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Multi-directional Newmark Sliding Analysis with Compliant Materials

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The objectives of this study are consideration and verification of the dynamic response of the material above the sliding surface and the interaction of two components, and proposal of a new procedure of Newmark sliding analysis. In this study, the lumped mass-spring analogy is used to represent the flexibility of the sliding block. Further, circular influence curve for resistant strength (yield criteria) is used to the multi-directional movement of ground. The results of the proposed method show that the flexibility of the sliding block influences the permanent displacement and the tuning of the predominant frequency of the ground motion and the natural frequency of the soil layer is critical to the response. Also the multi-directional method in which the resisting strength changes due to the vector angle yields the larger displacement.

Key Words: Newmark sliding analysis, Flexibility, Multi-direction, Permanent displacement

1. Introduction

Newmark sliding block analyses (hereafter referred to as "original method" or "rigid block model") are widely used for estimation of permanent displacements of slopes in earthquake. However, the original method (1) neglects the dynamic response of the material above the sliding surface (uses a rigid (non-compliant) block), and (2) considers only a single direction, in other words neglects the multi-directional interaction.

Since Newmark[1] deduced the method in 1965 and it was named after him, Seed and Martin[2], Ambraseys and Sarma[3] and Makdisi and Seed[4] developed simplified procedures for predicting the permanent displacements of dams due to earthquake motion, by applying Newmark's sliding block concept.

Furthermore, to represent the flexibility (compliance) of the sliding block, Lin and Whitman[5] modeled the seismic response of an earth dam using a multi degree of freedom system consisting of vertical stacks of lumped masses connected by springs and viscous dashpots. A horizontal sliding element was introduced within the model at varying locations to simulate shallow, intermediate and deep sliding wedges. Gazetas and Uddin[6] used two dimensional finite element analyses with slip elements to investigate the de-coupling assumption. Kramer and Smith[7] developed a modified method by taking a multi degree of freedom system into Newmark concept, which is a simple method compared to the previous two methods.

Though various methods are proposed now, all of these points are not taken into account. Regarding to these points, (1) is partially considered by the above-mentioned studies and (2) is not considered until now. The purpose of this study is consideration and verification of these points and proposal of new procedure of Newmark sliding analysis. (In this study, the up-down component of motion is not taken into account.)

2. Method of Seismic Slope Analysis

2.1 Newmark method[1]

The post earthquake stability of a slope is fundamentally related to the permanent deformations that developed during earthquake. Newmark developed a simple procedure for estimation of permanent slope displacement due to earthquake shaking. This Newmark method assumes that the block starts to move, if the driving force F_D is greater than the resisting force F_R (See Fig. 1). By assuming the material above the failure surface to be rigid, Newmark showed that the seismic slope stability problem was analogous to the problem of a rigid block resting on an inclined plane. The driving force in the Newmark method is written as:

$$F_D = -M\ddot{u}_b + Mg \sin \alpha \qquad (1)$$

where M is the mass of the de-coupled block, u_b is the ground

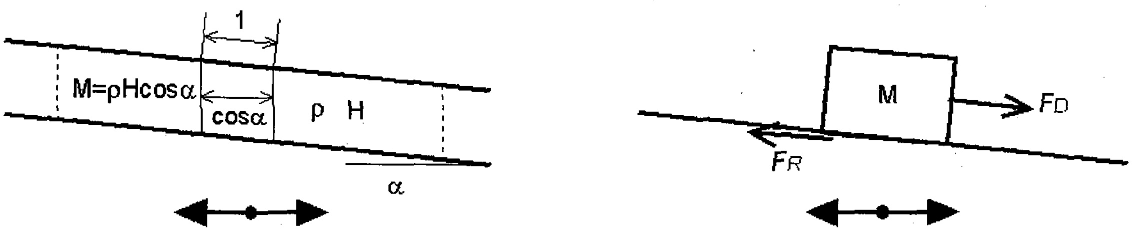


Fig.1 Schematic Image of Newmark model

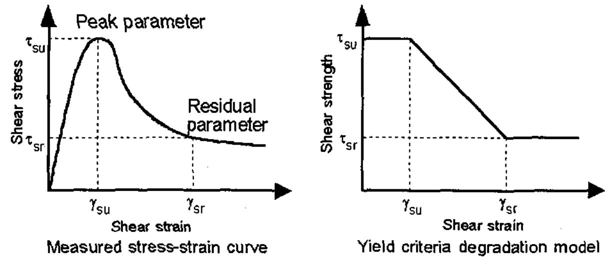


Fig. 2 Strain based yield criteria

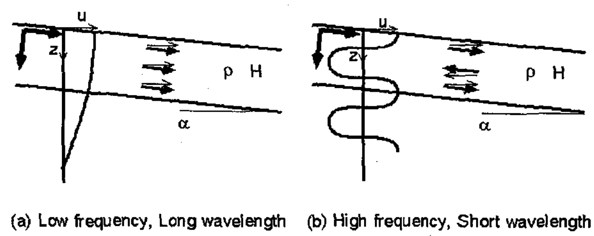


Fig. 3 Effects of failure mass compliance

motion along the slope, g is the acceleration of gravity and α is the slope angle. Regarding to the soil layer, the mass per unit length along the slope is given by

$$M = \rho H \cos\alpha = \frac{\gamma}{g} H \cos\alpha \qquad (2)$$

where ρ, γ and H are density, unit weight and thickness, respectively. Substituting Eq. (2) into Eq. (1), the driving shear stress based on the original method is expressed as follows.

$$\tau_D = -\frac{\gamma}{g} H \cos\alpha \cdot \ddot{u}_b + \gamma H \cos\alpha \sin\alpha \qquad (3)$$

In the case of infinite slope, the static shear stress due to the slope (gravity) may be considered with the effective stress (mass)[8]. So, Eq. (3) will be expressed as follow,

$$\tau_D = -\frac{\gamma}{g} H \cos\alpha \cdot \ddot{u}_b + \gamma' H \cos\alpha \sin\alpha \qquad (4)$$

and there is a resisting shear strength between the sliding surface and the block.

2.2 Yield criteria[9]

Regarding to the resisting shear strength, the results of vane shearing field test, which was done at some coastal sites, are used in this study. As shown in Fig. 2, these values were referred to peak and residual shear strength. Using these values, it is assumed that degradation of the initial value of resisting strength starts when the calculated strain (permanent displacement) reaches an initial threshold strain (displacement). For the coastal area and landfill, this initial threshold strain value corresponds to the peak of the stress-strain curve, as shown in Fig. 2. After reaching the initial threshold strain, the shear strength degrades linearly with increasing the calculated strain until the second threshold strain value. This second value corresponds to the strain at which the residual strength of the material is reached.

The above relations are expressed as followings: the shear strength τ_r is given a constant value of τ_{su} for strains ranging between 0 and γ_{su}, and a constant value of τ_{sr} for strains larger than γ_{sr} between strains of γ_{su} and γ_{sr}, τ_r decreases linearly. These equations are shown by Eq. (5). This yield criteria is used in the analysis from next chapter correspondent to change of the shear strain during calculation.

$$\tau_r = \begin{pmatrix} \tau_{su} : \gamma < \gamma_{su} \\ \dfrac{\tau_{su} - \tau_{sr}}{\gamma_{su} - \gamma_{sr}} * (\gamma - \gamma_{su}) + \tau_{su} : \gamma_{su} \le \gamma < \gamma_{sr} \\ \tau_{sr} : \gamma_{sr} \le \gamma \end{pmatrix} \qquad (5)$$

2.3 Properties of ground

The materials that comprise most slopes are compliant (flexible) rather than rigid. As an earthquake motion propagates through a slope, different parts of the slope move by different amounts and with different phases. The extent to which the compliance of the slope deviates from the original Newmark assumption of rigidity depends primarily on the relationship between the wavelength of the motion and the size of the potential failure mass. As shown in Fig. 3, for thin failure masses and/or long wavelength, the effects of failure mass compliance are likely to be small. For thick failure masses and/or short wavelength, the effect may be considerable. In such cases, motions within the potential failure mass will vary in amplitude and phase. Some different parts of the failure mass may be moving in different directions at a given instant in time. From next chapter, our new proposed method considering the flexibility of the materials is described.

3. Modified Newmark Sliding Analysis

In this chapter, the flexibility of the sliding block is considered. In the original method, it is assumed that the sliding block is rigid. However, if the soil layer on the sliding surface is very thick or very soft, the flexibility of the block influences the maximum and permanent displacement and it must be considered. Hereafter this method is referred to as "flexible block" model (or "compliant block" model or "modified method") in this paper.

3.1 Methodology

The lumped mass presentation is applied to the modeling of the shear deformation of the soil layer[10]. N layered system is converted to a lumped mass system (Fig. 4), by lumping one-half of the mass of each layer at the boundaries. The masses are connected by shear springs and dashpots, and the masses M_i (i=1, 2,$\cdots$, n) and the spring constants K_i (i=1, 2,$\cdots$, n) are expressed by

$$M_1 = \frac{1}{2}\rho_1 H_1 = \frac{1}{2g}\gamma_1 H_1\,, \quad M_{n+1} = \frac{1}{2}\rho_n H_n = \frac{1}{2g}\gamma_n H_n$$

$$M_i = \frac{1}{2}\left(\rho_i H_i + \rho_{i-1} H_{i-1}\right) = \frac{1}{2g}\left(\gamma_i H_i + \gamma_{i-1} H_{i-1}\right) \quad (6)$$

$$K_i = \frac{G_i}{H_i} \quad (7)$$

where ρ_i , γ_i , G_i and H_i are the density, unit weight, shear modulus and thickness of the i-th layer. Now taking the lumped mass model into the Newmark method, the sliding block is converted as shown in Fig. 5. Although single degree of freedom system is used in this paper, in the case that the layered system is thick or the soil properties are not uniform, multi-degree of freedom system could be applied to represent the dynamic response of the soil layer.

The forces acting on the system are shown in Fig. 6. I_0 and I_1 are the inertia force (inertia shear stress) acting on the lower and upper mass respectively, S is the force (shear stress) due to the shear spring, D is the force (shear stress) due to the dashpot, and W is the static force (gravity) and there is a resisting force (shear strength) between the lower mass and the sliding surface. Further, u_b is the base ground motion along the slope, u_0 is the relative displacement of the lower mass to the ground and u_1 is the displacement of the upper mass relative to the lower mass. Considering these forces, the equation of motion for the upper mass is

$$M_1\ddot{u}_1 + C\dot{u}_1 + Ku_1 = -M_1\left(\ddot{u}_b + \ddot{u}_0\right) \quad (8)$$

where M_1 = 0.5$\rho H\cos\alpha$ = 0.5$\gamma H\cos\alpha/g$, K = G/H. The shear stress driving the lower mass is

$$\tau_D = -\frac{\gamma}{2g}H\cos\alpha\cdot\ddot{u}_b + \gamma' H\cos\alpha\sin\alpha + \left(C\dot{u}_1 + Ku_1\right) \quad (9)$$

and the resisting shear strength is given by the field test results. The average acceleration method[11] is applied to the solution of Eq. (8) and it is processed with the Newmark method in the same time step during the calculation. The process after here is same with the original method. If the driving shear stress is greater than the resisting shear strength, the block starts to slide and continues to move until $\dot{u}_0 = 0$. If the spring constant $K = \infty$, the result will be same with that of the rigid model.

The parameters of the analytical model are shown in Table 1 and the shear modulus G that is necessary to determine the shear spring constant K is calculated by this equation[12],

$$G = 90\frac{(7.32-e)^2}{1+e}\sigma_0'^{\,0.6}\ (\mathrm{kN/m^2}) \quad (10)$$

where σ_0' is the effective stress at the middle of the soil layer and e is the void ratio and the coefficient of viscous damping C is based on the assumption of the damping constant h = 0.05, and is obtained by

$$C = 2h\sqrt{M\cdot K} \quad (11)$$

where M is the mass and K is the spring constant.

3.2 Comparison of two methods and parametric studies

In this paragraph, we verified the two models, named rigid block model and flexible block model. The parameters of the

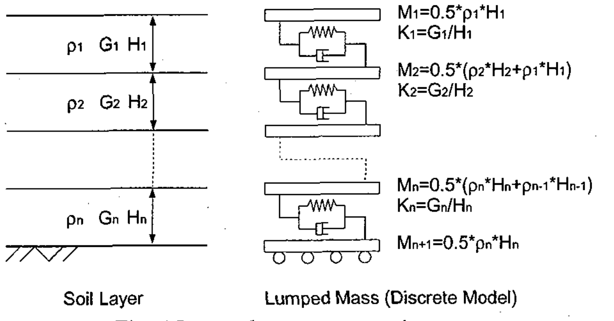


Fig. 4 Lumped mass presentation

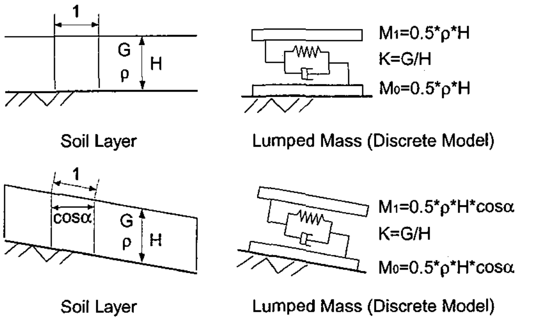


Fig. 5 Flexible block consideration

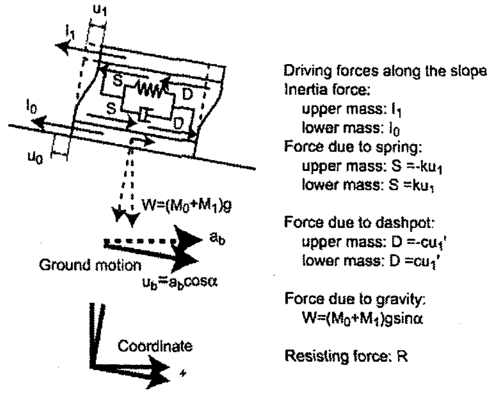


Fig. 6 Acting forces on the flexible block system

Table 1 Parameters of the ground

Thickness of soil layer: H	6.0m
Water table: dw	0.0m
Unit weight: γ	17.6kN/m^3
Slope angle: α	1.0 degree
Effective stress at middle of layer: σ_0'	23.5kN/m^2
Effective stress at bottom of layer: γ'H	47.0kN/m^2
Void ratio: e	2.5
Shear strength (peak): τ_{su}	18.8kN/m^2
Shear strength (residual): τ_{sr}	8.4kN/m^2

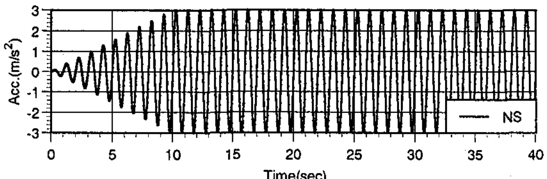


Fig. 7 Input motion (Harmonic wave, sine function)

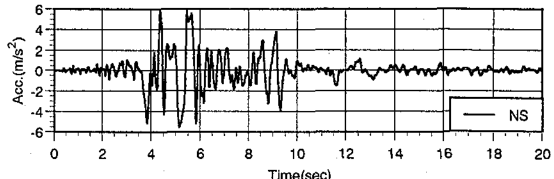


Fig. 8 Input motion (Newhall record, NS component)

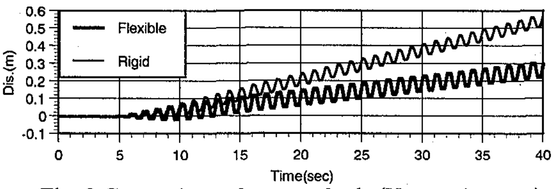


Fig. 9 Comparison of two methods (Harmonic wave)

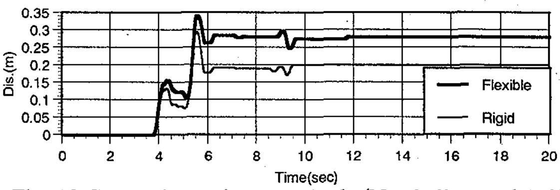
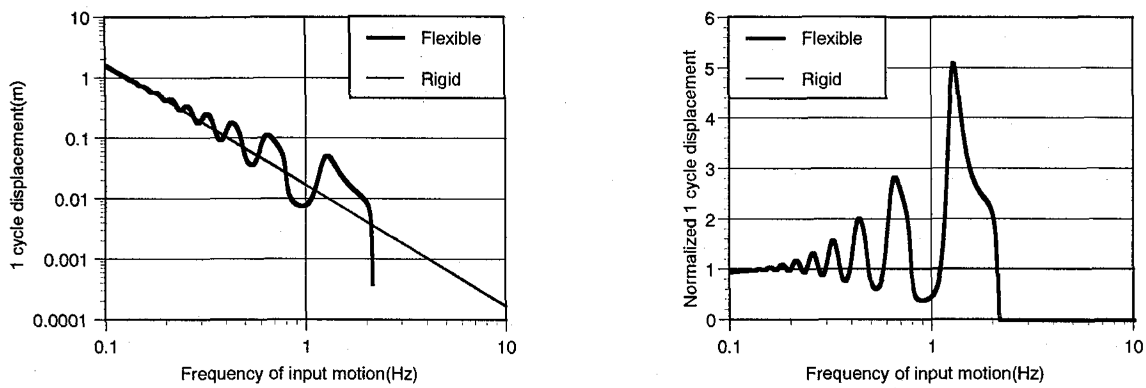


Fig. 10 Comparison of two methods (Newhall record, NS)



(1) 1 cycle displacement

(2) Normalized 1 cycle displacement

Fig. 11 Results of parametric study (harmonic wave)

analytical model from the typical ground condition of San Francisco Bay area are shown in Table 1. The input motions are harmonic wave and NS component of the Newhall record from the 1994 Northridge Earthquake (See Figs. 7 and 8). In the step by step calculation, $\ddot{u}_{0_{t-\Delta t}}$ is applied to Eq. (8) instead of $\ddot{u}_{0_t}$, because it can not be obtained. It will be calculated at time step t of the calculation.

The results shown in Figs. 9 and 10 are for the rigid and flexible method. These figures show that the results for flexible block model are different from those of the rigid model. It is obvious that the flexibility of the sliding block influences the maximum and permanent displacement. The permanent displacement would be changed according to the frequency characteristics of input motion.

In order to examine the relationship between the displacement and the frequency (0.1Hz - 10.0Hz) of the input motion (harmonic wave with a similar shape given in Fig. 7), parametric study is executed. Since the response acceleration of the single degree of freedom system (SDF) against harmonic wave fluctuates during the initial several cycles, the averaged displacements of the last 10 cycles are compared between two methods. Further, the displacements normalized by the results of the rigid block method are plotted against the frequency of input motion as shown in Fig. 11. The natural frequency of the soil layer is about 1.7 Hz for this analytical model. This figure shows that the resonance effects to the results of the flexible block method, and the permanent displacement are large when the predominant frequency of the ground motion approaches the

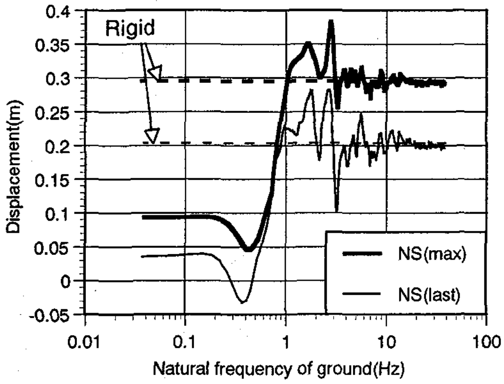


Fig. 12 Results of parametric study
(Newhall, NS)

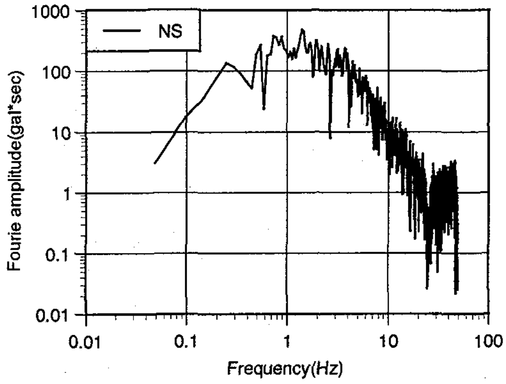


Fig. 13 Fourier spectrum
(Newhall, NS)

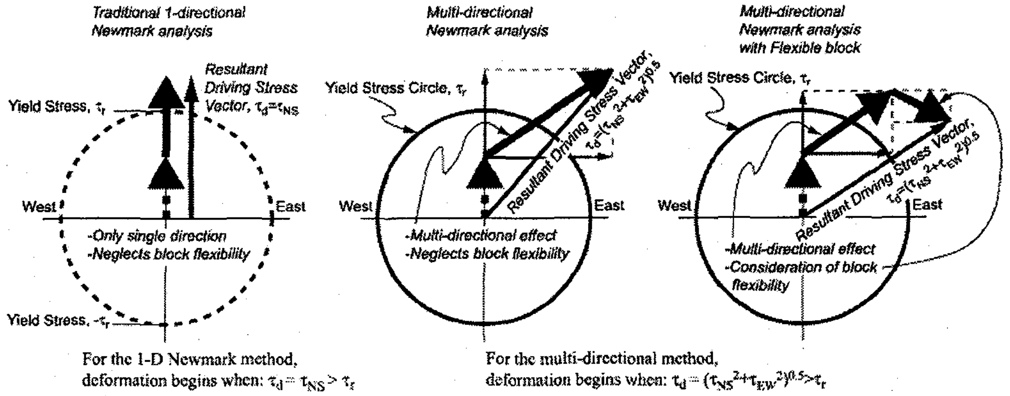


Fig. 14 Consideration of multi-directional method

natural frequency of the soil layer. Further, this figure shows several peaks due to change of frequency characteristics of the input motion to the upper block. After sliding, the mixed motion of ground motion and lower block motion is input to upper block, which is shown in Eq. (8). At that time, the mixed input motion has similar predominant frequency to 1.7Hz.

When the frequency of input motion is above 2.2 Hz, the displacement is zero. This corresponds to very soft (flexible) system in regard to the frequency of input motion. The response acceleration is reverse to the input motion ($M_1\ddot{u}_1 \approx -M_1\ddot{u}_b$), the response velocity and displacement are negligible ($C\dot{u}_1 + Ku_1 \approx 0$), the driving shear stress, except for the static stress due to the gravity, approaches half of the value produced by the rigid model. This is the reason why the displacement is zero. On the other hand, as the frequency decreases, the results for the flexible block method are close to those of the rigid block model. This is correspondent to so stiff system in regard to the frequency of input motion. The upper mass moves in unison with lower mass and this condition is same with the rigid model.

Another parametric study is done by using various shear moduli of the ground calculated by the Eq.(12) due to change of the shear wave velocity.

$$G = \rho V_S^2 = \frac{\gamma}{g} V_S^2 \qquad (12)$$

where ρ is the density, γ is the unit weight and V_S is the shear wave velocity. The input motion is the Newhall record shown in Fig. 8. The results shown in Fig. 12 depict the displacement against the natural frequency of the ground. The figure shows the maximum and final displacement calculated by the analysis. The Fourier spectrum of the input motion is shown in Fig. 13 revealing that the predominant frequency of input motion is about 1.43 Hz. From this figure, it is concluded that the displacement becomes large at the natural frequencies of the ground similar to the predominant frequency of input motion. As the ground becomes stiff, or the results get similar to those of the rigid model. This tendency observed in the previous study by the harmonic waves is confirmed to hold for earthquake input as well.

4. Multi - Directional Newmark Analysis

The transient ground motion is only in a single direction. However during earthquakes the ground shakes at any direction and multi-directional effect should be considered in the Newmark analysis. In the single directional method, if $\tau_x > \tau_r$ or $\tau_y > \tau_r$, the block starts to move. But in the multi directional method the block starts to move, if the composed driving shear stress $\sqrt{\tau_x^2 + \tau_y^2}$ is greater than the resisting shear strength τ_r (Fig. 14).

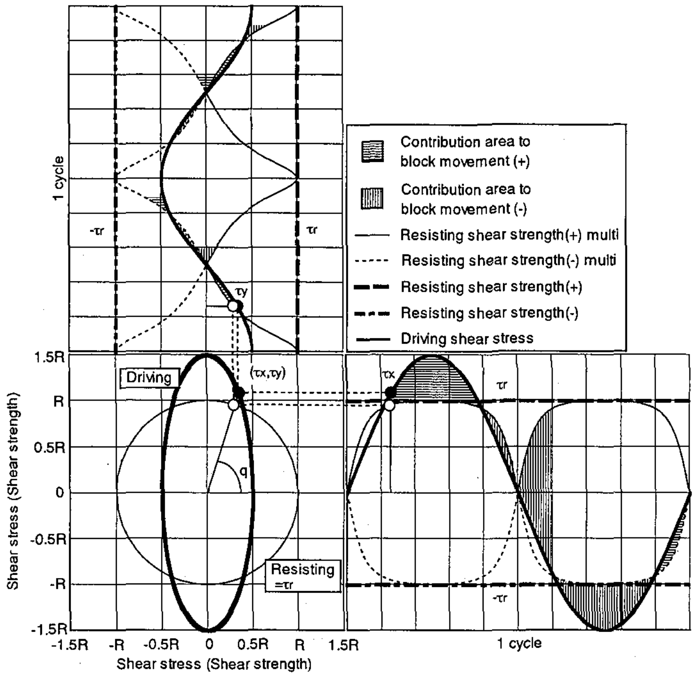


Fig. 15 Image of multi-directional effect

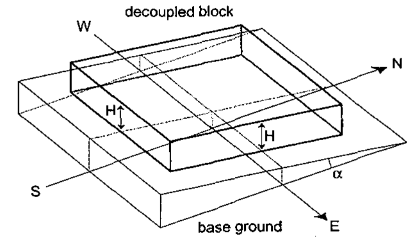


Fig. 16 Ground condition

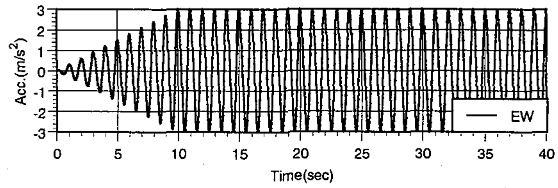


Fig. 17 Input motion (Harmonic wave, cosine function)

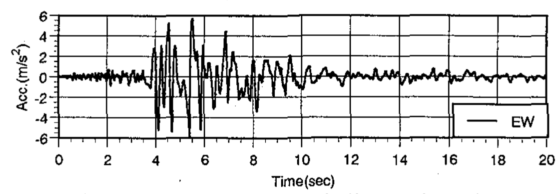


Fig. 18 Input motion (Newhall record, EW)

4.1 Methodology

In a simple example shown in Fig. 15, one cycle of the driving shear stress of a certain time history is discussed. For the multi-directional model, the area of contribution to the sliding is hatched in this figure.

Compared to the single directional method, the contribution of the resisting shear strength to each components changes according to the angle θ made by the two directions of driving shear stress. Further, one of the important points of the multi-directional analysis is shown here. When only x component of motion is examined, displacement does not occur at all because the driving shear stress (maximum: $0.5R$) does not exceed the resisting shear strength R. Similarly to the y component, it is obvious that the multi-directional effect is same.

Here, ground conditions shown in Fig. 16 are considered. In the NS direction where there might be slope, the driving shear stress is expressed by Eqs. (4) and (9) for the rigid and flexible block model, respectively. On the other hand, about the EW component, the driving shear stress is expressed by Eqs. (4) and (9) without the term of static shear stress. If the slope angle is zero, the two components of the driving shear stress are written by the same equation.

4.2 Multi directional method versus composed single directional method

In this paragraph, the difference between the multi directional and the composed single directional method is verified. Composed single directional method means composition of two components of the single directional method discussed at the previous two chapters.

The parameters of the analytical model are those in Table 1, and the shear modulus G for shear spring constant K and viscous damping C used in the flexible block model are identical with those used in the previous chapter. One of the input motions is harmonic wave. Its NS component is shown in Fig. 7 and Fig. 17 shows the EW component. The other is the Newhall record from the 1994 Northridge earthquake. Its NS component is shown in Fig. 8 and Fig. 18 shows the EW component.

For harmonic input motion, the displacement results of the multi-directional and composed single directional method are shown in Figs. 19 and 20, respectively. The block does not return to the original position because of the static stress due to the NS component of the gravity. However, in the composed single directional method, where both components do not influence each other, permanent displacement does not occur in the EW direction, which has no slope. In the multi-directional method, permanent displacement is generated, even if there is no slope in the EW direction. Therefore, the multi-directional effect must be considered.

The results for earthquake motion are shown in Fig. 21 and 22. These figures also show the results of the multi- directional method are quite different from those of the composed single directional method. The former method calculates larger displacement than the latter for both models on the given ground condition. It is concluded that the multi directional method, in which the contribution of the resisting shear strength to each components changes (basically decreases) due to the vector angle, would produce the largest displacement.

4.3 Parametric study by harmonic wave in multi-directional method

The rigid and flexible block models are examined and

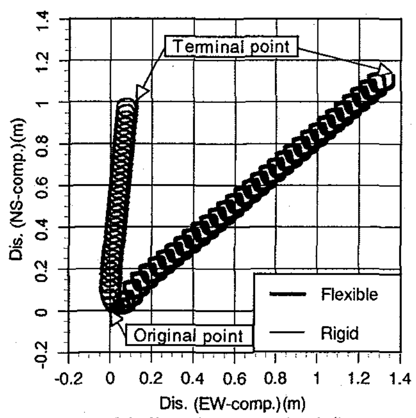


Fig. 19 Results of multi-directional method (harmonic wave)

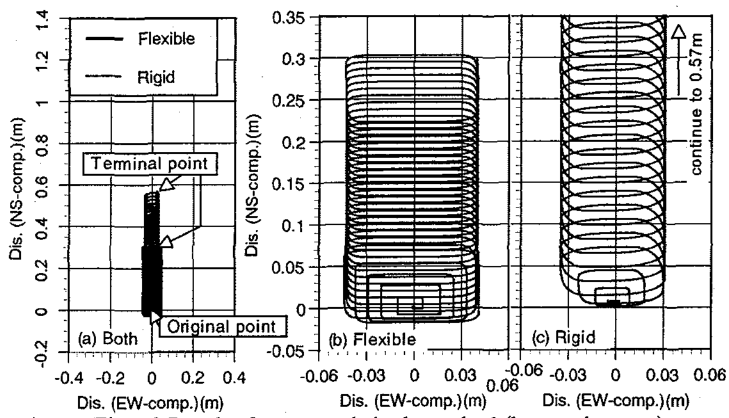


Fig. 20 Result of composed single method (harmonic wave)

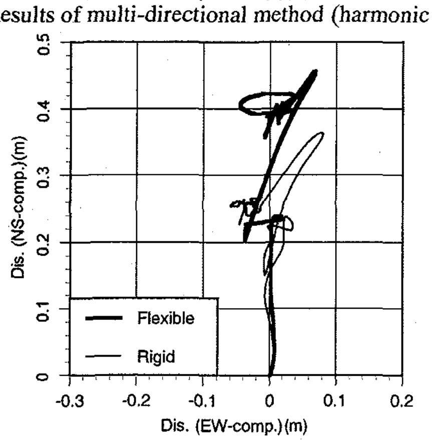


Fig. 21 Results of multi-directional method (Newhall record)

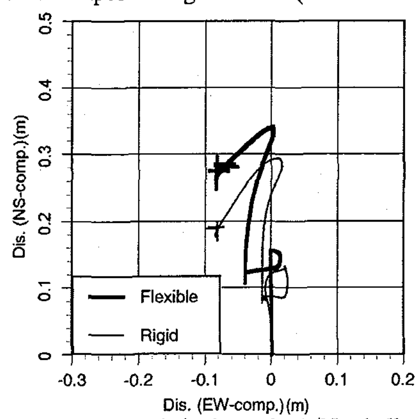


Fig. 22 Result of composed single method (Newhall record)

compared here. For the input motion in the NS direction (Fig. 7), the sine function is selected and the cosine function is applied for the EW component (Fig. 17) as in the previous paragraph. The analytical model of the ground is the one used in the previous paragraph (Fig. 16 and Table 1).

In order to examine the relationship between the displacement and the frequency of input motion, parametric study is performed in the same manner as for the single directional method study discussed before. One cycle of displacement after the steady condition is compared for the two models, and further the displacement normalized by the results of the rigid block model against the frequency is shown in Fig. 23.

The resonance effects on the results of the flexible block model are similar to those of the single directional calculation. This shows that the permanent displacement is large when the predominant frequency of the ground motion is close to the natural frequency of the soil layer.

When the frequency of input motion is above 2.2, the displacement is zero. On the other hand, as the frequency decreases, the results of flexible block model are close to those of the rigid block model. These tendencies are also similar to those revealed by the single directional method.

4.4 Parametric study by the multi-directional method using earthquake input

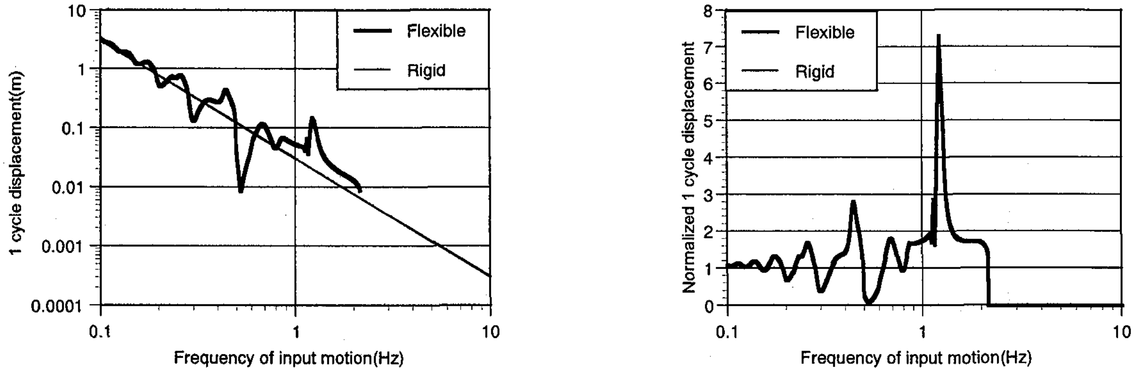
The Newhall record is applied as input motion and various shear stiffness of ground is used. As in the single direction, the relationship between the displacement and the natural frequency of the ground is examined. The maximum and final displacements are compared. The results are shown in Fig. 24. The Fourier spectrum of the EW component is shown in Fig. 25. The predominant frequency of this motion is at 1.67 Hz. So, considering the NS component, the range of dominant frequencies is 1.4 to 1.7 Hz.

In Fig. 24, the displacement has a peak at the natural frequency which is a little different from the predominant frequency of input motion. However, it is evident that the displacement becomes large at around a natural frequency of ground similar to the predominant frequency of input motion. The tendency from previous studies still holds.

5. Conclusions

This study introduces multi-directional Newmark sliding analysis with compliant materials to seismic stability of slopes and shows that the multi-directional effect and the flexibility of the decoupled block should be considered. It is thought that the proposed method is quite simple and applicable to engineering use. The main conclusions can be summarized as followings.

1) The results of the flexible block method are very different from those of the rigid method. It is obvious that the flexibility (compliance) of the sliding block influences the



(1) 1 cycle displacement (2) Normalized 1 cycle displacement

Fig. 23 Results of parametric study (multi-directional method, harmonic wave)

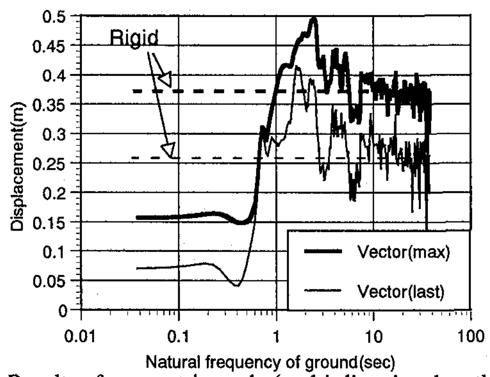


Fig. 24 Results of parametric study (multi-directional method, Newhall)

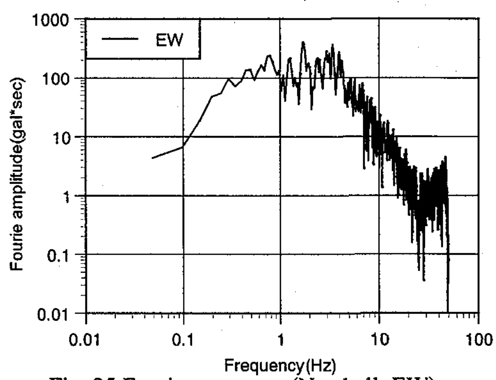


Fig. 25 Fourier spectrum (Newhall, EW)

permanent displacement and resonance effects the results of the flexible block method.

2) Tuning of the predominant frequency of the ground motion and the natural frequency of the soil layer is critical and the permanent displacement becomes large.
3) Since both components of motion do influence each other (for example, permanent displacement is generated during the harmonic input motion, even if there are no angles in that component), the multi-directional effect must be considered.
4) The results of the multi directional method are quite different from those of (composed) single directional method at almost all cases in this study, and it is considered that the former method produces different result from the latter.
5) Furthermore, the multi directional method in which the resisting shear strength changes (basically decreases) due to the vector angle would produce the larger displacement.

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