Statistical Damage Localization and Assessment of Frame Structures from Static Test Data

Jing CAI* Zhishen WU**

*Dr. Cand. of Eng., Dept. of Urban & Civil Eng., Ibaraki Univ., Nakanarusawa-cho 4-12-1, Hitachi Ibaraki, 316-8511
Dr. of Eng., Assoc. Prof., Dept. of Urban &Civil Eng., Ibaraki Univ., Nakanarusawa-cho 4-12-1, Hitachi Ibaraki, 316-8511

In this paper we develop two algorithms for identifying the member constitutive parameters of a structure with determined geometry and topology from static test data. The proposed algorithms are based on the concept of minimizing an index of discrepancy between the analytical model and the real structure, the first one using the errors in displacements at the measurement sites and the second one using the error in nodal forces. The mathematical model is set up as a nonlinear least-squares minimization and is solved with Gauss-Newton method. By using Monte Carlo method, the effectiveness and stability of two algorithms are investigated in detail when the measured data are polluted by noise. The first algorithm is distinguished by that the sensitivity matrix is not affected by the measurement error of displacements. The identified constitutive parameters are at the element level, and a statistical approach by a hypothesis test is introduced to locate and assess damage of elements. Numerically simulated examples are presented to demonstrate the validity and accuracy of the SI-based damage assessment algorithm.

Key Words: constitutive parameter, sensitivity matrix, system identification, damage assessment

1. Introduction

Damage due to an extreme event is possible in a structure at some time during its design life. A well-designed structure may survive a damaging event, but its function could decreases greatly, and then its safety cannot be guaranteed. Damage assessment methods based on system identification (SI) techniques have been investigated to detect damage in structural systems during the last decade (Sanayei and Onipede[1], Hajela and Soeiro[3]; Hjelmstad and Shin(1997) [8], Yeo. and Shin[10]. A SI-based damage assessment algorithm consists of two parts, (1) system identification; (2) damage assessment. First, the constitutive properties of given structure are estimated by a SI algorithm, and then damage of a structure is located by comparing changes of those properties of the structure. Therefore, a stable SI algorithm is essential for a reliable damage assessment. It will be reasonable to say that SI lays an important foundation for the establishment of theory of maintenance.

Considerable researches have been performed in the area of parameter estimation and are divided into two major categories: dynamic and static according to the properties of the measured test data. The intent of parameter estimation is to adjust the parameter of the analytical finite element model (FEM) to match the real structure with measured data.

Although there have been successful examples of dynamic parameter identification in civil engineering, there are several disadvantages to this kind of methods. Firstly, a large amount of dynamic data is needed to derive an accurate response of the structure. In many cases, an estimated damping matrix must be used, which induces error into the system identification. Finally, the identification process usually does not occur at the element level, so we cannot determine the damage location.

Now researchers pay more and more attention on static parameter estimation. There are two ways to measure the discrepancy between the analytical model and the real structure. The first one is displacement error estimator and the second one is force error estimator. Based on them, several methods have been proposed in Ref.[1], [3], [5], [8],

[10]. From those papers we can see the main difference among those methods is to choose which estimator to express the discrepancy between the analytical model and the real structure, how to deal with incomplete measurements or measurement sparsity problem and then what schemes are used to solve the minimization problems. In the paper of Banan and Hjelmstad[5], the unknowns comprise both constitutive parameters X and unmeasured displacements. Thus the number of unknown parameters increases and the stability of calculating process decreases. Sanayei and Onipede[1] proposed the algorithm in which it condensed the unmeasured displacements, whose limitation is that the degrees of freedom of measured displacements are fixed in all load cases and didn't discuss the problem of measurement error. The measurement error and measurement sparse is still two main problems. Although output-error indicates the errors of whole calculation process, it mainly comes from input-error and the problem about the relationship between input-error and output-error remains.

In this paper, first we develop two identification algorithms based on two estimators respectively. By using Euler Coordinate, the system stiffness matrices in the global coordinate can be linearly parameterized in terms of kernel matrices that have solid physical basis and easy to be assembled. Then by using Monte Carlo simulation, we simulate the input data with measurement error and introduce several statistical indices as criteria to compare the solutions investigate its relationship between input-error and output-error from two algorithms in detail. After better one is determined, we assume the baseline distribution for the system parameters and apply hypothesis test to locate and assess damage of elements based on its results.

2. Formulations of Static Parameter Estimation

2.1 Governing Equilibrium Equations

Consider a structure is variously subjected to nlc static load cases. Each case of forces should be neither equal to any other case nor a linear combination of the previous cases of applied forces. Through the finite element method, the force-displacement relationship of a linear, finite-element model of the structure under the *i*th load case is described as:

$$\{f_i\} = [K(p)] \cdot \{u_i\} \quad i=1,2,......\text{nlc} \qquad (1)$$

where $\{f_i\}$ (N×1) is the vector of applied forces for the *i*th load case, $\{u_i\}$ (N×1) is the corresponding response vector at N degrees of freedom in the finite element model of system, $[K(p)]$ is the structural stiffness matrix, and nlc is the number of load cases in a load set.

To perform parameter estimation, the unknown parameters are selected to be identified variables and the rest of the parameters is assumed to be known with a high level of confidence. If nup stands for the number of unknown parameters, so the dimension of constitutive parameters $\{p\}$ is nup×1.

2.2 Two Kinds of Error Estimators

In the identification problem, we need an index to express the discrepancy between the measured data of a real structure and the calculated data from the analytical model. There are two kinds of definitions of the error functions which are illustrated in Fig.1.

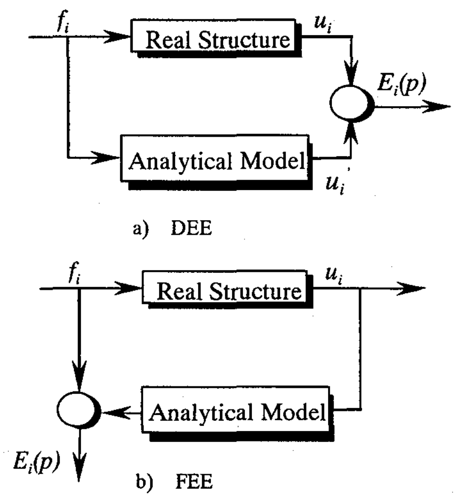


Fig. 1 Two kinds of error definitions

The first one is about nodal displacement. We define the discrepancy between measured of real structure and the calculated displacements of analytical model as a error function of constitutive parameter of system $\{p\}$, and it is called as the displacement error estimator(DEE), or output error estimator.

$$\{E_i(p)\} = [K(p)]^{-1} \cdot \{f_i\} - \{u_i\} \qquad (2)$$

The second one is about nodal equilibrium, in which the error function is defined as the discrepancy between the applied nodal forces and the calculated nodal forces according to the analytical model. Therefore it is called as the force error estimator (FEE).

$$\{E_i(p)\} = [K(p)] \cdot \{u_i\} - \{f_i\} \qquad (3)$$

where $\{E_i(p)\}$ are the error vectors under ith load, and they are the function of unknown constitutive parameters of the structural system. If the parameters $\{p\}$ are exactly captures the properties of the system, then both of $\{E_i(p)\}$ will be zero, otherwise they will not be zero.

For nlc load cases, there are nlc pairs of $\{f_i\}$ and $\{u_i\}$. So there are nlc error vectors $\{E_i(p)\}$. These error vectors can be concatenated vertically into a new vector $\{E(p)\}$. It is as follows:

$$\{E(p)\} = \begin{Bmatrix} \{E_1(p)\} \\ \vdots \\ \{E_i(p)\} \\ \vdots \\ \{E_{nlc}(p)\} \end{Bmatrix} \qquad (4)$$

where the dimension of the vector $\{E(p)\}$ is $(n \times nlc) \times 1$.

2.3 Incomplete measured test data

As for a large-scale system, it is hard to measure the complete response of a structure (e.g. when part of the structure is inaccessible). In fact, not all displacements need to bee measured. To help resolve this inherent problem, we partition the displacement vector $\{u_i\}$ into two parts in each case as follows:

$$\{u_i\} = \begin{Bmatrix} \{u_{ai}\} \\ \hline \{u_{bi}\} \end{Bmatrix},$$

where u_{ai} is the part of measured displacements, u_{bi} is the part of unmeasured ones, and it can be described as the follows:

$$\{u_{ai}\} = [Q_i]\{u_i\}$$

where $[Q_i]$ is a ith Boolean matrix that extracts the measured response u_{ai} from the complete displacement vector u_i. We can assume that the partitioning is different for the different load cases. Corresponding with this division of displacements, the applied force vector is also divided into two parts and the stiffness matrix is divided into four parts. Then Eq. (1)can be rewritten as:

$$\begin{Bmatrix} \{f_{ai}\} \\ \{f_{bi}\} \end{Bmatrix} = \begin{bmatrix} [k_{aa}] & [k_{ab}] \\ [k_{ba}] & [k_{bb}] \end{bmatrix} \cdot \begin{Bmatrix} u_{ai} \\ u_{bi} \end{Bmatrix} \qquad (5)$$

After being condensed out the unmeasured displacements $\{u_{bi}\}$, the Equ. (5) can be rewritten as follows:

$$\{f_{ai}\} = \left[[k_{aa}] - [k_{ab}]\cdot[k_{bb}]^{-1}\cdot[k_{ba}]\right]\cdot\{u_{ai}\} + [k_{ab}]\cdot[k_{bb}]^{-1}\cdot\{f_{bi}\} \qquad (6)$$

Therefore, based on the DEE, the displacement error function $\{E_i(p)\}$ is expressed as:

$$\{E_i(p)\} = Q_i[K(p)]^{-1}\{f_i\} - \{u_{ai}\} \qquad (7)$$

On the other hand, based on FEE,we have:

$$\{E_i(p)\} = \left[[k_{aa}] - [k_{ab}]\cdot[k_{bb}]^{-1}\cdot[k_{ba}]\right]\cdot\{u_{ai}\} + [k_{ab}]\cdot[k_{bb}]^{-1}\cdot\{f_{bi}\} - \{f_{ai}\} \qquad (8)$$

Here the component of error vector $\{E(p)\}$ in Equ.(7), (8) is a nonlinear function of unknown parameters to be estimated $\{p\}=[p_1\ p_2\ \dots\ \dots\ p_{nup}]^T$. The dimension of $\{E(p)\}$ is $\sum_{nlc} n_i$, where n_i is the number of measurements at ith load case. We adopt the scalar of error vector as the criterion of judgment.

$$J(p) = \{E(p)\}^T\{E(p)\} \qquad (9)$$

In that case, the smaller the $J(p)$, the better accuracy of fitting we will get. The essence of parameter estimation is to find a set of parameters, which can minimize the difference $J(p)$. Although the structure is linear, because of the inversion of matrices, they change into nonlinear problems. Now they turn into an optimal problem about parameters. The mathematical model of this problem is

to find $\{p_i, i=1,2, \dots, nup\}$

make $J(p) = \{E(p)\}^T\{E(p)\} \to \min$

subject to $\{x_{i1} \le p_i \le x_{i2},\ i = 1,2,\dots,nup\}$

where x_{i1} and x_{i2} are lower bounds and upper bounds of unknown parameters respectively. The former is assumed to be zero and the latter is three times of the true values. The bounding constraints ensure that the parameters will not become negative or too large.

2.4 Estimation Algorithm

At the same time, this is a nonlinear least square problem. We will adopt Gauss-Newton method to solve this problem.

Assume the initial value of unknown parameters {p} as $\{p_0\}$ and $\{p\}=\{p_0\}+\{\triangle p\}$, then the problem of solving {p} can be settled by determining the corrector vector $\{\triangle p\}$.

Let the error vector $\{E(p)\}$ be deployed around $\{p_0\}$ according to Taylor series and omit the second and higher order terms. We get,

$$\{E(p)\}\approx\{E(p_0)\}+[S(p_0)]\cdot\{\Delta p\} \tag{10}$$

where,

$$\{E(p_0)\}=\{E(p)\}\big|_{\{p\}=\{p_0\}}$$

$$[S(p_0)]=[S(p)]\big|_{\{p\}=\{p_0\}}$$

in which $[S(p)]=\left[\dfrac{\partial\{E(p)\}}{\partial\{p\}}\right]$ is formed through differentiating $\{E(p)\}$ with respect to each unknown parameter. $[S(p)]$ is mathematically called Jacobi matrix. Here it would be rather called a Sensitivity Matrix following Sanayei et al.[1]. The dimension of the sensitivity matrix is $(\sum_{nlc} n_i)\times nup$, and nup stands for the number of unknown parameters. If we divide jth column of matrix $[S(p)]$ into nlc parts, according to the number of load cases, for the case of DEE, the ith part can be described as:

$$\{S_{ij}(p)\}=\left[-[Q_i][K]^{-1}\frac{\partial[K]}{\partial p_j}[K]^{-1}\{f\}_i\right]$$

$$j=1,2,\ldots\ldots nup \tag{11}$$

and for the case of FEE, we have:

$$\begin{aligned}\left[S_{ij}(p)\right]=&\Big[\frac{\partial[k_{aa}]}{\partial p_j}-\frac{\partial[k_{ab}]}{\partial p_j}[k_{bb}]^{-1}[k_{ba}]\\&-[k_{ab}][k_{bb}]^{-1}\frac{\partial[k_{ab}]}{\partial p_j}\\&+[k_{ab}][k_{bb}]^{-1}\frac{\partial[k_{bb}]}{\partial p_j}[k_{bb}]^{-1}[k_{ba}]\Big]\{u_{ai}\}\\&+\Big[\frac{\partial[k_{ab}]}{\partial p_j}[k_{bb}]^{-1}-[k_{ab}][k_{bb}]^{-1}\frac{\partial[k_{bb}]}{\partial p_j}[k_{bb}]^{-1}\Big]\{f_{bi}\}\end{aligned}$$

$$j=1,2,\ldots\ldots nup \tag{12}$$

Based on minimum criterion, we can get the following equations,

$$\frac{\partial J(p)}{\partial\{p\}}=\{0\} \tag{13}$$

From Equ.(9), Equ.(10), Equ.(13), we get

$$[S(p_0)]^T\left[E(p_0)+[S(p_0)]\{\Delta p\}\right]=\{0\} \tag{14}$$

Now they have changed into linear equations, we can get:

$$\{\Delta p\}=-\left[[S(p_0)]^T[S(p_0)]\right]^{-1}[S(p_0)]^T\{E(p_0)\}$$

$$\{p\}=\{p_0\}+\{\Delta p\} \tag{15}$$

Again by assuming the current $\{p\}$ as the initial value, a new corrector vector $\{\triangle p\}$ and $\{p\}$ can be achieved. This process can be repeated until the required accuracy is reached.

In order to identify a unique set of parameters from a given set of measurements, the number of independent measurements must be great than or equal to the number of unknown parameters. If the aforementioned condition does not hold, there may exist an infinite number of values of parameters that satisfy these measurements [1].

Besides, two criteria are chosen to check the algorithm for convergence. The first one is about the changes in the scalar error function, $J(p)$ and the second one is about changes in the parameters, $p_{i,n+1}/p_{i,n}$, where n is the iteration number. As to measure the goodness of fit between the real structure and analytical model, the first one is more suitable. Tolerance limits are set for two criteria. When any of the limits are reached the algorithm is considered to have converged. These limits can be used to control the desired accuracy in the identified parameters.

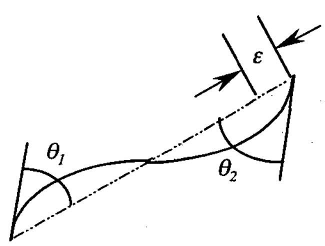
2.5 Parameterized Model

In order to use the above parameter estimation algorithms, we must specify a parameterized model of our system. The frame structures are widely used in civil engineering. Let us assume that the topology of the frame structure is known, and consider that each element or member of the structure possesses its own independent parameters. Firstly, the displacements and deformations of a beam element can be described schematically as Fig. 2 shows.

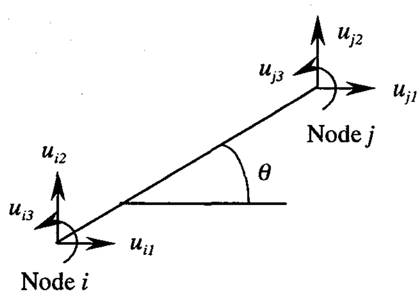
In the Euler coordinate as shown in Fig. 2(a), we can let the element constitutive relationship be expressed in the form:

$$[D_m(p)] = \sum_{l=1}^{M_m} P_{lm}[D_{lm}] \tag{16}$$

Where M_m is the total number of parameters associated with element m, P_{lm} is constitutive parameter, D_{lm} is parameter-independent constitutive kernel matrices, and D_m is characteristic matrix of element m.



a) Deformations of the element in Euler coordinate



b) Nodal displacements in global coordinate

Fig. 2 Two kinds of coordinates

For the problems of frame structure, there are two independent parameters, namely, $M_m = 2$, tensile stiffness EA, and bending stiffness EI. As to element m, we use l_m to express its length and we can get

$$P_{1m} = {(EA)_m}/{l_m}, \quad P_{2m} = {(EI)_m}/{l_m}$$

$$D_{1m} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad D_{2m} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 4 & 2 \\ 0 & 2 & 4 \end{bmatrix}$$

$$(axial) \qquad\qquad (flexural)$$

In finite element method, we use global coordinate. Two kinds of coordinates is shown schematically in Fig. 3 and their relationship between them can be expressed as :

$$\{a_m\} = [B_m]\{u_m\}$$

Where a_m and u_m is the deformation and displacement vector of element m in Euler coordinate, global coordinate respectively. B_m is called as translative matrix, and is derived as follows:

$$[B_m] = \begin{bmatrix} -\cos\theta_m & -\sin\theta_m & 0 & \cos\theta_m & \sin\theta_m & 0 \\ -\dfrac{\sin\theta_m}{l_m} & \dfrac{\cos\theta_m}{l_m} & 1 & \dfrac{\sin\theta_m}{l_m} & -\dfrac{\cos\theta_m}{l_m} & 0 \\ -\dfrac{\sin\theta_m}{l_m} & \dfrac{\cos\theta_m}{l_m} & 0 & \dfrac{\sin\theta_m}{l_m} & -\dfrac{\cos\theta_m}{l_m} & 1 \end{bmatrix}$$

Therefore the element stiffness matrix can be derived as follows:

$$[k_m(P)] = \sum_{l=1}^{M_m} P_{lm}[B_m]^T[D_{lm}][B_m]$$

According to the stiffness summation rule, we can get:

$$\begin{aligned} [K(p)] &= \sum_{m=1}^{N_m} [\lambda_m][k_m(p)] \\ &= \sum_{m=1}^{N_m}\sum_{l=1}^{M_m} [\lambda_m]p_{lm}[B_m]^T[D_{lm}][B_m] \\ &= \sum_{m=1}^{N_m} p_{1m}[\lambda_m][B_m]^T[D_{1m}][B_m] \\ &\quad + \sum_{m=1}^{N_m} p_{2m}[\lambda_m][B_m]^T[D_{2m}][B_m] \end{aligned} \tag{17}$$

Where N_m is the number of the elements, and $[\lambda_m]$ is the location matrix of element m. When the structure is given, $[B_m]^T[D_{lm}][B_m]$ will be a constant matrix depending on element geometry only. The parameterized finite element model is formulated by decomposing the stiffness matrix into constitutive parameters and constant matrices for each element.

3. Statistical Parameter Estimation

The formulas above don't involve any measurement errors. Practically, measured data always contain certain levels of noise. Those noises include not only true measurement errors, but also those caused by difference in support and connection between the actual structure and assumed model. The modeling error, as it known, may also include other effects such as manufacturing inconsistencies, residual or thermal stresses, or material flaws. Modeling error is not the topic of this discussion, and is not considered in this paper.

Now we only deal with the errors of input data that consist of force vectors and displacement vectors. If the force is applied on only one dimension of the freedom space at one time, then except for that forced dimension, all other dimensions apparently will have zero force vector components. On the other hand, the displacement will be formed along all dimensions. As a result, the force can be assumed as having no errors in the process of analogical (simulating) measuring and only displacement vector is added with noise.

3.1 Modeling of Input Error

Although we consider that there are noises only in the measured displacements, it is difficult, if not impossible, to mathematically model measurement noise. However, for numerical experimentation, we can simulate them by varying the calculated displacement measurement values slightly as shown in Fig. 3 with a known probability distribution.

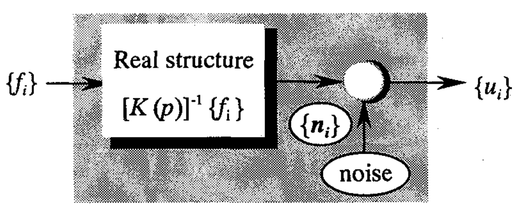


Fig. 3 Simulated measured displacements

The most commonly used distribution is the normal distribution which represents a higher probability of noise level closer to the mean, and a lower probability of larger noise. To range the distribution, it is convenient to use a 95% confidence interval. Therefore, $k_{\alpha/2}$, or I_e equals to 1.96 σ with 95% confidence. Namely, the variance of measurement noise is

$$\sigma^2 = \frac{I_e^2}{1.96^2} \approx \frac{I_e^2}{4} \tag{18}$$

Having determined the distribution parameterized on I_e, Equ.(18) is used to generate a set of normal random vector $\boldsymbol{n}_i$ with a mean of zero and deviation σ. Then,

$$\{u_i\} = [Q_i][K(p)]^{-1}\{f_i\} + \{n_i\} \tag{19}$$

where $\{p\}$ is the vector of true values of real structure. The response $\{u_i\}$is taken as the measured response of the real structure.

3.2 Statistical Indices

In a noisy environment, the parameters we are estimating behave as random variables. Through Monte Carlo simulation we produce a sample of random solutions $\{p\}$ from noisy data whose statistics are completely known to us. To study our proposed estimation algorithms and to find trends in the behavior of these estimators, we will use statistical indices to characterize our results.

For noisy response $\{u_k$, k=1,2, ..., $NOBS$ $\}$, the estimation simulation develops a sample $\{p_{j,k}$, k=1,2, ..., $NOBS\}$ for every variable, where $p_{j,k}$ represent the Kth observation of parameter j and $NOBS$ stands for the number of observations. So the sample size of every variable is $NOBS$. By increasing the sample size NOBS and using the method of Maximum Likelihood, the mean and deviation of the sample converge to the mean and deviation of the population.

Taking $p_{j,t}$ to be the true value of the parameter j, the percentage error of the kth observation of the jth parameter is

$$E_{j,k} = 100\frac{p_{j,k} - p_{j,t}}{p_{j,t}} \tag{20}$$

The mean and deviation of the percentage error of the jth parameter is

$$ME = \frac{1}{NOBS}\sum_{k=1}^{NOBS} E_{j,k} \tag{21}$$

$$SD = \sqrt{\frac{1}{NOBS}\sum_{k=1}^{NOBS}(E_{j,k} - ME)^2} \tag{22}$$

For each unknown parameter p_j, there will be $NOBS$ values. In all, there will be $NUP \times NOBS$ estimated parameters. It is desirable to reduce this large number to single grand mean percentage error (GM), and a single grand deviation percentage error (GSD) for ease of comparison. GM and GSD will be used to compare the input-output error behavior of various sets.

$$GM = \frac{1}{NOBS \cdot NUP} \sum_{j=1}^{NUP} \sum_{k=1}^{NOBS} E_{j,k} \quad (23)$$

$$GSD = \sqrt{\left[\frac{1}{NUP \cdot NOBS - 1}\right] \sum_{j=1}^{NUP} \sum_{k=1}^{NOBS} (E_{j,k} - GM)^2} \quad (24)$$

In the same sense that a sample size of 1 is not valid statistically, reducing all these experiments to just two scalar values is not an accurate representation; in particular, the *GM* does not show maximums or minimums, but is merely a mean.

3.3 Damage Assessment Using a Hypothesis Test

After the mean and deviation values for each member of current structure have been obtained from the data perturbation trials, normally distributed parameters can be assumed. Suppose that measurements are obtained under exactly the same conditions for both the current structure and the associated undamaged structure. Therefore the statistical distributions of system parameters in the undamaged structure can be reasonably assumed based on the characteristics of the above Monte Carlo simulation. The assumed normal distribution $N_b(1, \sigma^2)$ will be called the baseline distribution for the system parameter, wherein σ is the same as those of current structure and *1* represents the intact status of member in undamaged structure. Let we assume a random variable $X=p_{j,k}/p$, where *p* is the intact Young's modulus of element. A hypothesis test can be applied to determine damaged members by useful properties of the normal distribution. The hypothesis test is defined as follows:

H_0: $\mu=1$
H_1: $\mu<1$
Statistic: X
Rule form: Accept H_0 if $X \geq C$
Otherwise, accept H_1
Significance level: α
Acceptance region: $P(-k_\alpha \leq \frac{X-1}{\sigma}) = 1-\alpha$

Result: $C = 1 - k_\alpha \cdot \sigma$

where C is the critical value to classify the damaged or undamaged elements. Using the hypothesis test, the damage status of a member in the current structure is evaluated as Fig. 4 illustrates. A member that accepts H_0 is taken as

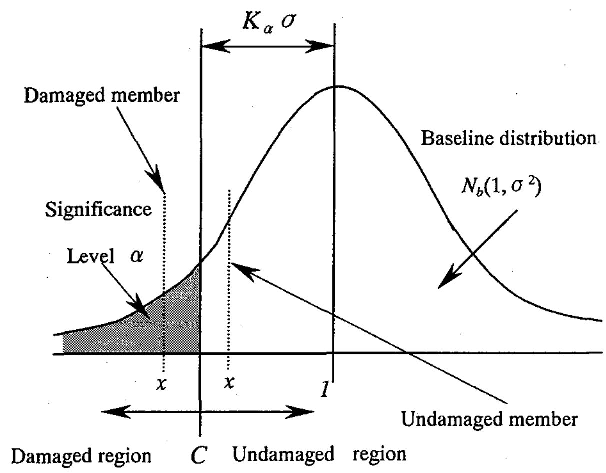


Fig. 4 Interval estimation for damage assessment

undamaged with 100×(1−α)% confidence; in the same way, a member that accepts H_1 is taken as damaged with 100×(1−α)% confidence. The damage index I_D, which represents the damage status of a member with the significance level of α, is defined as follows:

$$I_D = \begin{cases} 0 & if\ H_0\ acceptd\ (x \geq c) \\ 1 & if\ H_1\ acceptd\ (x \leq c) \end{cases} \quad (25)$$

The severity of damage S_D, which indicates how serious a member is damaged with the significance level of α, is defined as a relative distance of the estimated one from the intact value

$$S_D = (1-x) \times I_D \times 100\% \quad (26)$$

4. Numerical Simulation

In this section, we wish to estimate the parameters of a frame structure using the two algorithms described above.

The 5-story, two-bay steel frame shown in Fig.5 was used in the study. The frame was divided into 25 frame elements. Nodes were assigned at every joint, and each node provided three degrees of freedom. Elements 1-15 make up the columns, and elements 16-25 make up the beams of the frame. The cross-sectional areas, moment of inertias of elements is listed in table 1 and the modulus of elasticity are as follows.

a. Undamaged structure
Young's modulus for all elements=206.8 Gpa
b. Current structure (or real structure)

Damage in the structure is simulated with reductions in the Young's modulus of elements, details of which are listed in table 2. All of other elements are the same as before.

Table 1 Cross Sectional Properties

Member	Area (cm^2)	Moment of inertia (cm^4)
1~15	1065	442246
16~21	1606	442246
22~25	1406	422246

Table 2 Damage Situations

Member	Damage
5	50%
13	40%
21	25%

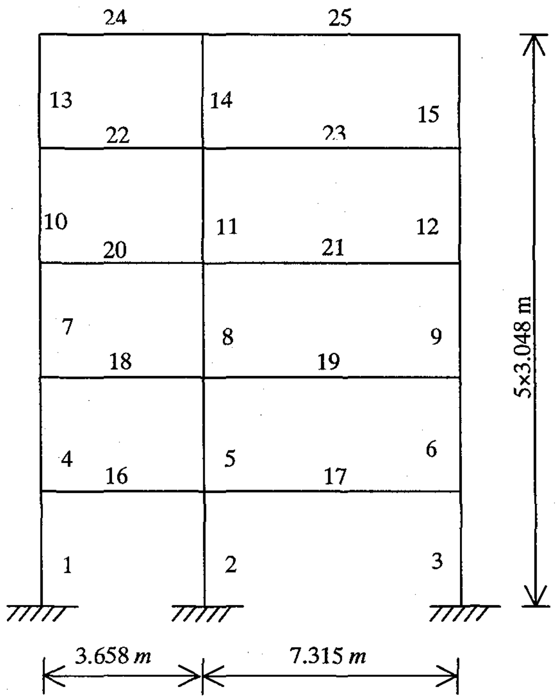


Fig.5 5-story, two-bay steel frame

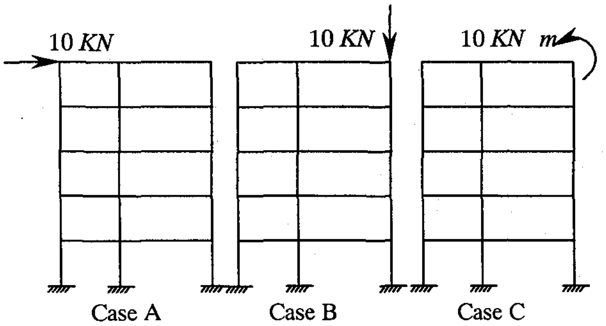


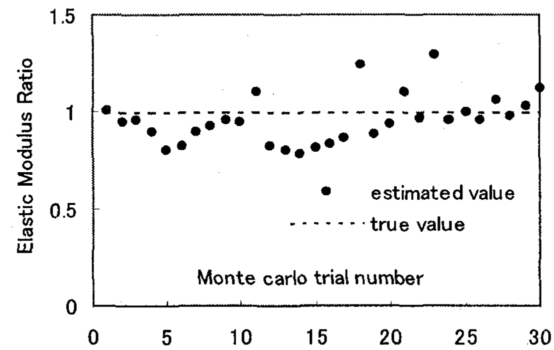
Fig. 6 A typical load set

In the present study we use three typical load cases as Fig. 6 shows. The measurements consist of three parts: the horizontal displacements of all nodes under load case (a), the vertical displacements of all nodes under load case (b) and the rotational angles of all nodes under load case (c).

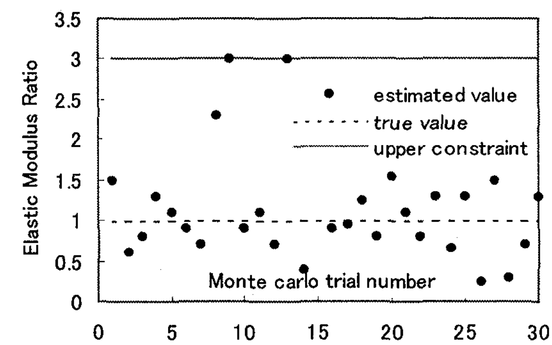
The accuracy of Monte Carlo simulation depends on the sample size, which should be large enough to establish significant estimates. We set *NOBS*=30.

We take the Young's modulus of element as a parameter to be estimated.

4.1 Comparison of Estimator Performances



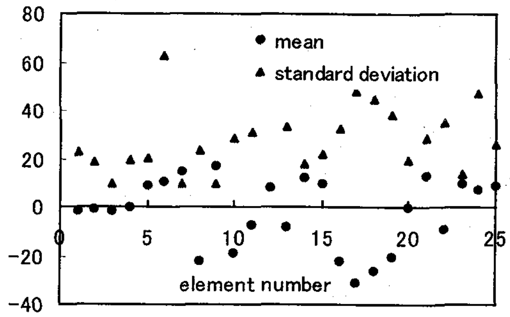
a) The results from the first algorithm



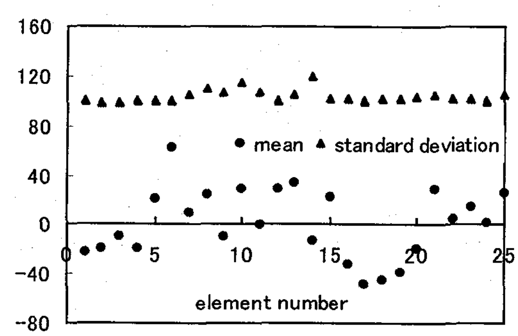
b) The results from the second algorithm

Fig. 7 The sample of estimated elastic modulus ratio ($p_{j,k}/p_{j,t}$) of member 3 with 5% input error

Because there are two kinds of error definitions, we have developed two methodologies to identify the element properties, the first one based on DEE and the second one based on FEE. Therefor we can obtain two sets of solutions. Fig.7 shows one of nup solution samples from those two algorithms with 5% input error respectively. They are the samples of element 3, and others are more or less alike. In Fig. 7(a), all the iteration computations stop on the first convergence criterion. So we can see the identification results would not change if there are no upper and lower constraint. However, as shown in Fig. 7(b), two values move toward the upper bounds and we can see the results would change as the upper bound increases. Part of computation processes stop on the second convergence criterion.



a) The results from the first algorithm



b) The results from the second algorithm

Fig. 8 Mean and standard deviation of the percentage error of all members with 5% input error

Fig.8 shows the means and deviations of all members with 5% input error. We can clearly see that the deviations in (b) are much larger than those in (a).

To compare the behavior of the proposed algorithms, we need a criterion. The mean defined here indicates the bias between the estimated parameters and the true values, and thus provides one of suitable measures of goodness of fit. An unbiased estimation is, practically speaking, more desirable than a biased estimation. The expected value of mean for an unbiased estimator is equal to zero. However, an estimator with a small bias and a small deviation might be preferred to one that is unbiased but has a large deviation. From this, the first one is much better.

4.2 Relationship between Input-error and Output-error

Fig. 9 shows the relationship between the input-error and output-error from the first algorithm. The grand means and grand variations are plotted against the input error as noisy amplitude increases. Although errors in the identified parameters are always magnified, if the input error is small enough to utilize the linear input-output error behavior range, it is possible to use the identified parameters for damage

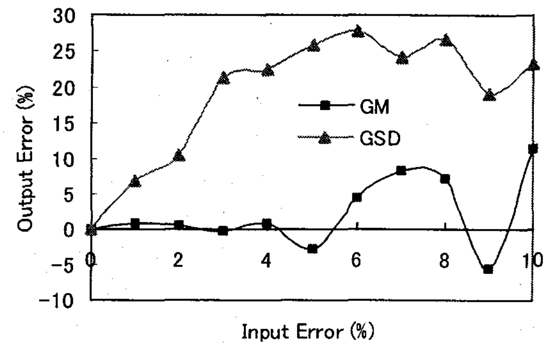


Fig.9 Variations of statistical indices for DEE with noise amplitude

assessment. Even if we known that the results depend on cases, we can determine the stability of algorithm approximately based on them.

4.3 Damage Localization and assessment

We set α=5%, σ=25%

so k_α=1.65, C=1−1.65σ=0.59

In practical engineering application, it is difficult to measure input data many times and then use the average value of them. In simulation study, we only choose one of the samples $\{u_k\}$ with 5% measurement error, and then we get corresponding sample, $\{p_j$, j=1,2, ..., nup$\}$ estimated parameters.

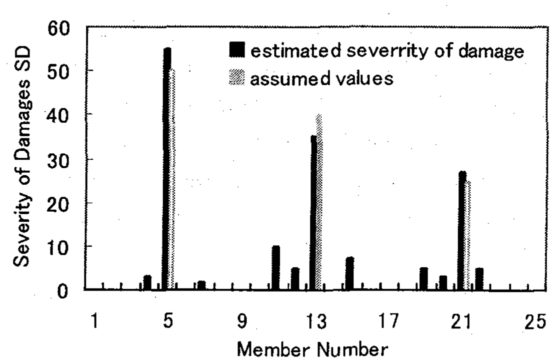


Fig. 10 Severity damage charts

By using this sample, the damage severity of all members is presented in Fig. 10. One damaged members are identified as damaged and main part of undamaged members are identified as undamaged by the hypothesis test. In the Fig. 10, when x<1, we also calculated their severity of damage. So two other damaged members are found. However, since the damage severity of other members is very small compared to that of the damaged members, it is concluded that there is little chance of damage in those members. Those results have 95%

confidence.

5. Conclusion

This paper presents two methodologies to identify the element properties based on the previous works of many researchers. By using Monte Carlo methods, the performances of two algorithms in presence of measurement errors are investigated in details. Based on Eqs.11 and 12 and numerical simulations, the following conclusions can be drawn:

1) If there is free from the measurement error, both of them can reconstruct the element rigidities of frame structures accurately. Under the situation of complete measurements, the second algorithm based on the force-error estimator is more effective because its mathematical model is linear least-square problem.
2) The first algorithm is distinguished in the presence of measurement noise. The quality of sensitivity matrix plays important part in the iteration computation. The sensitivity matrix in the first algorithm is not affected by the measurement noise. Just because of this, it enhances the stability of this algorithm largely.
3) In terms of uniqueness of the identified parameters, there is no mathematical proof ensuring that the identified parameters are unique. However, based on the works of the writers, when the algorithm converged on the first criterion, it converged on the whole minimum, not on a local minimum under the condition that the amplitude of noise is bellow 10%.,
4) All the results listed here are the solutions of complete identification. Under the condition of part identification, the performances of two algorithms will improve as the number of variables decreases.

In this paper, all works are theoretical derivations and numerical simulation. The first algorithm presented here is an excellent tool to reconstruct the element rigidities of steel frame structures. Future work can include laboratory testing to validate the proposed technique, and then apply it outside of the laboratory to a full damage assessment system in engineering.

References

1.) Sanayei, M., and Onipede, O., "Damage assessment of structure using static test data" AIAA J., Vol. 29, No.7, 1991, 1174-1179.

2.) Benjamin, J.R., and Cornell, C. A. Probability, Statistics, and Decision for Civil Engineers. McGraw-Hill, 1970, New York.

3.) Hajela, P., and Soeiro, F. J., "Structural damage detection based on static and modal analysis." AIAA J., 28(6), 1990, 1110-1115.

4.) Hajela, P., and Soeiro, F. J., "Recent developments in damage detection based on system identification methods", Structural Optimization 2, 1990, 1-10.

5.) Hjelmstad, K.D., and Clark, S.J., "Mutual residual energy method for parameter estimation structure" ASCE, J. of Structural Engineering, Vol. 118, No. 1, 1992, 223-241.

6.) Banan, M.R., and Hjelmstad, K.D., "Parameter estimation of structures from static response, Ⅰ. Computational Aspects", Journal of Structural Engineering, Vol.120, No.11, November, 1994, 3243-3258.

7.) Banan, M.R., and Hjelmstad, K.D., "Parameter estimation of structures from static response, Ⅱ. Numerical Simulation Studies", Journal of Structural Engineering, Vol.120, No.11, November, 1994, 3259-3283.

8.) Hjelmstad, K.D. and Shin, S., "Damage detection and assessment of structures from static response", Journal of Engineering Mechanics, Vol. 123, No.6 June, 1997, 568-576.

9.) Liu, Pei-Ling, and Chian, Chung-Chi, "Parametric identification of truss structures using static strains" Journal of Structural Engineering, Vol. 123, No. 7, July, 1997, 927-933.

10.) Yeo, Inho and Shin, Soobong, "Statistical damage assessment of framed structures from static responses", Journal of Engineering Mechanics, Vol. 126, No. 4, Aprial, 2000, 414-421.

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