APPLICATION OF RIGID BODIES SPRING MODEL TO PRECAST MEMBERS PRESTRESSED WITH INTERNAL AND EXTERNAL TENDONS

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This paper presents the experimental and analytical results on the bending behavior of a structure made of five precast concrete members prestressed by internal and external tendons. The analysis method used in this study is the Rigid Bodies Spring Model[1] in which the prestressing steel tendons and the deviators were introduced as the beam element model, for the concrete and reinforcement steel, the plate element model was used . The analytical results were in good agreement with the experimental results.

It was confirmed that the Rigid Bodies Spring Model (hereinafter written RBSM) was effective for analyzing the bending behavior of a precast structure prestressed by internal and external tendons.

Key Words: Precast member, prestressed concrete, external tendon, RBSM

1. INTRODUCTION

Prestressed concrete has been widely used in the field of construction for quite a long time, especially in bridges. In almost all structures, the prestressing system has been internal prestressing. Recently the use of external prestressing has increased remarkably all over the world, most extensively in building new structures and reinforcing old bridges, especially box girder bridges. Many studies, experimentally [2,3] and analytically [4,5], on the behavior of the external tendons were done. However, many obscure problems concerning the use of external prestressing system in reinforcing old structure have remained to be unsolved.

In this study, there were two main purposes. One was to investigate experimentally the behavior of old existing structures after represtressing with external tendons. The other was to apply the RBSM method, which is suitable for nonlinear problems, for analyzing the externally prestressed structure.

2. TEST SPECIMENS AND EXPERIMENTAL METHOD

2.1 Test Specimens

Experiments were done on a segmental concrete beam consisting of five members with a box shape cross-section.

The members were reinforced with a minimal reinforcement steel, and prestressed with internal or external tendons or both. In the joint, four shear keys and an epoxy resin were used. Fig. 1 shows the layout of the test beam. Based on the purpose of this research, experiments were divided into three cases as shown in Table 1.

Case 1: The beam was prestressed with four internal tendons bonded to the concrete by a cement grout. The load was applied up to failure.

Case 2: The beam was prestressed with four internal tendons bonded to the concrete and two external tendons, using steel deviators. The load was applied up to failure.

Case 3: The beam was prestressed with four internal tendons bonded to the concrete, the load was applied up to the first cracks, after release of the load the beam was represtressed with two external tendons and finally the load was applied up to failure.

2.2 Loading method and measured items

Loading method and measured items are shown in Fig. 2. Taking the span length of 580 cm long as a simple supported beam, two symmetrical loading points were provided. For cases 1 and 2, the load was increased monotonically by the load control method until the beam reached final failure. For case 3, the load was increased in the same way as cases 1 and 2 until the first cracks occurred in the center block or the joint started to open clearly by the bending moment. After this first loading, the beam was once unloaded and represtressed with two external tendons. Then, the beam was reloaded monotonically up to the failure of the beam.

The measured items were: displacements of five points of the beam by a displacement meters; compressive strains in the upper edge of the beam; tensile strains at the lower edge of the beam and strains at the center of the cross-section by electric resistance strain gauges; the opening of the joint by arch displacement meters.

The loading force was measured by load cell and the prestressing force was measured at the end of the tendon by center hole type load cell.

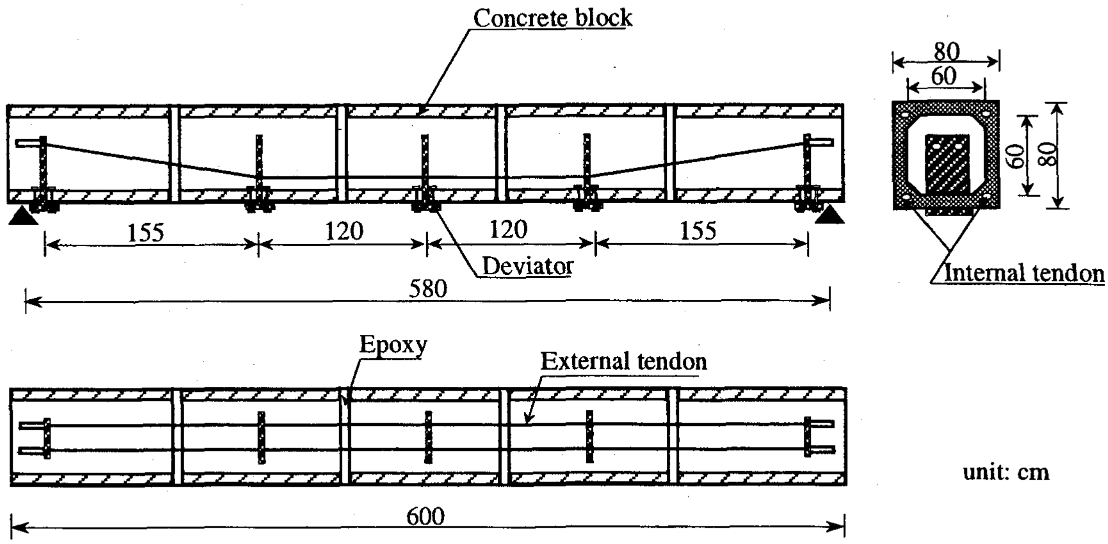

Fig. 1 Layout of Test Beam

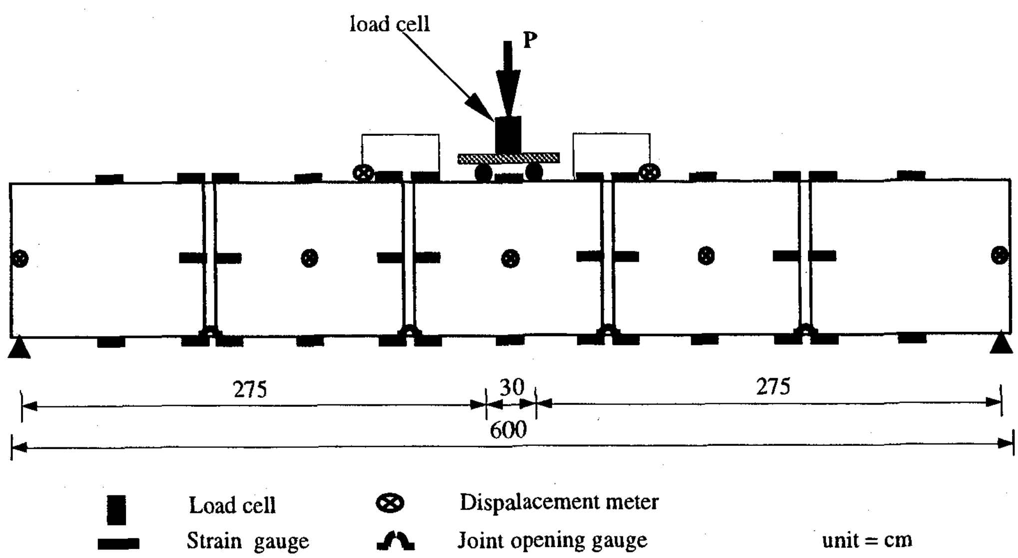

Fig. 2 Loading Method and Measured Items

Table 1 Cases of Experiment

Case	Tendon layout	Prestressing force (kN)	Remarks
1	internal	200	up to failure
2	internal + external	200 + 100	up to failure
3	internal + external	200 + 100	after the first cracks external prestressing was applied

3. ANALYTICAL METHOD

RBSM [1], which was proposed by Prof. KAWAI in 1977, is a numerical analysis method developed as a limit analysis of structures, in which the structure is idealized as an assemblage of rigid body elements connected by two kinds of distributed springs as shown in Fig. 3. This analysis can easily express the discontinuous phenomenon due to cracking and failure [6].

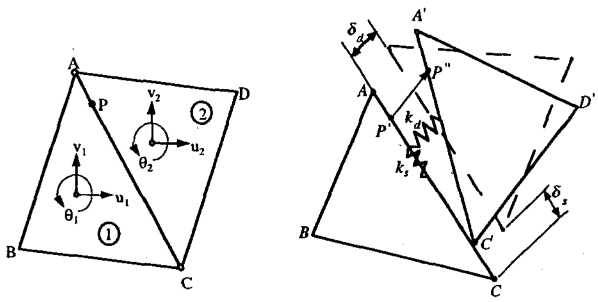

Fig. 3 Rigid Triangular Elements

3.1 Brief Description of Two Dimensional RBSM

The rigid displacement field is assumed in each element in terms of the displacement (u,v,θ) of the centroid as shown in Fig. 3. The relative displacement vector $\{\delta\}$ of the arbitrary point P can be given by the following equations:

$$\{\delta\} = [M]\,[R]\,[Q]\,\{u\} = [B]\,\{u\} \qquad (1)$$

where, $\{\delta\}=[\delta_d,\delta_s]^T$, $[Q]$ is a matrix to derive the displacements of points P' and P'' from the centroidal displacement $\{u\}=[u_1,v_1,\theta_1;u_2,v_2,\theta_2]^T$, $[R]$ is a coordinate transformation matrix and $[M]$ is a matrix to derive the relative displacement from the displacements of P' and P''.

The spring constants k_d and k_s which resist normal and tangential forces respectively on the contact surface between element ① and ② can be determined systematically by using the finite difference equation for strain components $\{\varepsilon\}=[\varepsilon_d,\gamma_s]^T$ as follows:

$$\{\varepsilon\}=\begin{Bmatrix}\varepsilon_d\\ \gamma_s\end{Bmatrix}=\frac{1}{h_1+h_2}\begin{Bmatrix}\delta_d\\ \delta_s\end{Bmatrix}=\frac{1}{h}\{\delta\} \qquad (2a)$$

$$\{\sigma\}=\begin{Bmatrix}\sigma_d\\ \tau_s\end{Bmatrix}=\begin{Bmatrix}\dfrac{(1-v)E}{(1-2v)(1+v)}\varepsilon_d\\ \dfrac{E}{(1+v)}\gamma_s\end{Bmatrix} \qquad (2b)$$

where E is the young's modulus, ν is the Poisson ratio and $h=h_1+h_2$ is the projected length of a vector connecting centroids along the line perpendicular to $\overline{AC}$, as shown in Fig. 4.

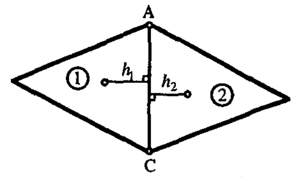

Fig. 4 Definition of Projected Length

Substituting Eq. (2a) into Eq. (2b), the following relation can be obtained:

$$\{\sigma\}=\begin{bmatrix}k_d & 0\\ 0 & k_s\end{bmatrix}\begin{Bmatrix}\delta_d\\ \delta_s\end{Bmatrix}=[D]\,\{\delta\} \qquad (3a)$$

$$k_d=\frac{(1-v)E}{(1-2v)(1+v)h}, \qquad k_s=\frac{E}{(1+v)h} \qquad (3b)$$

Based on the above preliminaries, the strain energy V stored in the spring system on $\overline{AC}$ will be given as the following matrix equations:

$$V=\frac{t}{2}\int_A^C\{\delta\}^T[D]\{\delta\}ds \qquad (4a)$$

$$V=\frac{t}{2}\{u\}^T\int_A^C[B]^T[D][B]ds\{u\} \qquad (4b)$$

where t is the element thickness. By applying Castigliano's theorem to Eq. (4b) the following stiffness equation can be derived:

$$\frac{\partial V}{\partial\{u\}}=[K]\{u\}=\{F\} \qquad (5)$$

where $[K]$ is the stiffness matrix and $\{F\}$ is the centroidal load defined by the following equation:

$$\{F\}=[X_1,Y_1,M_1;X_2,Y_2,M_2]^T \qquad (6)$$

3.2 Modelization of Tendons and Deviators Using Beam Element Model

The beam element model was used to introduce the prestressing tendons and deviators into the RBSM. This model considers the deformation of two rigid bars connected by a system spring as shown in Fig. 5 [7].

The horizontal and vertical displacement at the connection point of the two bars can be given by the Eq.(1). Using the same process described in paragraph 3.1, the expressions for k_d and k_s can be derived as follows:

$$k_d=\frac{E}{L}, \quad k_s=\frac{E}{2(1+v)L} \qquad (7)$$

where $L=\dfrac{L_1+L_2}{2}$

Here, the strain energy V stored in the spring system is given by the following equation:

$$V=\frac{1}{2}\{u\}^T\int_A[B]^T[D][B]dA\{u\} \quad (8)$$

where A is the cross-section area of the bar element.

In this study we assumed, for the external tendons and deviators, the model shown in Fig. 6, in which tendon and deviator were supposed to be connected by a pin element with vertical spring $k_d=\infty$ and shearspring $k_s=0$. Therefore, for the internal tendons the proposed model is shown in Fig. 7, where the bond-slip model between the grout and tendons is introduced into the shear spring factor.

3.3 Materials Characteristics

a. Concrete

The dashed line in Fig. 8 shows the uniaxial stress-strain relationship of the concrete. In this analysis, the relationship is approximated according to the plane line[6]. For the tension side, the tensile stress is assumed to be lineary released as tension-softening effect.

b. Reinforcement Steel and Tendon

After yielding, the elastic-perfectly plastic material is assumed for the reinforcement steel and tendon.

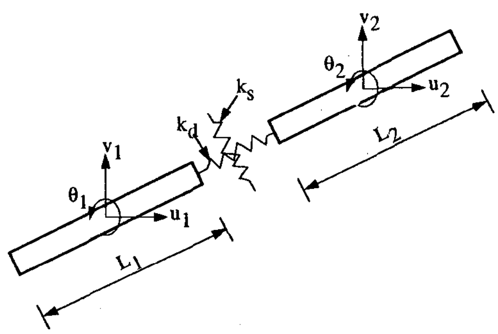

Fig. 5 Beam Element Model

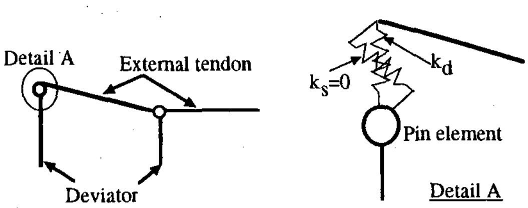

Fig. 6 Assumed Model for External Tendon

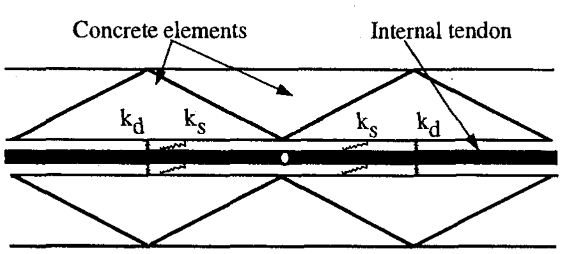

Fig. 7 Element of Internal Tendon

3.4 Nonlinear Analysis Algorithm

In this paper, the analysis was carried out according to an incremental displacement method. The algorithm shown in Fig. 9 was proposed for material nonlinear analysis[8], in which Rmin method was modified to add the released force to the remaining load while counting the applied load, and to take into account the cracking and compressive failure.

3.5 Mesh Division for Analysis

A mesh division of a half test beam is shown in Fig. 10, where the non-smudgy triangular element indicates the concrete element, the smudgy triangular element corresponds to the reinforcement steel and concrete layers. The thick line corresponds to the external tendons, deviators and internal tendons. The line element as a particular triangle was used for the support point, loading point and laterally immovable plane at midspan.

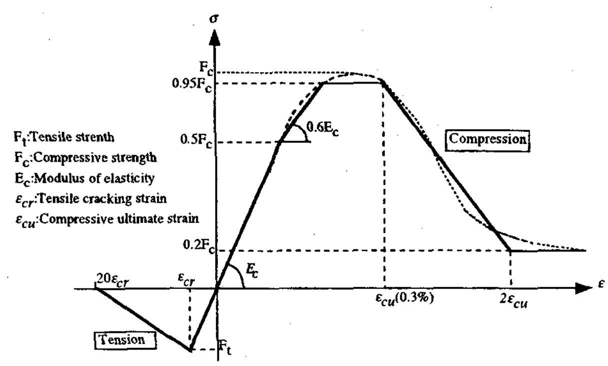

Fig. 8 Stress-Strain Relationship for Concrete

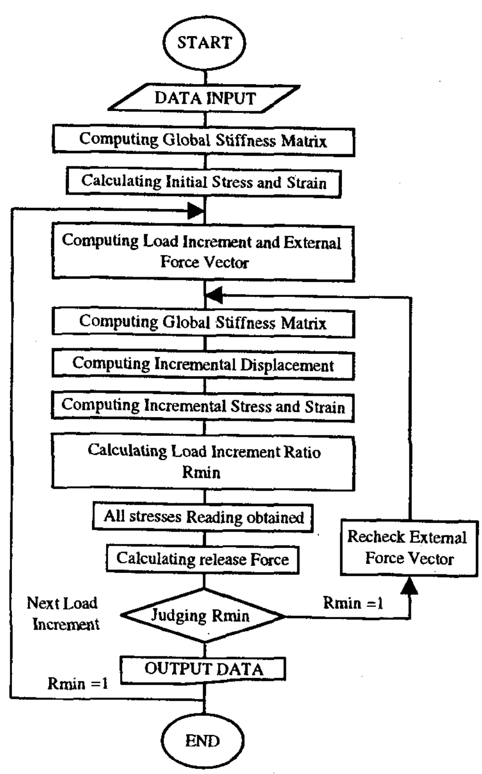

Fig. 9 Flow Chart

Table 2 Material Properties

Concrete		
Young's Modulus	GPa	31
Poisson's Ratio		0.2
Compressive Strength	MPa	32
Tensile Strength	MPa	3
Reinforcement Steel		
Young's Modulus	GPa	210
Poisson's Ratio		0.3
Yield Stress	MPa	300
Prestressing Steel		
Young's Modulus	GPa	190
Yield Stress	MPa	1660

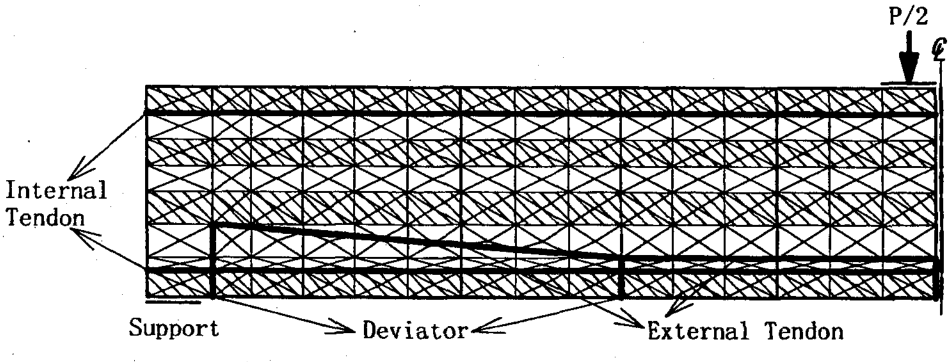

Fig. 10 Mesh Division for Half Part of Specimen

The mechanical properties used in the analysis are shown in Table 2.

4. RESULTS AND DISCUSSION

4.1 Load-Strain Behavior in The Joint

Fig. 11 and Fig. 12 show the strain behavior on contact surface in the joint of the bottom flange of the tested beams. Within the range of the opening-load, the relationship between the applied load and concrete strain changes proportionally. However in the stage exceeding the opening-load, it is observed that the strains do not increase and go down due to the opening of the joints. these results were approved by the analysis.

4.2 Load-Opening Behavior

Fig. 13 shows the opening of joints close to the center of the span. The epoxy resin and shear keys of joint were sufficient to resist up to the final stage of the experiments with no slip between blocks.

For case 1, the incremental rate of the joint opening rapidly becomes larger within 100 kN, however for case 2 and case 3, which present the same behavior, this load was about 50 kN greater than case 1. The analytical curves are well fitted with the experimental ones.

4.3 Load-Deflection Behavior

Fig. 14 shows the load vs. the deflection at midspan for the three cases. There is an obvious difference in the deflection behavior between the first loading stage and the second loading stage of case 3.

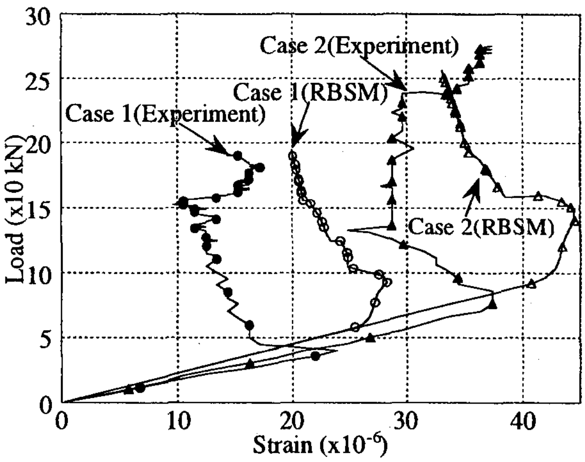

Fig. 11 Load-Strain Behavior (Tension Side)

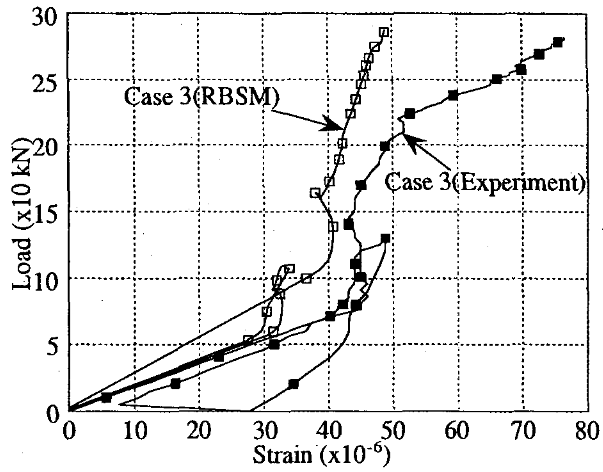

Fig. 12 Load-Strain Behavior (Tension Side)

In the first loading stage the load-deflection behavior is the same as in case 1, and the behavior of the second loading stage after represtressing with the external tendons is almost same as the one of case 2.

It is considered that it is directly due to the effect of prestressing force applied to the structure through only the internal tendons during the first loading stage and both of the internal and external tendons during the second loading stage. Also it can be observed that the analytical results are in good agreement with the experiments data.

Fig. 15, Fig. 16 and Fig. 17 illustrate typical longitudinal deflection profiles with increase of the loading. After cracking, large displacements are observed in both cases. The deflection seems to be presented by two curves divided at the center point, because of the development of opening cracks.

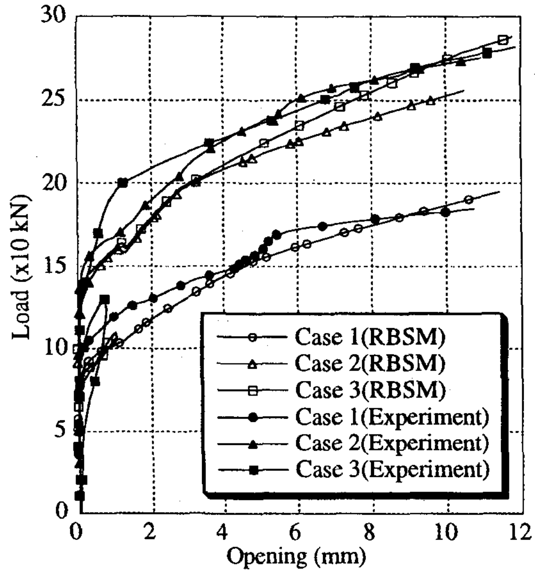

Fig. 13 Load-Opening Relation

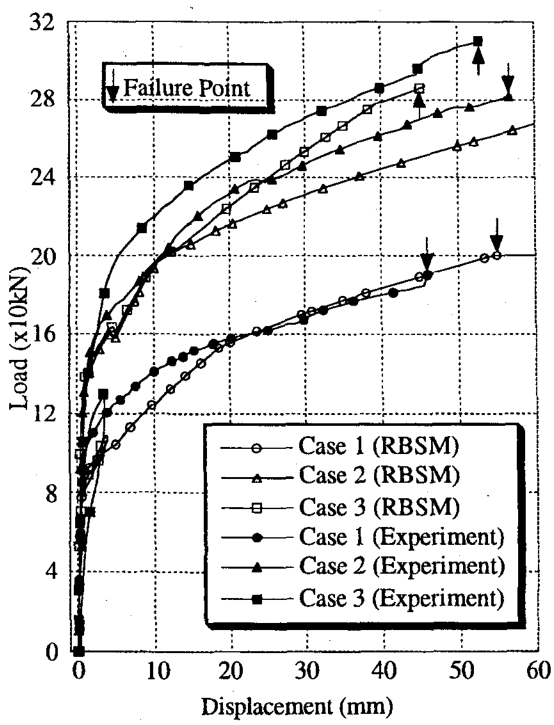

Fig. 14 Load-Deflection Behavior

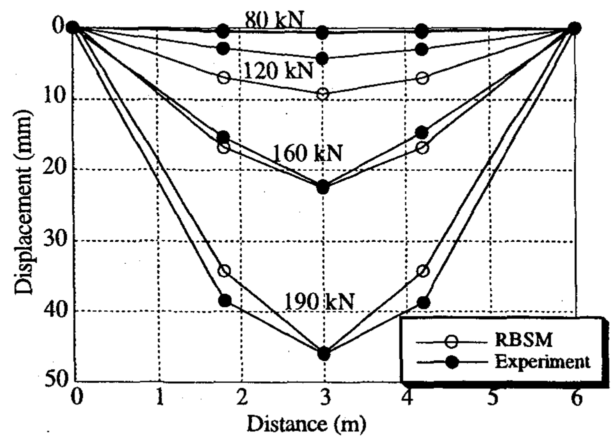

Fig. 15 Load-Deflection Profile (Case 1)

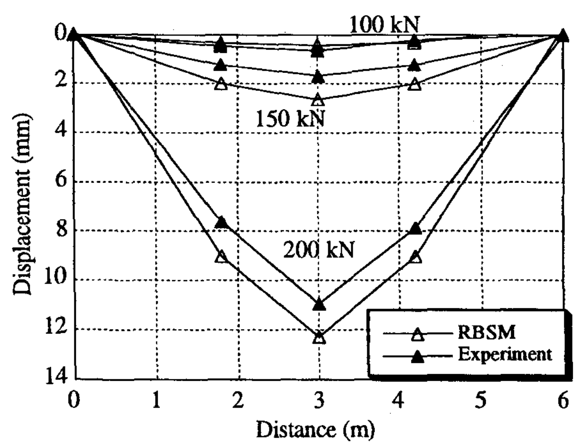

Fig. 16 Load-Deflection Profile (Case 2)

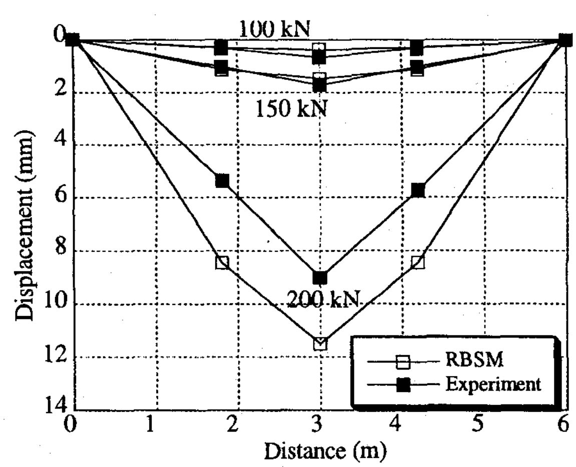

Fig. 17 Load-Deflection Profile (Case 3)

4.4 Load-Prestressing Force of External Tendons

Fig. 18 shows the load vs. the prestressing force of the external tendon in case 2. The analytical results fitted with the experimental ones up to the incremental rate of opening of the joint start to become larger, after that a different behavior was observed, which is due to friction between deviators and tendons. So it is necessary to consider friction problem in the future analysis, which can be solved by introducing a friction coefficient into the shear spring constant.

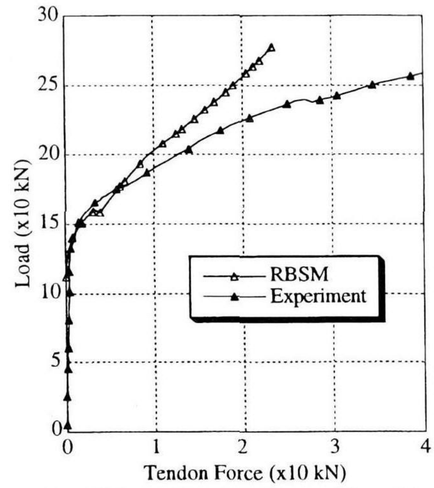

Fig. 18 Load-Tendon Force (Case 2)

4.5 Crack Pattern

Fig. 19, Photo 1 and Photo 2 show the crack layout of the different cases observed during the experiments and Fig. 20 shows the crack pattern obtained from the RBSM analysis.

In all cases it was almost the same crack layout. The cracks were in the upper flange of the central block due to

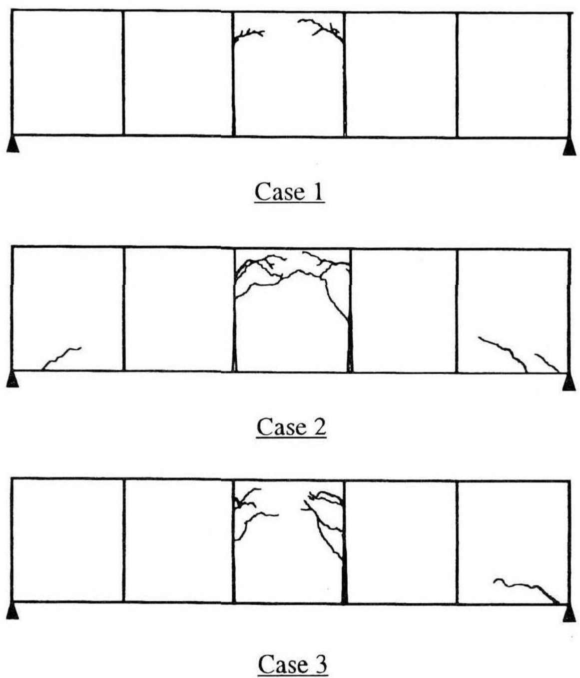

Fig. 19 Crack Layout (Experiment)

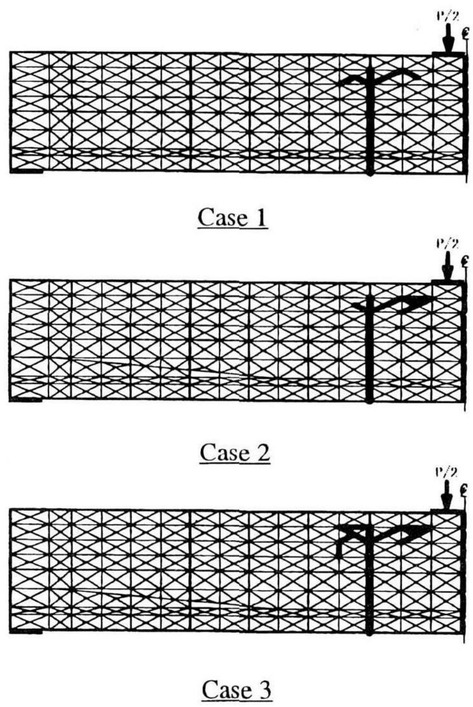

Fig. 20 Crack Layout (RBSM)

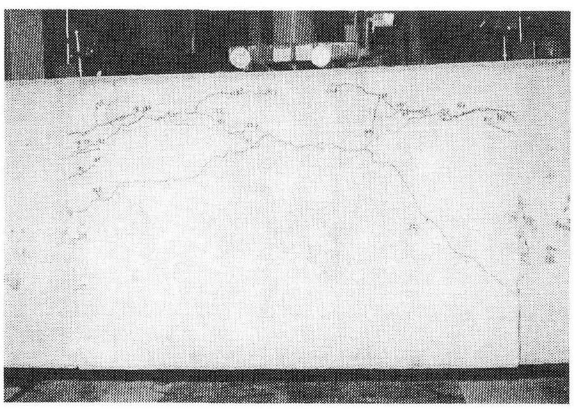

Photo 1 Crack Layout (Case 2)

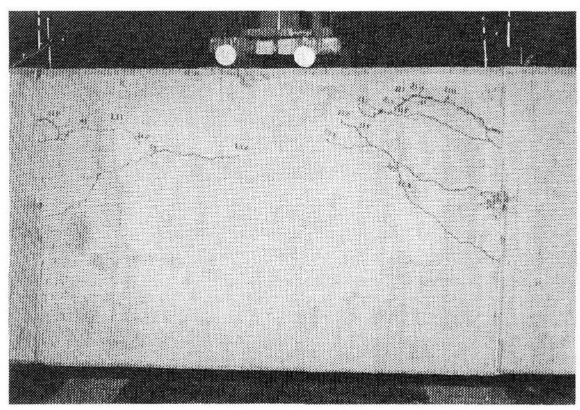

Photo 2 Crack Layout (Case 3)

compression of the concrete. Also it can be observed that the experiments and analysis gave almost the same crack pattern and joint opening length.

5. CONCLUSIONS

The experimental and analytical investigations in the bending behavior of a precast concrete beam reinforced with internal and external tendons were carried out. The analytical results of the RBSM were in good agreement with the experimental results, also the difference between the use of internal tendons and external tendons was observed. The following conclusions could be drawn from the above study:

1) The RBSM is an effective method to analyze the behavior of a precast structure prestressed with internal and external tendons.
2) It is possible to express the behavior of the tendons and diviators by introducing the beam elements method into the RBSM.
3) The bending behavior of a precast concrete beam reinforced with external tendons after cracking does not differ from the behavior of a precast concrete beam initially prestressed with external tendons.
4) A damaged precast prestressed concrete member can be restored or reinforced for a greater carrying load by represtressing with external tendons.

References

(1) Kawai, T., New Element Models in Discrete Structural Analysis, Journal of the Society of Naval Architects of Japan, No.141, pp. 174-180, 1977.
(2) Naaman, A. and Breen, J., External Prestressing in Bridges, ACI, SP.120, 1990.
(3) Fouré, B. et al., Flexural Behavior of Externally Prestressed Concrete Beam Monolithic or Made of Precast Segment, Workshop AFPC, St-Remy, France, pp. 163-174, 1993.
(4) Virlogeux, M. and M'rad, A., Flexural Behavior of Externally Prestressed Structures for Ultimate Loads, Workshop AFPC, St-Remy, France, pp. 185-205, 1993.
(5) Mutsuyoshi, H. et al., Flexural Behavior and Improvement of Ductility of Precast Segmental PC Beams with External Tendons, Journal of JSCE, Vol. 544, pp. 155-165, 1996-8.
(6) Kawai, T. et al., Discrete Limit Analysis of Reinforced Concrete Structures (1-3), Seisan-Kenkyu, Monthly Journal of Institute of Industrial Science, Vol. 38, No. 4-6, 1986.
(7) Kawai, T. and Chen, C.N., Discrete Element Analysis of Beam Bending Problems Including the Effect of Shear, Seisan-Kenkyu, Monthly Journal of Institute of Industrial Science, Vol. 30, No. 4, pp. 165-168, 1978.
(8) Ueda, M., Takeuchi, N., et al., Discrete Limit Analysis of Reinforced Concrete Structures Including the Effects of Tensile and Compressive Failure, Journal of Structural Engineering, JSCE, Vol. 36A, pp. 315-323, 1996.

(Received September 26,1997)