Sag Ratio and Side Span Ratio Study on Critical Flutter Speed and Cost of Long Span Suspension Bridge

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This paper presents some preliminary design considerations of long span suspension bridges. Comparison study of bridge configurations such as side span ratio and sag ratio to resist flutter instability in relevance to whole bridge design is conducted. As most decision in the construction project involves economic consideration, decision analysis in this research is also focused on minimizing total cost. To provide database of critical flutter speeds for various bridge configurations, several bridge lengths with various sag ratios and side span ratios are taken into consideration. 3-D Direct Flutter Analysis with Mode Tracing Method is applied to determine the critical flutter speed. It was found from the analytical results that sag ratio and side span ratio has significant effect on critical flutter speed.
Keywords: long span bridge configuration, critical flutter speed, aerodynamic

1. Introduction

Long span suspension bridges are extremely flexible structure in which wind effects play dominant role during design process. Since the failure of the Tacoma Narrow Bridge in 1940, aerodynamic stability has become one of the most important factors in design of a modern suspension bridge. Wind effects on long span suspension bridges can lead to instability of the whole bridge structure. Therefore consideration of wind effects to the whole structure starting from preliminary design process of the long span bridge project is important in order to get the appropriate design configuration. Wind engineer should be involved in the project from the initial stage so that wind-structure effects can be incorporated from the preliminary stage.

Selection of the structural configuration as a result of Preliminary Design process is a very important step in the bridge project design process. Based on that configuration, more detailed analysis including the complicated process such as wind tunnel test will be conducted. As the project constraint such as topography condition and wind condition vary with location, the configuration result also will vary accordingly.

2. Importance of Sag Ratio and Side Span Ratio

During preliminary design process, total length of the bridge is a given factor based on the geographical condition of the site. Variables of a bridge configuration which affect bridge stability are sag ratio, side span ratio and deck type. Effects of the first two variables to bridge instability should be taken into account in determining the appropriate configuration. Unfortunately, most decisions on preliminary bridge configuration do not consider these effects properly. It should be noted that for some bridge projects, selection of side span ratio is determined by topographical condition.

Another important factor in configuration decision is the total cost of the project. As the sag ratio and the side span ratio vary, the total cost of the bridge project will vary accordingly. Therefore effects of the variables should be investigated from the viewpoint of flutter instability and cost simultaneously.

For future bridge projects, one of the most important aspects is cost effectiveness. As the project becomes huge and involves construction of large-scale structures that requires major investment, cost-effectiveness is the key to success of the project. One solution to optimize the cost is by utilizing the optimum bridge configuration with appropriate sag ratio and side span ratio value.

To evaluate the influence of such bridge configuration variables on flutter speed, the relation between side span ratio and sag ratio with flutter speed and total cost is important. Based on this information the decision on bridge configuration can be made by already taking into account the flutter speed factor and cost factor.

In some previous bridge projects sag ratio and side span ratio values have been used but the determination of their values did not always consider wind-engineering point of view. Table 1 shows values of sag ratios and side span ratios for some long span bridge projects. There is no clear argument about how sag ratio and side span ratio values were decided.

Study on bridge configuration concerning the pylon and main cable cost has been done [3]. The analysis were limited to the study on the sag ratio and were not considering its effect to the critical flutter speed as a dominant factor in long span suspension bridge design. The study indicated that the optimal main cable and pylon cost occurs at sag ratio value 0.15. However this value is much higher than existing bridges in Table 1.

In this research the truss deck of Akashi Kaikyo Bridge is considered for long span suspension bridge model, on which the flutter analysis and cost analysis will be done. The critical flutter speed will be determined by 3-D Direct Flutter Analysis with Mode Tracing Method for each bridge configuration. Cost comparison will be conducted and for this purpose, structural element cost will be based on previous long span suspension bridge projects.

Table 1 Configuration of some bridge projects

	Total Length	Deck type	Sag ratio	Side span ratio
Akashi Kaikyo	3911.1	Truss	0.1	0.48
Great Belt E.	2694	Box	0.11	0.33
Verrazano N.	2039	Truss	0.09	0.285
Mackinac	2625	truss	0.092	0.474
Golden Gate	1965	Truss	0.112	0.267
Bisan Seto	1610	Truss	0.091	0.276

3. Case Studies

The behavior of flutter speed and cost under different sag ratio and side span ratio is investigated by considering 2 cases of total bridge length: 4000 meter and 5000 meter. In this study, the pylon is assumed made of steel. The truss deck consists of 6 lanes for all cases. The typical section of truss deck girder can be seen in Figure 1.

To provide a database of critical flutter speeds and construction costs, several configurations of long span bridge with different sag ratios and side span ratios are considered. The configurations are made by assuming that the total length of the bridge is constant, therefore different side span ratios are determined by different pylon position while different sag ratio are determined by varying the pylon height. General notation of 3-span suspension bridge is presented in Figure 2.

As the sag ratio and side span ratio vary, dimension of the main cable and pylon will vary accordingly. Therefore for different bridge configurations, the dimension of the cable and pylon will also be different. The formula from reference [3] is used to determine preliminary dimension of the main cable.

Case study considered in this research is summarized as can be seen in Table 2.

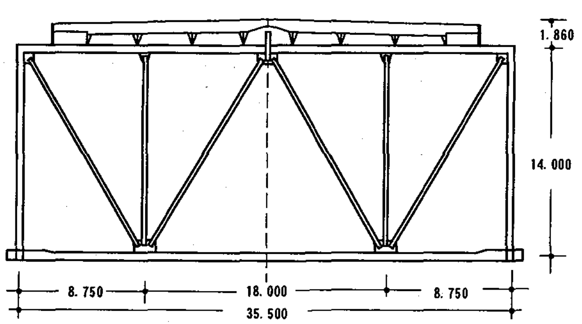


Figure 1. Truss Deck Section

In this research the allowable tensile stress for main cable steel is 8000 kg/cm2 and for pylon material is 2450 kg/m2. Based on these the dimension of the main cable and pylon are calculated for each bridge configuration.

Table 2 : Case Studies of Bridge Configuration

Deck type	Truss
Number of lane	6 lanes
Total bridge length	[4000] [5000] meter
Side span ratio	[0.25] [0.30] [0.35] [0.40] [0.45] [0.50]
Sag ratio	[0.080] [0.085] [0.090] [0.095] [0.100] [0.105] [0.110]

For flutter analysis, it is necessary to provide the truss deck section properties. Detail of the deck section properties for bridge model is shown in Table 3.

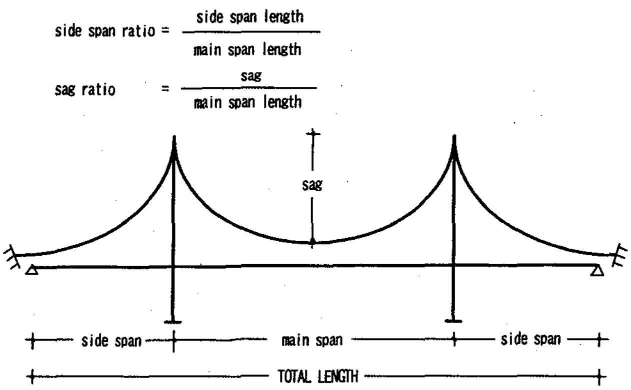


Figure 2 : General Lay out of 3-span suspension bridge

Table 3 : Deck Section Properties

Area	0.448 m^2
Inertia -X	14.185 m^4
Inertia -Y	154.25 m^4
Inertia -Z	23.99 m^4
Mass	29560 kg/m
Mass Moment of Inertia	5912000 kg m^2/m

4. Direct Flutter Analysis

3-D Direct Flutter Analysis with Mode Tracing Method is applied to determine the critical flutter speed. In Direct Flutter Analysis method, introduced by Miyata and Yamada [7], the bridge structure is modeled as a 3-D frame on which the flutter analysis is performed directly. The aeroelastic effects by means of self-excited forces are modeled by a set of unsteady coefficient of flutter derivatives. These self-excited forces are incorporated directly into the bridge matrices. As these forces are in complex form, complex eigenvalue analysis is required during the analysis process.

As the bridge is modeled more accurately the number of equation also increases significantly. Solution by Direct Finite Element method for such a large system would require a lot of computing time. Development of Direct Flutter analysis is proposed by Dung [2], which is called Mode Tracing Method, is used to reduce the computing time for large system. This method considers only one mode at a time and then traces the revolution of modal properties, with step by step increase of wind speed.

More detail explanation of 3-D Direct Flutter Analysis with Mode Tracing method can be found in reference [2] and [7].

4.1 Aeroelastic Forces

The equation of motion of a full-model bridge in the presence of aeroelastic forces can be expressed as

$$[M]\ddot{u} + [K]u = F_{ae} \tag{1}$$

where **M** and **K** are mass and stiffness matrix formed by finite element method, **u** is displacement vector, and $\mathbf{F_{ae}}$ is aeroelastic or self-excited forces, which depend on reduce frequency **k**

$$F_{ae} = \begin{bmatrix} L_{ae} \\ D_{ae} \\ M_{ae}/B \end{bmatrix} = \rho\pi B^2\omega \begin{bmatrix} L_{YI} & L_{ZI} & L_{\alpha I} \\ D_{YI} & D_{ZI} & D_{\alpha I} \\ M_{YI} & M_{ZI} & M_{\alpha I} \end{bmatrix} \dot{w} + \rho\pi B^2\omega^2 \begin{bmatrix} L_{YR} & L_{ZR} & L_{\alpha R} \\ D_{YR} & D_{ZR} & D_{\alpha R} \\ M_{YR} & M_{ZR} & M_{\alpha R} \end{bmatrix} w \tag{2}$$

in which $\mathbf{L_{ae}}$, $\mathbf{D_{ae}}$, and $\mathbf{M_{ae}}$ are aeroelastic or self-excited Lift, Drag and Moment Forces respectively and contain a set of unsteady coefficient which depend on the reduce frequency only, α is torsional angle, **B** is the full with of bridge deck, ρ=0.125 [kg.sec^2/m^4] is air density, ω is frequency and **w** is displacement vector in local coordinate.

$$w = \begin{Bmatrix} y \\ z \\ \alpha B \end{Bmatrix} \tag{3}$$

4.2 FEM Modeling

In Direct Flutter Analysis Method, the bridge structure is modeled as a frame of beam and truss with unsteady aerodynamic forces located at shear center of the bridge. As the wind effect in the direction along the deck axis is negligible, the aerodynamic forces at each node of shear center could take the form with 3 degrees of freedom corresponding to 3 self excited forces Drag, Lift, and Moment. The aerodynamic force of cable, which is formulated by quasi-static assumption, can also be incorporated.

Typical bridge mesh and the detail of FEM modeling of long span suspension bridge with aerodynamic forces can be seen in Figure 3 and Figure 4.

5. Configuration Analysis

To provide the cost data for configuration decision it is important to determine the volume of the bridge elements. In general, the main components of a suspension bridge super structure are main cable, stiffening girder, hanger cable and pylon. In this research only the main cable and the pylon are of interest since the volume of stiffening girder assumed constant if the total length of the bridge is constant.

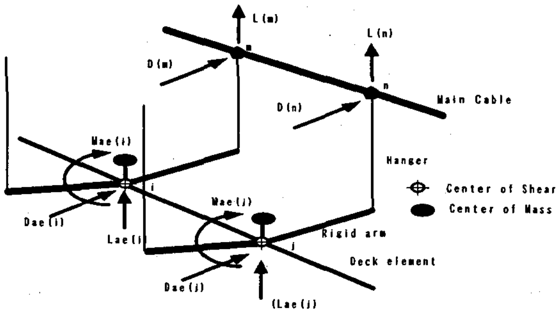


Figure 3 : Finite Element Model

Figure 4 : Typical Mesh of Bridge Model

As the sag ratio and side span ratio change, the force acting on the main cable and the pylon also will change. In turn this will affect the dimension of the main cable and pylon of the bridge. Their volume also changes accordingly and it affects the cost structure of the bridge.

The dimension of the bridge elements are affected not only by the dead load and live load but also by other kind of loads. However for simplicity, main cable diameter and pylon volume in this study are determined by dead load and live load effects only as was used in reference [3]

From the view point of flutter analysis, different sag ratio and side span ratio values will leads to the new value of stiffness matrix **[K]** and mass matrix **[M]** in Equation 1, as the dimension of the main cable, coordinate and its tensile force change. In turn it will affected the critical flutter speed of the bridge.

The general arrangement of the long span suspension bridge is shown in Figure 5 and Figure 6.

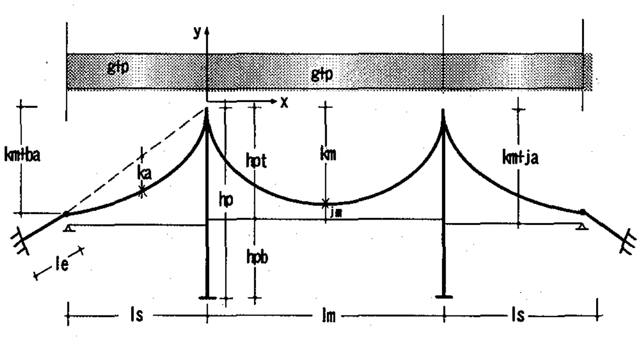


Figure 5. Three-span suspension bridge

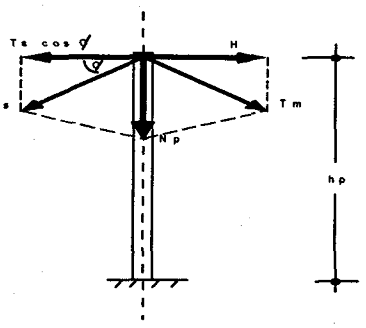


Figure 6. Forces at the pylon top

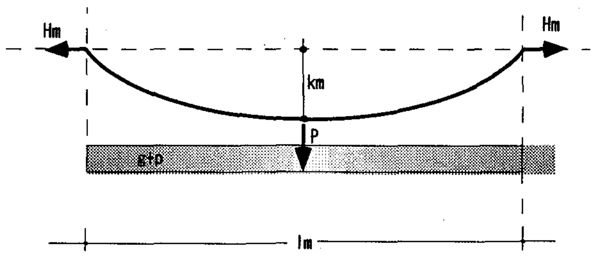


Figure 7 : Loading Cases for Maximum Main Cable Force

Notations used in figure 5, 6, and 7 are described as follows.

- g+p : distributed live load and dead load on stiffening girder
- P : point load
- l_m : main span length
- l_s : side span length
- k_m : main span sag
- k_a : side span sag
- b_a : the vertical difference between location of cable bearing and cable at mid-span
- ja : the vertical difference between mean level of hanger sockets at side span and main span
- l_e : the distance from bearing to anchorage block
- h_p : pylon height
- h_{pt} : pylon height above mean level of main span hanger sockets
- h_{pb} : pylon height bellow mean level of main span hanger sockets
- j_m : the distance from cable at center to mean level of main span hanger sockets
- T_s : main cable force at side span
- T_m : main cable force at main span
- N_p : normal force at pylon top
- H_m : horizontal force at pylon top

5.1 Size of Main Cable

The maximum force $\mathbf{T_m}$ of the main cable in the main span can be determined by assuming the dead load of the stiffening girder and the distributed traffic load acting uniformly in the entire main span and concentrated forces acting at mid-span. The dead load of hangers might be neglected, as their contribution is quite insignificant.

Determination of main cable area at center span and side span follows the formulation by Gimsing [3], in which the center span main cable area is calculated by

$$A_m = \frac{[(g+p)l_m + 2P]\sqrt{l_m^2 + 16k_m^2}}{8f_{cb}k_m - \gamma_{cb}l_m\sqrt{l_m^2 + 16k_m^2}} \tag{5}$$

and side span cable area required is determined by

$$A_s = \frac{T_s}{f_{cb}} = \frac{H_m}{f_{cb}\cos\phi_s} \tag{6}$$

5.2 Volume of Main Cable Steel

The volume analysis of main cable is divided into two parts. They are analysis of main cable in the main span and in the side span which are subject to distributed load **g+p** acting on the stiffening girder as indicated in Figure 5. The cable curve equation used in this study is second order parabola [3]. The equation for the center span cable curve is

$$y = 4k_m \frac{x}{l_m}\left(\frac{x}{l_m} - 1\right) \tag{7}$$

and the equation for the side span cable curve is

$$y = 4k_s\left(\frac{x}{l_s}\right)^2 + (4k_a + k_m + b_a)\frac{x}{l_a} \tag{8}$$

By using simple numerical program the curve length of the above equations can be determined

5.3 Volume of Pylon

An accurate calculation of the pylon volume is of considerable complexity as the required dimension of the pylon depends not only from the in-plane forces of the cable system, but also on the wind and other lateral forces as well.

The following formula to determine the pylon quantities is taken into account by stipulating an effective stress f_p that sustains the vertical forces from the cable system to the foundation [3]

For the pylon shown in Figure 6 the quantities is

$$Q_p = \frac{(g+p)+Q_{cm}}{8}\left\{\frac{k_m + 4k_a + b_a}{k_m}\frac{l_m}{l_s} + 4\right\}\left\{\exp\left(\frac{\gamma_p}{f_p}h_p\right) - 1\right\} \tag{9}$$

where $\mathbf{Q_{cm}}$ is the volume of main cable in main span, and $\boldsymbol{\gamma_p}$ and $\mathbf{f_p}$ are the density of the pylon material and allowable compressive stress of the pylon material respectively.

For effectively designed pylon structure, the value of $\mathbf{f_p}$ will be between 60% and 80 % of the allowable compressive stress of the material used in the pylon [3]. When comparing with the total weight of Akashi Kaikyo Bridge, Equation 9 with 60% allowable compressive stress gives smaller value. Therefore in this research the value of $\mathbf{f_p}$ is assumed 50% of its allowable value. Further detail of this formula can be found in reference [3].

6. Unit Cost Data

The unit cost of bridge elements will generally vary with the location of the project, the construction method, the erection procedure, and total volume of the bridge elements.

Furthermore it also varies from one part of the structure to another depending on the position of the element to be installed. As this analysis is intended to be used as guidance for the preliminary stage design process, each of the unit cost is stipulated as a constant value per unit volume of the bridge element.

Special attention should be given to main cable element since the unit cost of the element also will vary with the quality of the material used. In the case of the long span suspension bridge, the material quality of the main cable become one of the main concerns as new types of high quality cables but is more expensive have been developed. It application will reduce the volume of the main cable work. Another point is that the construction cost of the cable has significant contribution to the unit cost of the cable. In this research for the sake of simplicity the quality of main cable is assumed constant and the unit cost of cable is independent from the total volume of the cable.

Since the only deck type considered here is truss deck, the cost component are only main cable and pylon which are both made of steel. The cost of the deck is assumed constant for constant total bridge length

6.1 Unit Cost of Main Cable

Unit cost for main cable with 8000 kg/cm2 allowable stress is assumed roughly 1.400.000 yen per ton. This value is assumed to have included the material cost, construction cost and corrosion protection. The ratio between material and construction cost is assumed 1:1. Therefore if the quality of the cable increases, it will reduce the volume of the cable, but the material cost per unit volume will increase. In turn the unit cost of main cable per unit volume will change accordingly.

6.2 Unit Cost of Steel Pylon

From previous long span bridge project unit cost of steel pylon is assumed roughly 1.150.000 yen per ton. This price is assumed to have included the material cost, construction cost, and transportation cost as well.

7. Flutter Analysis Result

7.1. Natural Frequency

The dynamic characteristic of the bridges in terms of natural frequency can be seen in Figure 8 and Figure 9. Natural frequencies of symmetric lowest vertical bending and torsion modes at still air are plotted for different bridge configurations.

It is clear that first symmetric vertical bending natural frequencies are significantly decreases with increase in side span ratio and sag ratio values as well. However it can be seen that the value changes relatively small with sag ratio value. Different with vertical bending frequency, First symmetric torsional frequency increases with increase in side span ratio and it also increases with increase in sag ratio value.

As sag ratio decreases cable tension will increase and it will increase the required dimension of the main cable. The cable mass will increase accordingly. As the side span ratio increases the cable dimension decrease due the shorter main span length and thus, reducing the cable mass.

7.2 Critical Flutter Speed

Flutter analysis due to the effect of side span ratio and sag ratio has been carried out by Direct Flutter Analysis with Mode Tracing Method. The structural dumping is assumed equal to 0.02. Figure 10 shows the relation between side span ratio, sag ratio and the critical flutter speed for long span suspension bridge with total length 4000 and 5000 m.

It can be seen from Figure 10 that the critical flutter speed increases as the sag ratio increase. It is also can be seen that higher flutter speed occurred at side span ratio 0.3 or higher. For 4000 meter total bridge length, the optimum side span ratio are in the range of 0.3 to 0.4. However for 5000-meter total bridge length the critical flutter speed does not vary so much with side span ratio of 0.3 to higher values.

Another interesting point from Figure 11 is that the critical flutter speed drop quite significant at side span ratio of 0.25 and less for both 4000 m and 5000 m total bridge length. Therefore it can be concluded that from flutter instability point of view, side span ratio less then 0.25 is not favorable.

8. Cost Analysis Result

By keeping the total length of the bridges to remain constant, the cost of truss deck is assumed constant for different sag ratio and side span ratio values. Therefore cost comparison in this study were involved the cost of cable and cost of pylon only. Cost comparison for each bridge configurations can be seen in Figure 11, 12, and 13.

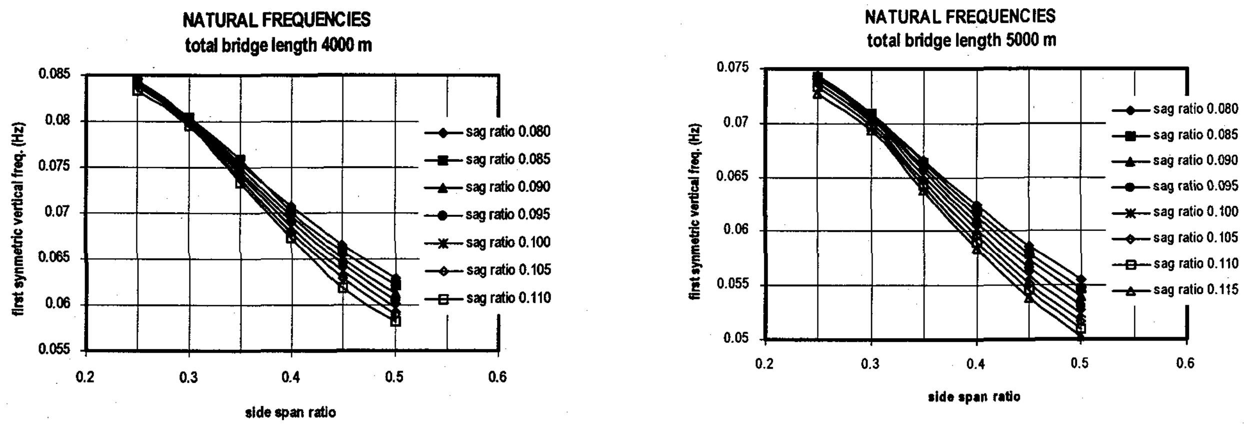
The cost of main cable steel for various configurations can be seen in Figure 11. It is clear that larger side span ratio will give a smaller main cable cost and the cost also decreases with increase in sag ratio value.

As sag ratio decreases, the cable tension will increase, and as the consequence, the main cable dimension will increase accordingly. However as the sag ratio increases the total length of the main cable will increase which also can result in volume increase. Figure 11 shows that that the cost effect of increases in main cable dimension due to decrease in sag ratio predominates over that due to reduction of cable length. Therefore in the viewpoint of cable cost, the higher the sag ratio and side span ratio are the better.

The cost of the pylon for the 4000 meter and 5000 meter total bridge length can be seen in Figure 12. It is evident that the cost of the pylon does not vary so much with sag ratio, but it will change significantly with side span ratio. Higher side span ratio and lower sag ratio value gives the smallest value of pylon cost.

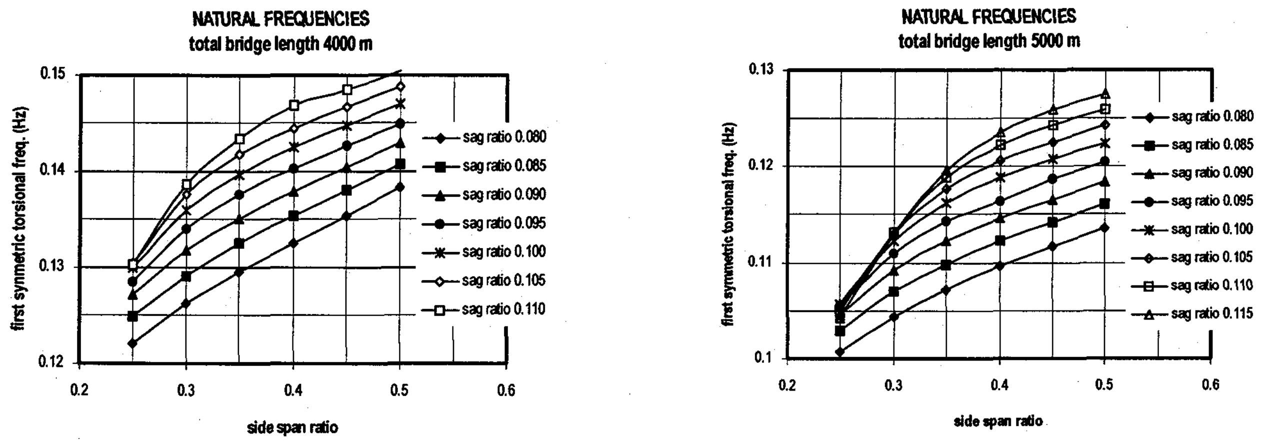
The relation between cost of pylon & main cable, sag ratio, side span ratio, and critical flutter speed is presented in Figure 13a & 13b. It can be seen that the total cost from pylon & main cable has similar trend with main cable cost. It indicates main cable cost portion is much higher than pylon cost.

Additional critical flutter speed curves were made. The curves are connecting the bridge configurations, in term of sag and side span ratio, with the same critical flutter speed value. Interesting point from Figure 13a & 13b is that higher values of sag ratio and side span ratio that will give lower cost turn out to have better ability to resist flutter instability.



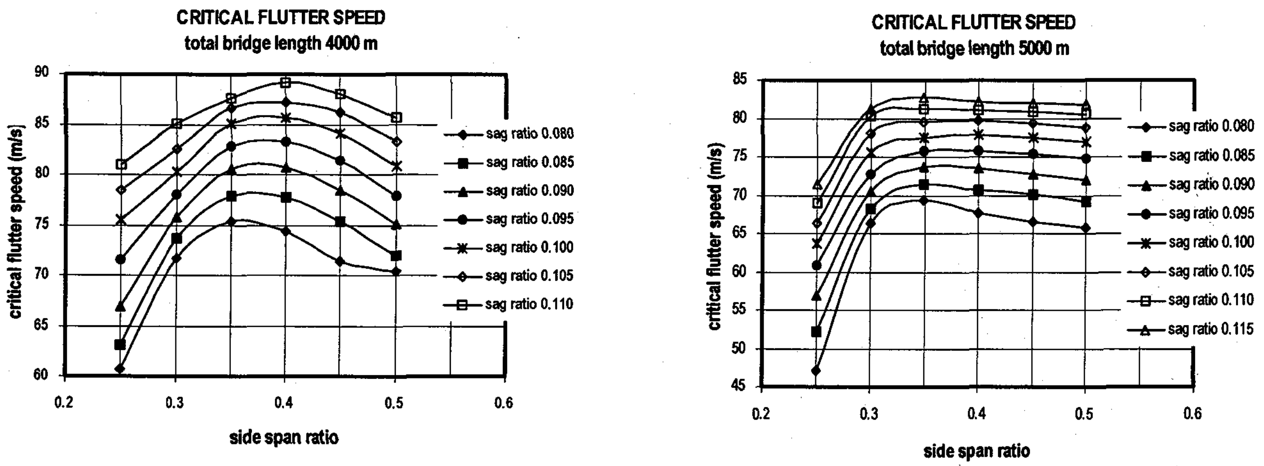
a. total bridge length 4000 m *b. total bridge length 5000 meter*

Figure 8 : First Symmetric Vertical Bending Natural Frequencies



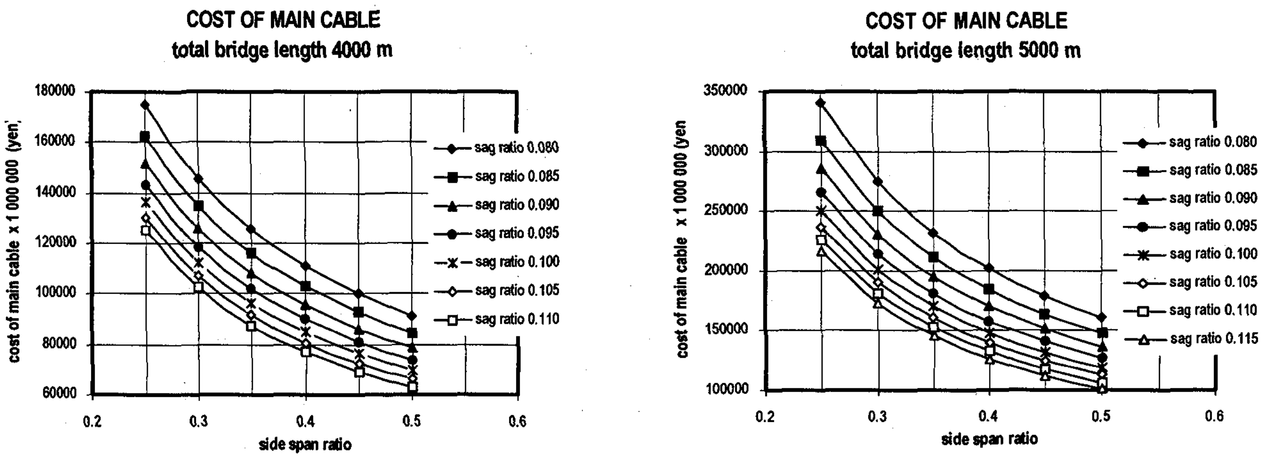
a. total bridge length 4000 meter *b. total bridge length 5000 meter*

Figure 9 : First Symmetric Torsional Natural Frequency



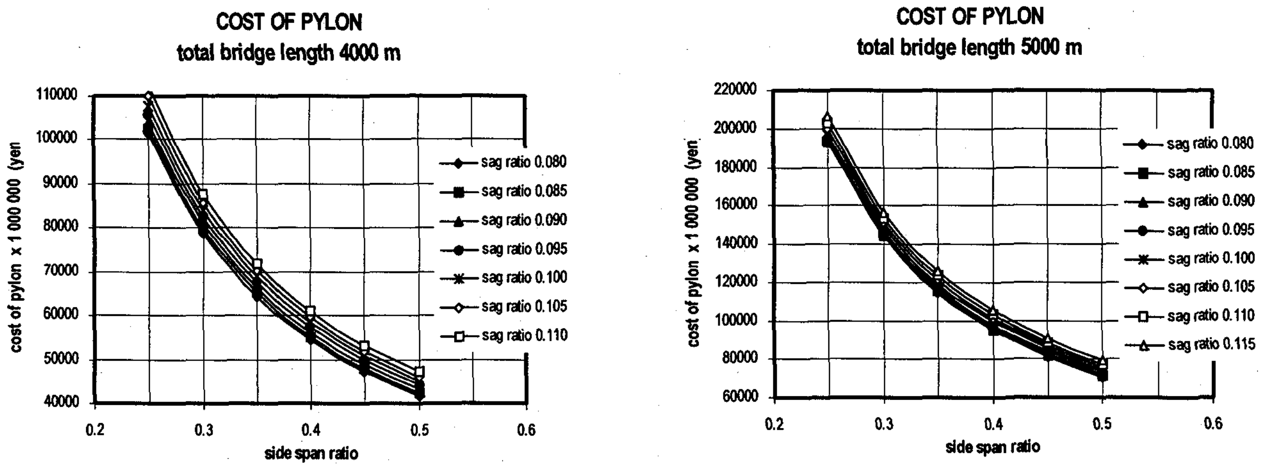
a. total bridge length 4000 meter *b. total bridge length 5000 meter*

Figure 10 : Critical Flutter Speed



a. total bridge length 4000 meter *b. total bridge length 5000 meter*

Figure 11 : Main Cable Cost Comparison



a. total bridge length 4000 meter *b. total bridge length 5000 meter*

Figure 12 : Pylon Cost Comparison

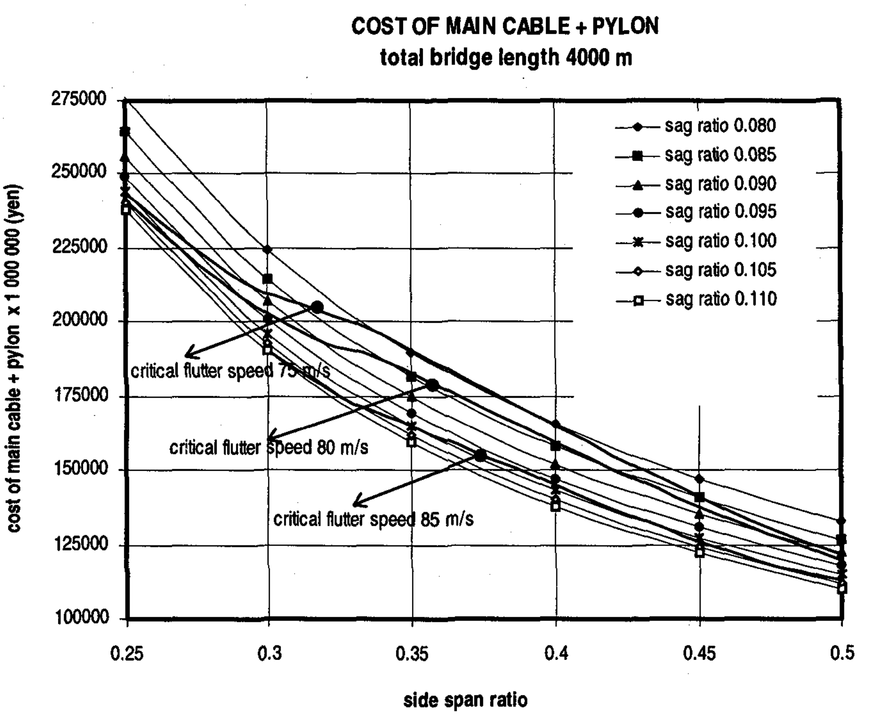


Figure 13.a : Main Cable + Pylon Cost Comparison (total bridge length 4000 m)

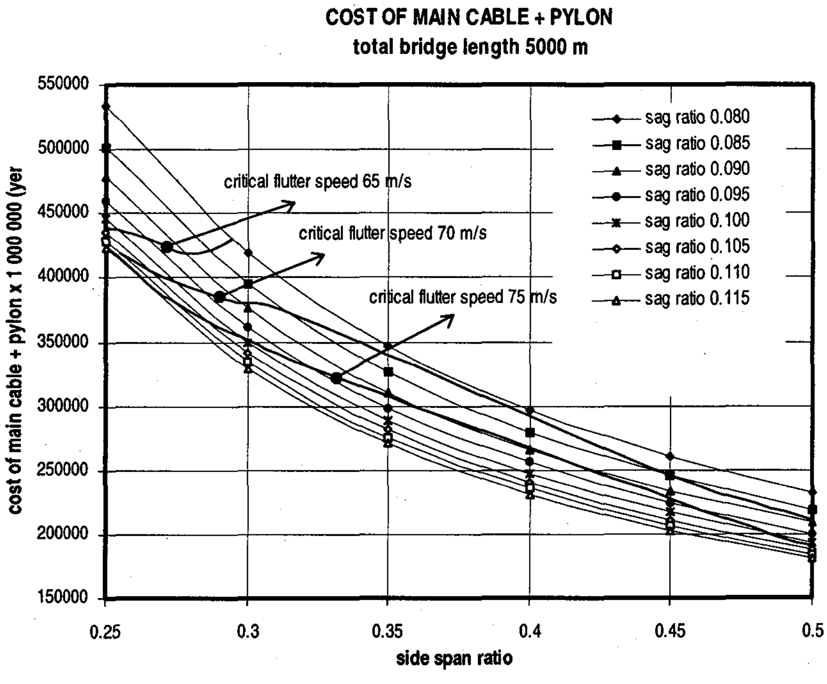


Figure 13.b : Main Cable + Pylon Cost Comparison (total bridge length 5000 m)

9. Conclusion

Many bridges projects are likely to be build in the near future. At the first design phase of the project, the side span ratio and sag ratio should be determined. This paper has shown how those parameters affect the stability of the bridge and also the main cable and pylon cost of the bridge project.

From assumptions and case studies of this research, it can be concluded that

- Selection of appropriate sag ratio and side span ratio is one of the methods to increase flutter speed of long span suspension bridge.
- Higher flutter speed can be obtained from larger sag ratio value
- Higher flutter speed can be obtained from larger side span ratio value.
- Lower main cable and pylon cost can be obtained from larger sag ratio value
- Lower main cable and pylon cost can be obtained from larger side span ratio value.

References

1) Dung, N. N., "*Active Control against Flutter Response in Long Span Bridge*", Doctoral Dissertation, Dept. of Civil Engineering, Yokohama National University, Japan, 1996
2) Dung, N. N., Yamada H., Miyata T., " *Flutter Response of Long Span Bridge by Complex Method* ", Proc. 4th asia-Pasific Symposium on Wind Engineering, Australia,1997,pp. 103-106.
3) Gimsing, N. J., "*Cable Supported Bridges: Concept and Design*", 2nd Edition, John Wiley and Sons, 1997
4) Miyata, T., Yamada, H., Kazama, K.," *On Application of The Direct Flutter FEM Analysis for Long Span Bridges* ", Proc., 9th International Conference, New Delhi, India, 1995, pp. 793-802.
5) Simiu, E., Scanlan, R.H., " *Wind Effect on Structures* ", 3rd Edition, John Wiley and Sons, 1996
6) Miyata, T.,Sato, H.,Toriumi, R.,Kitagawa, M.,Katsuchi, H.," *Full Model wind Tunnel Study on The Akashi Kaikyo Bridge* ", Proc., 9th International Conference, New Delhi, India, 1995, pp. 793-802.
7) Miyata, T., Yamada H., " *Couple Flutter Estimation of Suspension Bridge* ", Journal of Wind Engineering, International Colloquium on Bluff Body Aerodynamic and Its Applications no 37, 1988, pp. 485-492.

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