

# RELATIONSHIP BETWEEN TRANSFERRED SHEAR FORCE AND RELATIVE DISPLACEMENT OF SHEAR CONNECTOR IN STEEL-CONCRETE SANDWICH BEAM

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This study investigates the behavior of the shear connector in steel-concrete sandwich beam under two-point loading system. The most significant aim is to clarify the relationship between the transferred shear force and the relative displacement of shear connector. The shear connector continues to transfer shear force even after shear cracking in the core concrete has taken place at the top of the shear connector. In order to explain the relative displacement of the shear connector the equivalent stiffness of the shear connector was introduced. The effects of location and height of the shear connector on this relationship were studied. The equations obtained by empirical formulation of the transferred shear force and relative displacement were introduced to predict the relative displacement of the shear connector.

**Keywords :** shear connector, transferred shear force, relative displacement, equivalent stiffness of shear connector.

## 1. INTRODUCTION

Recently, the application of composite structures has become increasingly popular. A new type of composite structures, which is the steel-concrete sandwich structure, has been greatly developed to fulfill complicated structural requirements. The sandwich member has proved its high load carrying capacity and high ductility in bending and shear.

As shown in Fig. 1, the steel-concrete sandwich member is composed of core concrete, steel skin plate, shear reinforcing steel plate, and shear connector (i.e., stiffener in the figure). The shear connectors are required for transfer of shear force between concrete and steel element in order to develop the composite action. The shear connectors are also provided for stiffening the structural steel plate which also acts as formwork. It is important to understand the behavior and capacity of the shear connector as a basic knowledge for the design of composite structure. While shear transfer capacity of shear connector has been studied and well understood, researches on shear connector were conducted merely by direct pull-out tests. In actual structures, the shear connector is not only subjected to the transferred shear force but also compressive force and the local bending deformation of the steel plate. The behavior and load carrying capacity of the shear connector may be different from those obtained by the direct pull-out test. Considering this point the present study was conducted with a series of tests on simply supported beams with a symmetric two-point load. The study attempted to predict experimentally and analytically the relationship between

transferred shear force and relative displacement of the shear connector in steel-concrete sandwich beams.

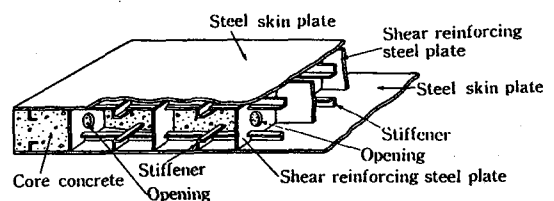


Fig.1 The steel-concrete sandwich member

## 2. EXPERIMENT

### 2.1 Description and preparation of specimens

The experimental work was carried out for the steel-concrete beams shown in Fig. 2. L-shaped shear connectors were provided at the interface between the concrete and the steel plate. The shear connectors were welded perpendicularly to the steel plate. The spacing of the shear connectors was 18.15 cm. Tests were carried out for three

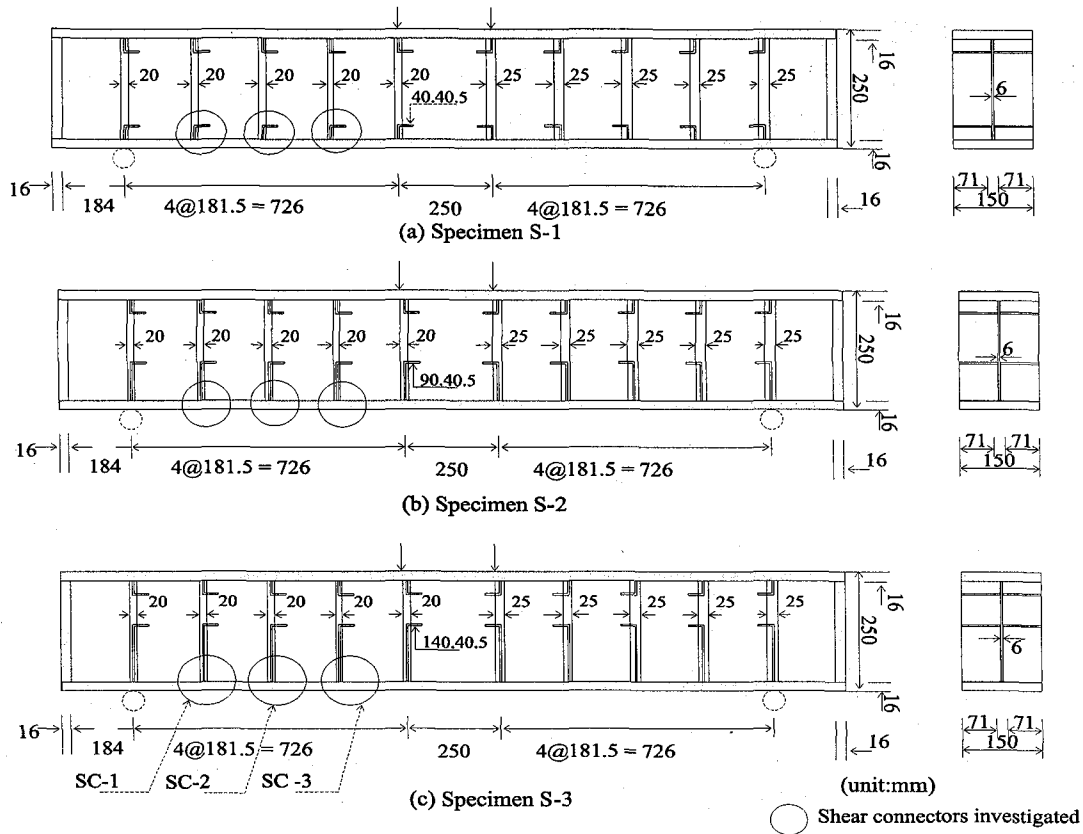


Fig. 2 Details of specimens

specimens. The sizes of the shear connectors for specimens S-1, S-2 and S-3 were 40x40x5 mm, 90x40x5 and 140x40x5 mm respectively. The details of the specimens are given in Fig. 2 and Table 1. Steel properties are given in Table 2. In order to estimate the vertical compressive force in concrete surrounding the shear connector, a tie plate was put at the same location of the shear connector. The vertical compressive force of the concrete is assumed to be the same as the tensile force of the tie plate. The specimens were designed to fail in the observed side of the shear span. Therefore, the area of the tie plate of the observed side was less than another side as shown in Fig. 2.

Table 1 Detail of specimens

Specimen	$t_w$ mm	$h$ mm	$b$ mm	$h_{sc}$ mm	$A_{tp-r}$ cm <sup>2</sup>	$A_{tp-l}$ cm <sup>2</sup>	$f'_c$ MPa
S-1	16	250	150	40	3.60	4.50	24.2
S-2	16	250	150	90	3.60	4.50	24.9
S-3	16	250	150	140	3.60	4.50	24.9

$t_w$  : thickness of flange plate     $h$  : height of beam  
 $f'_c$  : concrete compressive strength     $b$  : width of beam  
 $h_{sc}$  : height of shear connector  
 $A_{tp-r}$  : area of tie plate (right side)  
 $A_{tp-l}$  : area of tie plate (left side)

## 2.2 Instrumentation and test procedure

The experimental work of this study was performed by simply supported sandwich beams with a symmetric two-point loading system. The load was applied by a hydraulic jack and its magnitude was measured by an electrical load cell. Three electrical strain gauges were mounted on both sides of the vertical part of the shear connector to measure its deformation. In order to measure the transferred shear force between concrete and lower flange plate through the shear connector electrical strain gauges were mounted on both sides of the lower flange plate near the shear connector. The gauges were also mounted on both sides of the tie plate to measure the vertical compressive force of concrete surrounding the shear connector. Detailed arrangement of the electrical strain gauges is shown in Fig. 3.

The measuring system with contact gauges was used to measure relative displacement of the shear connector. For specimen S-1 the relative displacement was measured between the top of the shear connector and the lower flange plate. While for specimens S-2 and S-3 the relative displacement of the shear connector to the lower flange plate was measured at two points, at the height of 90 mm and 40 mm from the lower flange plate. The detailed arrangement of the contact gauges is given in Fig. 4. During the test, the deflection of the specimens, the relative dis-

placement of the shear connector and the strain of the lower flange plate, the tie plate and the shear connector were measured at every load step. Crack propagation was observed in detail.

Table 2 Steel properties

Component	Steel type	$f_y$ (MPa)	$f_t$ (MPa)	$E_s$ (GPa)
Flange Plate	SM 450 A	377.6	523.2	179
End Plate	SM 450 A	377.6	523.2	179
Tie Plate	SM 450 A	357.5	514.2	186
Shear Connector	SS 400	367.7	526.1	213

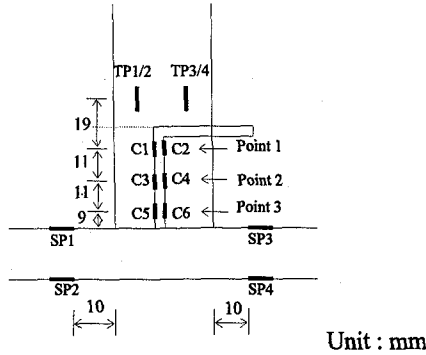


Fig. 3 Locations of strain gauges

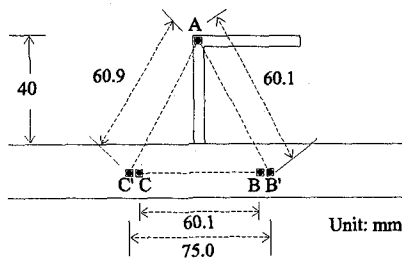


Fig. 4 Distance points measured by contact gauges for specimen S-1

### 3. TEST RESULTS AND DISCUSSION

#### 3.1 Load-carrying capacity and failure characteristics

Load-carrying capacity and crack patterns of the tested specimens are summarized in Table 3 and schematically shown in Fig. 5. In all specimens, concrete cracks occurred from the top of the shear connector.

#### 3.2 Strains in vertical part of the shear connector

For each specimens three shear connectors were investigated, namely SC-1, SC-2, and SC-3 (see Fig. 2).

Figure 6 shows the relationship between applied load and strain distributions along vertical part of the shear connector for specimens S-1.

Table 3 Experimental and calculated results

Specimen	Experimental Results		Calculation Results		
	$V_{dc}$	$V_{max}$	A	B	C
S-1	17.2	151.5	211.4	180.6	141
S-2	14.7	151.0	211.4	180.6	141
S-3	9.8	127.0	211.4	180.6	141

$V_{dc}$  : load at first diagonal cracking, kN

$V_{max}$  : ultimate load, kN

A : load corresponding to flexural capacity using the assumption of RC beam, kN

B : shear strength without yielding of the tie plate (JSCE<sup>4</sup>), kN

C : shear strength with yielding of the tie plate JSCE<sup>4</sup>), kN (failure side)

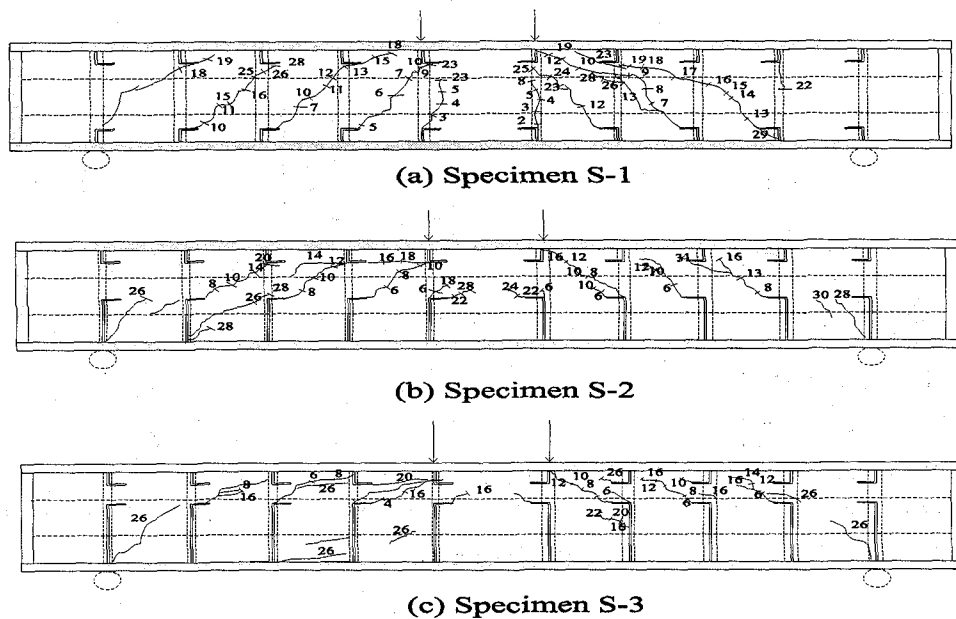


Fig. 5 Crack pattern (numbers indicating load in tf)

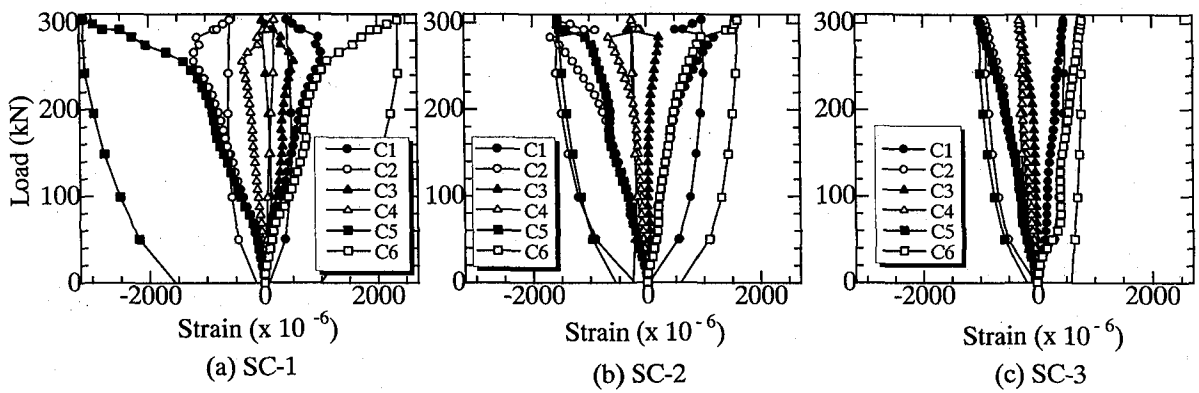


Fig. 6 Strain along vertical part of the shear connector, specimen S-1

It can be seen that strain gauges, C1 and C2 (point 1, near the top of the shear connector as shown in Fig.3), C5 and C6 (point 3, near the welded point) indicate a rather large strain and that the strain distributions were rather large in shear connector SC-1.

### 3.3 Stress distribution of lower flange plate

Figure 7 shows the measured stress distribution of the lower flange plate in the observed shear span of the tested specimens. The local bending of the lower flange plate occurred near the shear connector as indicated by the measured point b in Fig. 7 (b), and point d in Fig. 7 (c). To avoid the effect of the local bending of the lower flange plate near the shear connector, the measured stress of the lower flange plate that will be used to calculate the transferred shear force should be at some distance from a shear connector.

Judging from the above mentioned, the difference of measured stresses of points a-c, c-e and e-g were used to calculate the transferred shear force through the shear connector SC-1, SC-2 and SC-3 respectively.

### 3.4 The equivalent stiffness of the shear connector

In order to explain the experimental results of the relative displacement of the shear connector, the equivalent stiffness was introduced. The equivalent stiffness of the shear connector is the stiffness provided by the stiffness of the shear connector itself and the concrete surrounding the shear connector.

The shear connector element in the steel concrete sandwich beam was assumed as shown in Fig. 8. The fixed end at the bottom of the shear connector and no rotation at the top of the shear connector are assumed as the boundary conditions. Uniformly distributed load which balances the transferred shear force is assumed.

The basic equation used to calculate the displacement is:

$$(EI)_{eq} \frac{d^4 y}{dx^4} = q \quad (1)$$

where,  $(EI)_{eq}$  : the equivalent stiffness of the shear connector

$q$  : uniform load

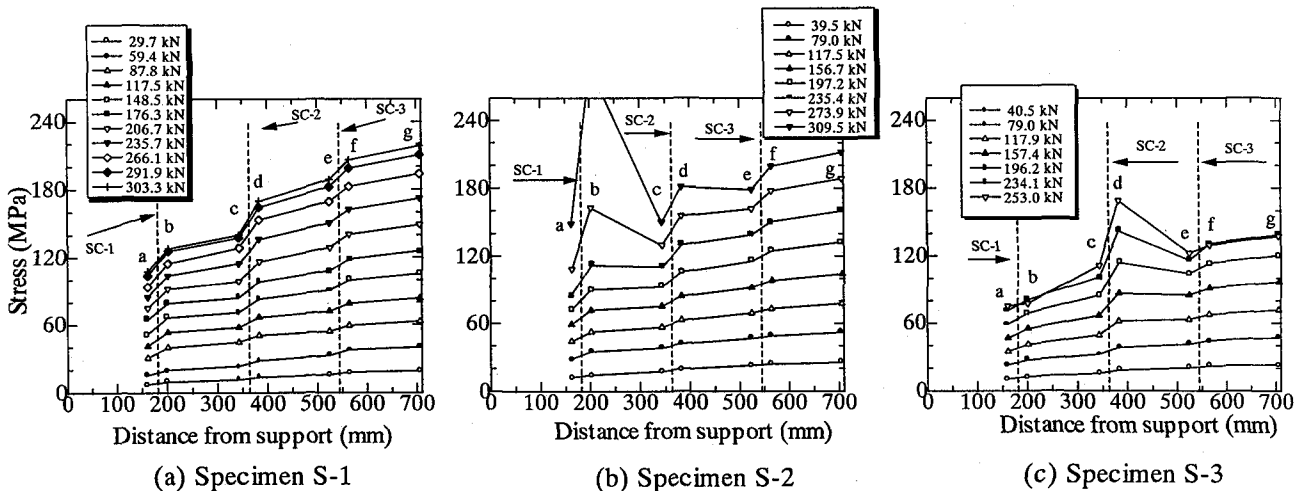


Fig. 7 Stress distribution of the lower flange plate

After solving Eq. (1) with the boundary conditions in Fig. 8, the formula for displacement of the shear connector can be expressed as follows :

$$y = \frac{1}{24(EI)_{eq}} \left( \frac{Q_a}{h_{sc}} x^4 - 4Q_a x^3 + 4Q_a h_{sc} x^2 \right) \quad (2)$$

where,

- $y$  : horizontal displacement at any point of the shear connector at distance,  $x$  from the bottom
- $Q_a$  : transferred shear force
- $h_{sc}$  : height of the shear connector

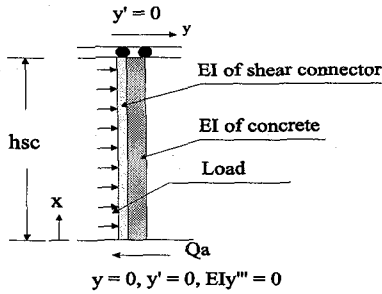


Fig. 8 Simplified shear connector and loading conditions

The curvature of the shear connector was calculated by the following equation :

$$\varphi = \frac{1}{(EI)_{eq}} \left\{ \frac{Q_a x^2}{2h_{sc}} - Q_a x - \left( \frac{Q_a h_{sc}}{6} - \frac{Q_a h_{sc}}{2} \right) \right\} \quad (3)$$

The equivalent stiffness of the shear connector in the elastic range may be estimated by substituting the experimental result of the transferred shear force and horizontal displacement at the top of the shear connector with

$Q_a$  and  $y$  in Eq.(2). The concept of introducing the equivalent stiffness in inelastic range is based on the assumption that the stiffness of the shear connector itself is constant and the effective stiffness of the concrete surrounding the shear connector reduces with an increase in the transferred shear force. Therefore, in the present study the following equivalent stiffness with a similar function to that of Branson's equation was adopted.

$$(EI)_{eq,r} = \left( \frac{Q_c}{Q_a} \right)^n (EI)_{eq,i} + \left[ 1 - \left( \frac{Q_c}{Q_a} \right)^n \right] (EI)_{sc} \quad (4)$$

where,

- $Q_c$  : transferred shear force at the beginning of inelastic range
- $Q_a$  : transferred shear force at which the relative displacement is being computed
- $(EI)_{eq,i}$  : initial equivalent stiffness
- $(EI)_{sc}$  : stiffness of shear connector itself
- $n$  : constant,  $n = 2$

As the first step, the transferred shear force at the beginning of the inelastic range was adopted from the experimental results. The initial equivalent stiffness of the shear connector was considered to be the same for the same sized shear connectors. Therefore, the average value of the initial equivalent stiffness among the initial equivalent stiffnesses of shear connectors SC-1, SC-2 and SC-3 was taken as the initial equivalent stiffness. The analytical results of the relationship between the transferred shear force and relative displacement for specimens S-1, S-2 and S-3 compared with the experimental ones are shown in Figs. 9, 10 and 11 respectively. The broken lines in the figures indicate the stiffness of the shear connector itself.

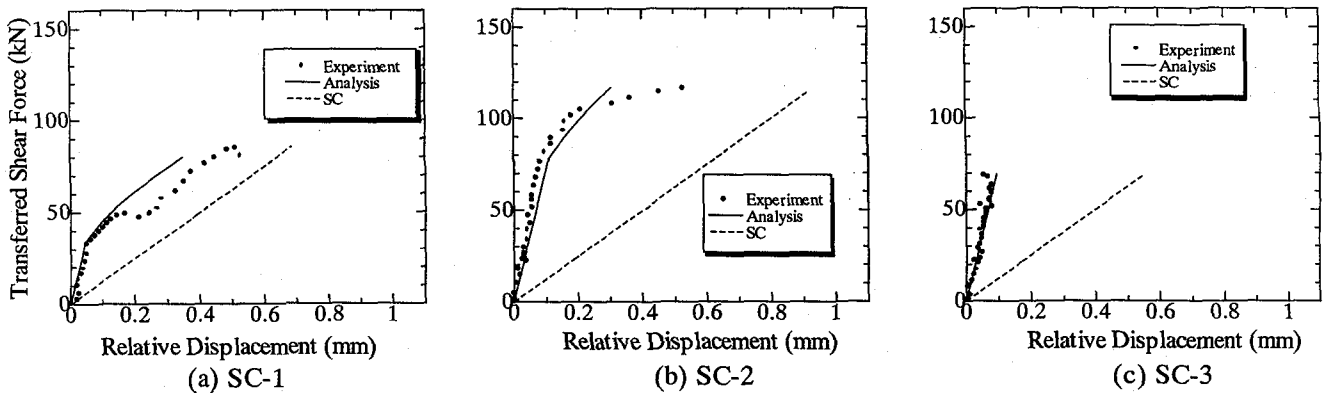


Fig. 9 The experimental and the analytical relationship between transferred shear force and relative displacement in specimen S-1

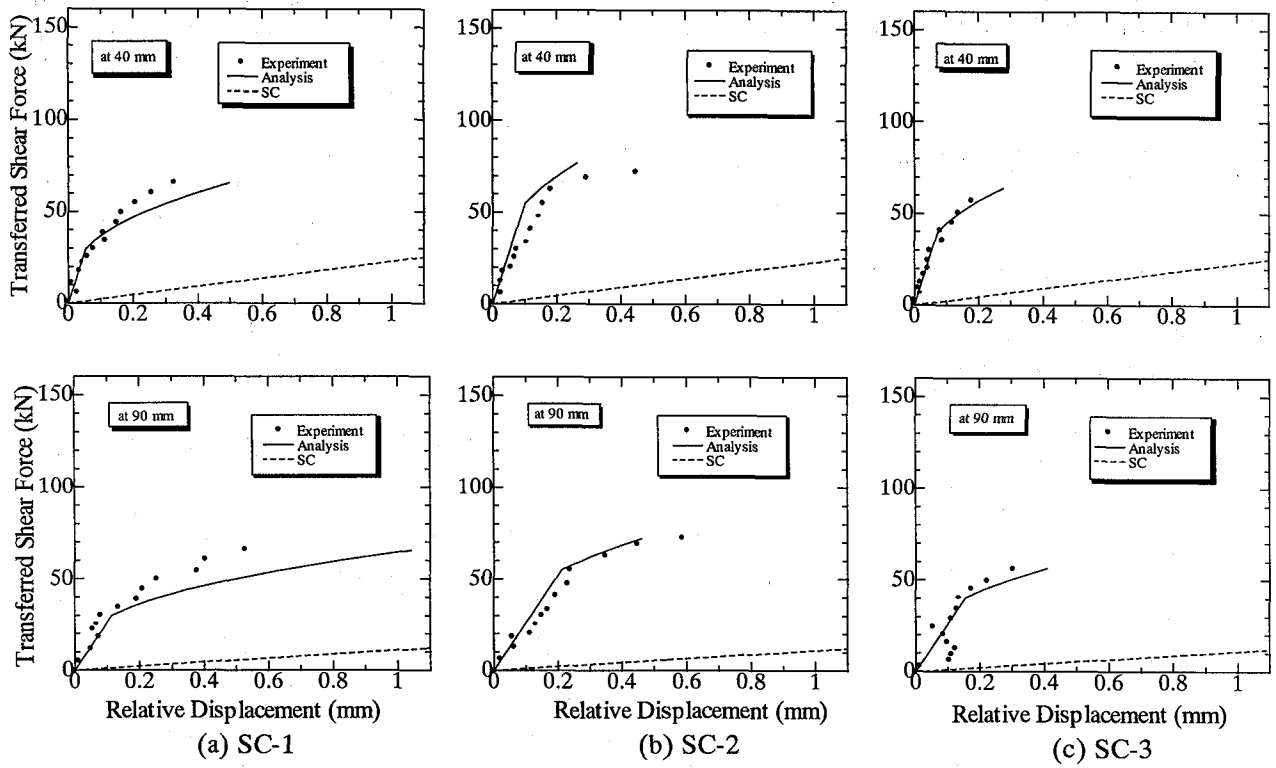


Fig. 10 The experimental and the analytical relationship between transferred shear force and relative displacement in specimen S-2

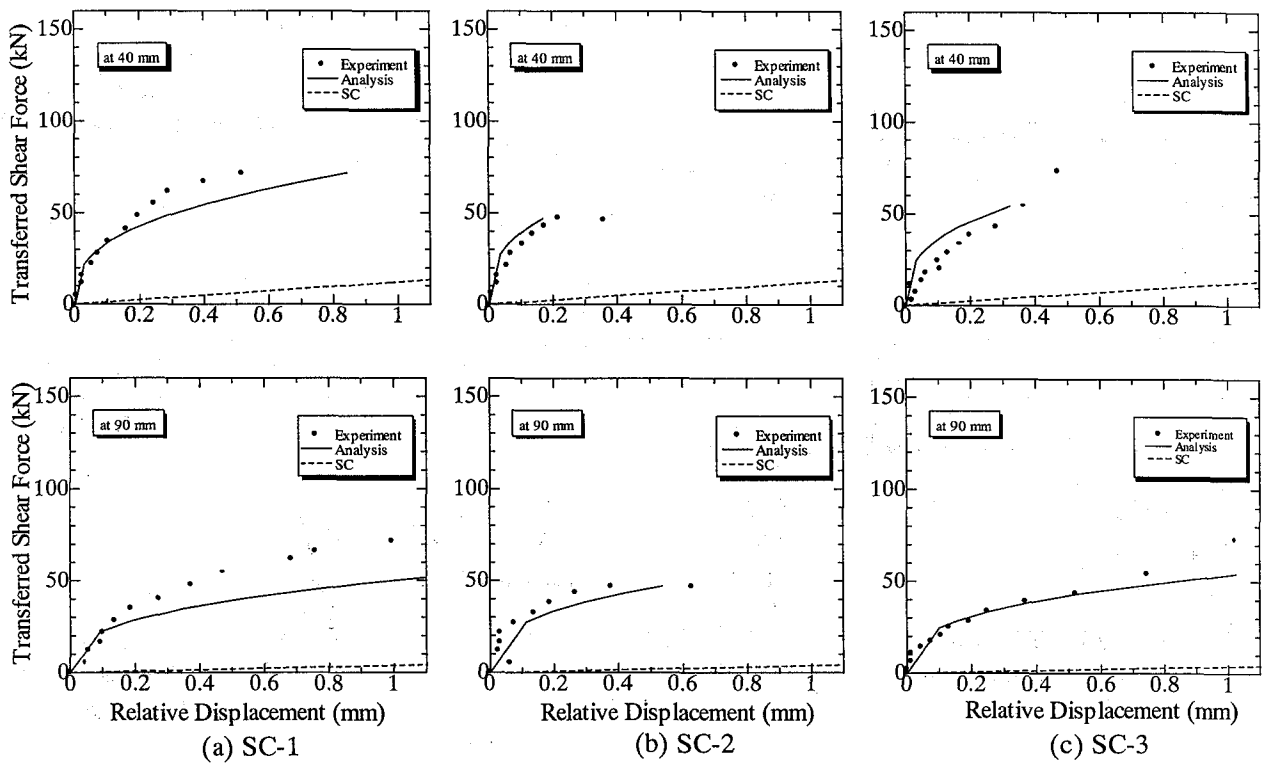


Fig. 11 The experimental and the analytical relationship between transferred shear force and relative displacement in specimen S-2

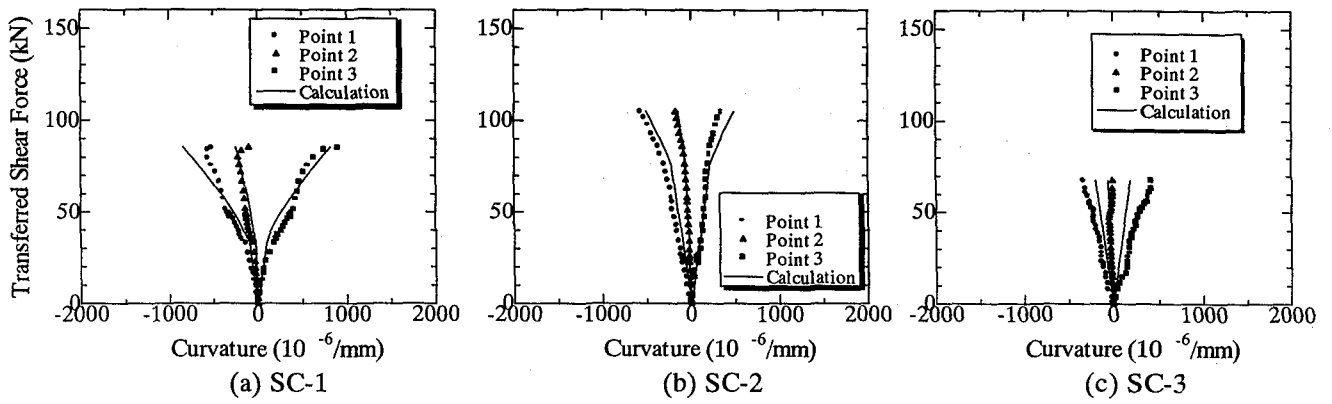


Fig. 12 The experimental and the analytical relationships between transferred shear force and curvature in specimen S-1

It can be seen that for all specimens except at the height of 90 mm measured point of the specimens S-2 and S-3 the analytical initial stiffnesses are in good agreement with the experimental ones. It can be concluded that the simplified shear connector and loading conditions may not be applicable to specimens S-2 and S-3 with 90 mm and 140 mm height of shear connectors.

Figure 12 shows the comparison between the analytical results and the experimental results of the curvature of the shear connector for specimen S-1. The curvature was calculated for three locations at the vertical part of the shear connector. The experimental one was calculated by dividing the difference between the strains in front of and behind the vertical part of the shear connector by the thickness of the shear connector. The curvature calculated by Eq. (3) gave a rather good agreement with the experimental results.

### 3.5 Determination of the initial equivalent stiffness of the shear connector

As already mentioned in previous section, the equivalent stiffness of the shear connector is the summation of the stiffnesses of the shear connector itself and the effective stiffness of the concrete surrounding the shear connector which can be expressed as follows:

$$(EI)_{eq,i} = (EI)_{sc} + (EI)_{con} \quad (5)$$

where,

$(EI)_{eq,i}$  : initial equivalent stiffness

$(EI)_{con}$  : initial effective stiffness of the concrete

$$\left( = E_c \frac{bt_c^3}{12} \right)$$

$E_c$  : Young's modulus of concrete

$t_c$  : effective thickness of surrounding concrete

$b$  : width of the shear connector

It was observed that the initial equivalent stiffness increased with increased height of the shear connector. The increase in the initial equivalent stiffness can be considered a consequence of increasing the initial effective stiffness of concrete surrounding the shear connector while the stiffness of the shear connector itself was constant. It is considered that there is a relationship between the height of shear connector  $h_{sc}$  and the effective thickness of the surrounding concrete  $t_c$ , which can be expressed as follows :

$$t_c = 1.42 h_{sc}^{0.675} \quad (t_c \text{ and } h_{sc} \text{ in mm}) \quad (6)$$

Figure 13 shows the relationship between the height of the shear connector and the effective thickness of the concrete. By knowing the effective thickness of the surrounding concrete, the effective stiffness of the concrete can be calculated. Likewise the stiffness of the shear connector itself can be calculated by knowing the steel properties of the shear connector. The initial equivalent stiffness, stiffness of the shear connector itself, initial effective stiffness of concrete (predicted and experimental results) and effective thickness of the concrete are summarized in Table 4.

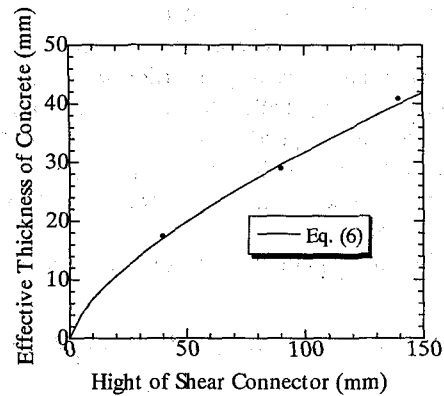


Fig. 13 The relationship between  $h_{sc}$  and  $t_c$

Table 4 Initial equivalent stiffness properties

Specimen	$(EI)_{eq,i}$ kN-mm <sup>2</sup>	$(EI)_{sc}$ kN-mm <sup>2</sup>	$(EI)_{com}$ (kN -mm <sup>2</sup> )		$t_c$ mm
			Exp.	Pred.	
S-1	$1.95 \times 10^6$	$3.33 \times 10^5$	$1.62 \times 10^6$	$1.58 \times 10^6$	17.3
S-2	$7.91 \times 10^6$	$3.33 \times 10^5$	$7.57 \times 10^6$	$8.16 \times 10^6$	28.9
S-3	$2.13 \times 10^7$	$3.33 \times 10^5$	$2.09 \times 10^7$	$2.00 \times 10^7$	40.6

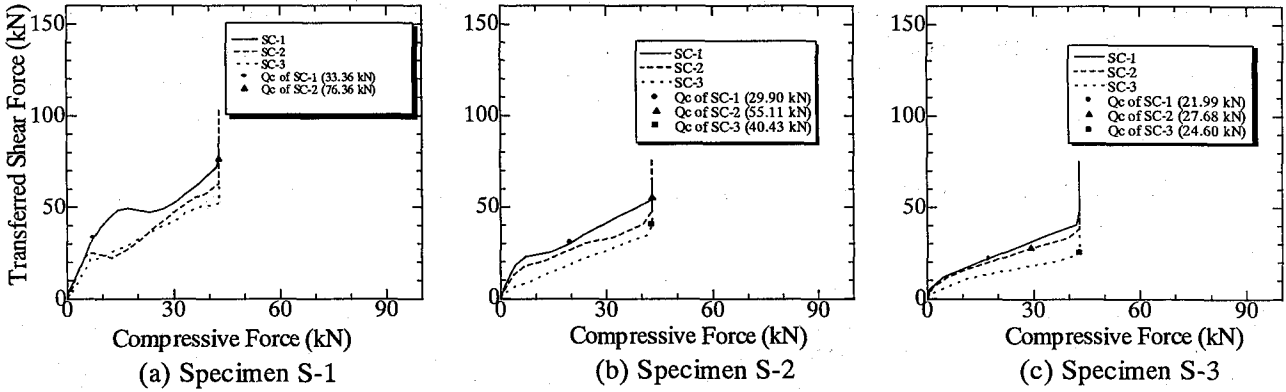


Fig. 14 The relationship between transferred shear force and compressive force

### 3.6 Determination of the $Q_c$ value

$Q_c$  is the value of the transferred shear force when sudden decrease of the equivalent stiffness of the shear connector starts. The shear connector continues to transfer shear force even after shear cracking has taken place at the top of the shear connector. Figure 14 shows the relationship between the transferred shear force and the compressive force for the tested specimens as well as  $Q_c$ .

Figure 15 shows relationship between  $Q_c$  and the corresponding compressive force. In order to estimate  $Q_c$  value it was assumed that  $Q_c$  increased in proportion to square root of the compressive strength of concrete,  $f'_c$ . It was also assumed that  $Q_c$  increased with an increase in the area of the surrounding concrete which is in proportion to its effective thickness,  $t_{sc}$ . From the experiment, it was found that  $Q_c$  increased when the compressive force,  $F'_{com}$  increased. Regarding the effect of the height of the shear connector, it was found that  $Q_c$  did not increase with an increase in the shear connector height,  $h_{sc}$  although  $Q_c$  was assumed be in proportion to  $t_c$  which increases with  $h_{sc}$  (see Eq. (6)). Therefore it was assumed that  $Q_c$  increased in proportion to the inverse of  $h_{sc}^{1.3}$ . The relationship between  $Q_c$  and its parameters is as shown in Fig. 14. The y-axis shows the normalized  $Q_c$  with its parameters.

Lastly, the following equation is proposed to estimate  $Q_c$ :

$$Q_c = (0.00993 F'_{com} + 0.265) f_c^{1/2} \frac{b_{sc} t_c}{h_{sc}^{1.3}} \quad (7)$$

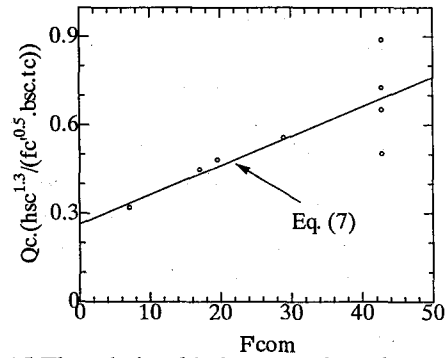


Fig. 15 The relationship between  $Q_c$  and corresponding compressive force

The predicted values of the transferred shear force with sudden decreasing of the equivalent stiffness of the shear connector,  $Q_c$  were compared with the experimental values (Fig.16). It can be seen that the predicted results are in good agreement with the experimental ones.

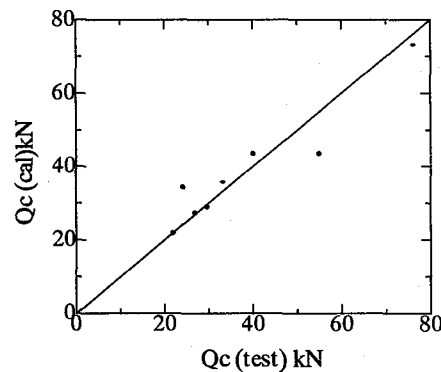


Fig. 16 Comparison between  $Q_c$  values predicted and measured



#### 4. CONCLUSIONS

This study investigates the relationship between the transferred shear force and the relative displacement of the shear connector in steel-concrete sandwich beams ( $Q_a$ - $y$  relationship). The relationship is rather linear until sudden decrease in the stiffness takes place.

The shear connector continues to transfer shear force even after the shear cracking has taken place at the top of the shear connector.

The initial linear stiffness can be calculated by assuming the effective area of the concrete surrounding the shear connector.

The transferred shear force at the sudden decrease of the stiffness in the  $Q_a$ - $y$  relationship,  $Q_c$  increases with increase of the compressive strength of concrete, width of the shear connector, effective thickness of surrounding concrete and the compressive force acting in the surrounding concrete, and decrease of the height of the shear connector.

The estimation of the relative displacement of the shear connector can be made by the following procedure:

1. Estimate the initial equivalent stiffness of the shear connector,  $(EI)_{eq,i}$  using Eq. (5) and Eq. (6).
2. Estimate the value of the transferred shear force at sudden decrease in the equivalent stiffness of the shear connector,  $Q_c$  using Eq. (7).
3. Calculate the reduced equivalent stiffness when the transferred shear force increases,  $(EI)_{eq,r}$  using Eq. (4).

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