9, it is noted that modulus of elasticity of the fill material has a marked effect in the linear elastic range as would be expected. The lower the modulus of elasticity, the lower the initial stiffness. However, the impact of modulus of elasticity reduces as the ultimate state is approached, and further diminishes in the post-ultimate range. In the elastic range, modulus of elasticity determines the load sharing proportion between the steel and the fill material. If the modulus of fill material is very low, then most of the load is carried by the steel tube, which leads to early buckling of the steel tube. However, the effect of modulus of elasticity diminishes in the ultimate state region because at this state the steel will have yielded, redistributing most of its load to fill material.

The effect of modulus of elasticity can also be viewed in combination with Poisson's ratio, through the bulk modulus (K) which measures the degree of incompressibility of the material defined as; K = E/3(1-2v). A high modulus of elasticity implies high bulk modulus with a condition of small Poisson's ratio, which implies high incompressibility; a condition that promotes confined strength for pressure dependent fill materials like cement concrete.

## 4.3 Effect of fill material Poisson's ratio

Poisson's ratio  $\nu$  was varied to cover the range of possible fill materials i.e. from 0.150 to 0.490, but the strength is kept to be same as normal concrete. The effect of Poisson's ratio alone on stiffness and strength of stub column is quite moderate as given in Fig. 10, the extent of which depends on the modulus of elasticity of the fill material. High Poisson's ratio of fill material, at low modulus of elasticity, leads to high ductility of the stub column, that is feasible only for polymer-based materials.

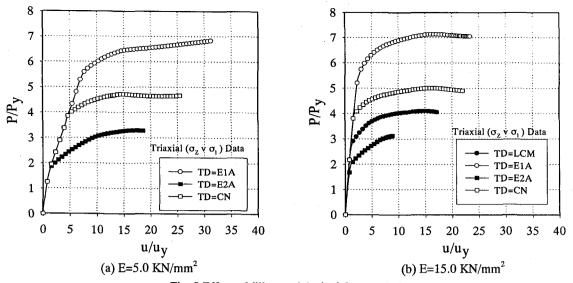


Fig. 8 Effect of fill material triaxial strength data

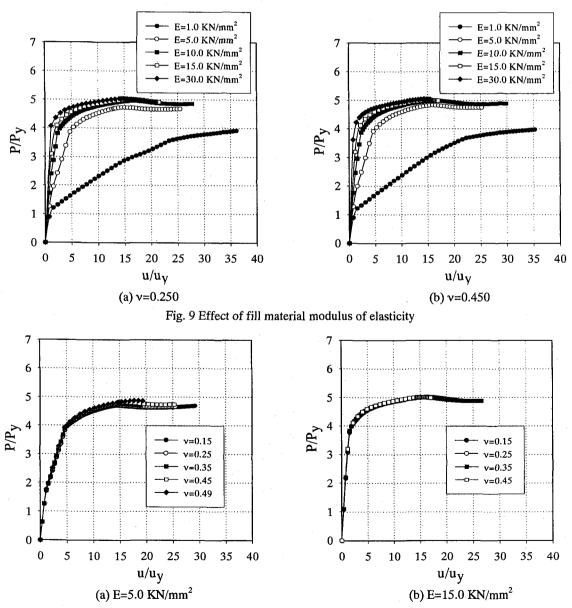


Fig. 10 Effect of fill material Poisson's ratio

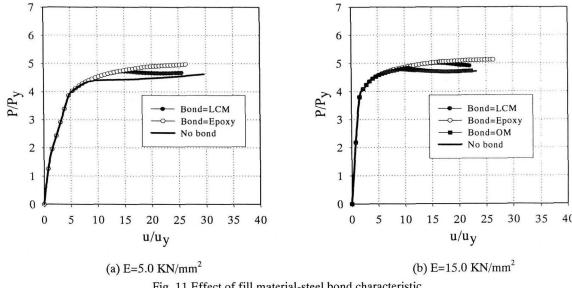


Fig. 11 Effect of fill material-steel bond characteristic

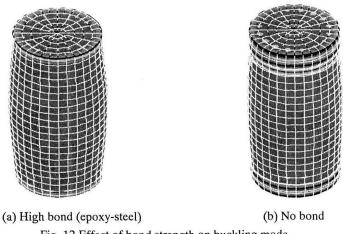


Fig. 12 Effect of bond strength on buckling mode

Poisson's ratio could also be analyzed when combined with modulus of elasticity in the form of shear modulus (G) given by the expression G = E/2(1+v) and bulk modulus K =E/3(1-2v). In this context, a very high Poisson's ratio would mean a low shear modulus or high flow capability and a high bulk modulus or high incompressibility, giving a behavior that tends to that of liquids such as exerting enormous pressure on the walls of the steel tube to expand outside. In experimental study, epoxy E2 was found to fall in this category.

#### 4.4 Effect of steel-fill material bond characteristics

Bond strength considered covered; latex cement mortar-steel, epoxy-steel, ordinary mortar-steel and the case of no bond between the fill material and the steel. But other properties are kept equal to those of normal concrete. The improved strength and ductility resulting from high bond strength, as that of epoxy-steel interface, is clearly evident in Fig. 11. Latex cement mortar-steel interface characteristic is also

seen to give high strength and ductility. On the other hand, the low bond strength between ordinary cement mortar and steel is seen to result in lower strength and ductility, punctuated with an abrupt drop in strength. In fact the bond strength of ordinary mortar-steel interface is seen to be equivalent to the case where no bond exists. High interface bond strength facilitates the transfer or sharing of loads and increases the buckling strength of the encasing steel tube as is manifested in the buckling mode of the specimens (Fig. 12). The high bond strength stub column has mainly a global-like buckling originating from the middle region of the column, while the low bond strength column has local buckling at the column ends.

#### 4. Conclusions

From the analytical studies conducted on filled steel stub columns, potential supplementary and complementary fill materials to normal cement concrete have been further identified in the form of latex cement mortar, epoxy polymers and epoxy polymer concretes, or other possible material design. Most importantly, it is determined that polymers and polymer-based materials present a wide array or variety of properties, unlike conventional cement concrete, which can be suitable tailored to meet specific construction requirements or needs. Through the wide range of properties offered by polymer-based materials, it is deduced that certain fill material properties have significant effect on the response of stub columns;

- (1) High triaxial or confined strength of the fill material leads to high stub column strength.
- (2) High modulus of elasticity of the fill material gives high elastic stiffness of the stub column.
- (3) High deformability or high Poisson's ratio, at low modulus of elasticity e.g. 5 KN/mm<sup>2</sup>, results in increased ductility.
- (4) Regardless of Poisson's ratio, extremely very low bulk modulus implying high compressibility such as the case for rubber, counteracts and delays the confining effect of steel tube on fill material, thus minimizing the strength benefit accrued from composite action.
- (5) High interface bond strength enhances both strength and ductility of the stub column due to the change in buckling wave length of steel tube.

This study only specifies the primary material properties to affect the strength and ductility of FSTs, not including strain hardening behavior of fill materials. The systematic parametric analysis considering various elasto-plastic relation and its quantitative evaluation is a future research need.

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