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# Prediction of Hysteretic Curve of Thin Steel Plates and Chaotic Time Series

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The accurate model of the constitutive relation of thin steel plate elements is necessary for the reliable nonlinear analysis of columns sustaining highway bridges and so on. However, accurate prediction of the hysteretic curve in conspicuously nonlinear domain is the difficult problem as well as the important matter. The main purpose of this paper is to propose a prediction method of hysteretic behavior of thin steel plate elements in more nonlinear strain level from that in relatively linear strain level. In order to achieve the prediction of the conspicuously nonlinear hysteresis, first, Momentary Deformation Modulus (MDM) is defined in this paper. Then, it is discriminated by the Lyapunov spectrum whether the time series of MDM behaves chaotically or not. If the behavior of the time series of MDM is deterministic chaos, application of a prediction method of chaotic time series results in good prediction accuracy, permitting the prediction of the hysteretic curve in the conspicuously nonlinear domain.

*Key Words: hysteretic curve, deterministic chaos, prediction*

## 1. Introduction

In order to grasp and predict the ultimate state of a structure / member, its nonlinear hysteretic behavior is important, which is found out of the constitutive relation of material and the general load-displacement relation considering the local instability [1), 2)]. Recently design methods proceed to the Limit State Design Method so that a method of modelling which is simplified and capable of precision is expected. So far, on various pieces of nonlinear behavior, many experimental data and case studies using finite element analysis of structural member and element are accumulated. Consequently if an appropriate modelling method is developed with aid of the accumulation and is effectively applied to the numerical analysis, it is convenient toward solving the problems to improve the accuracy and the efficiency for the nonlinear analysis.

In order to achieve an appropriate hysteretic model as described above, this paper is aimed at proposing a prediction method of the constitutive relation of thin steel plate elements, namely, flange plates under cyclic loading. That is to say, the main objective in this paper is to predict cyclic axial stress - strain relation of thin steel plates in more nonlinear strain level from that in relatively linear strain level. Herein, the axial stress and strain are average values of the plates under repetitive compressive-tensile loading in the axial direction.

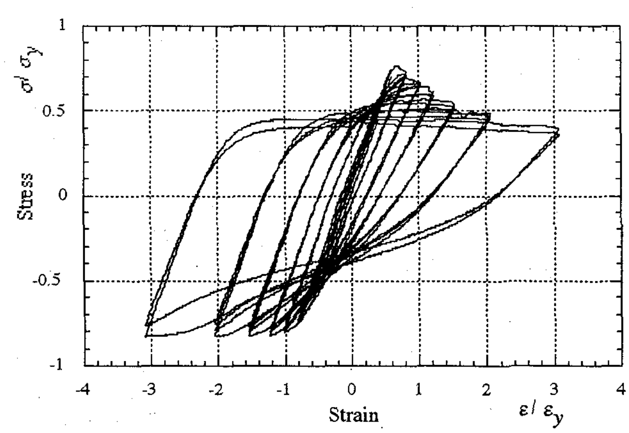



**Fig. 1** Hysteretic Curve

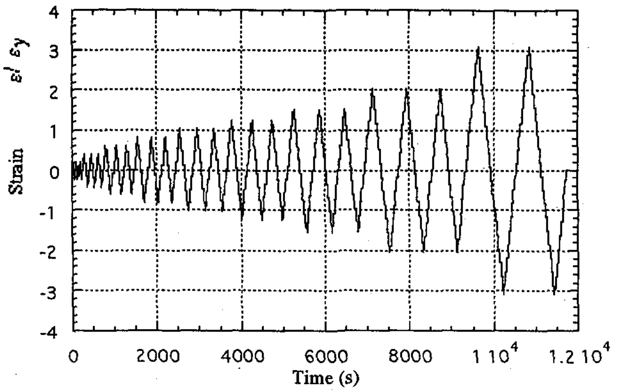



**Fig. 2** Time Series of Strain

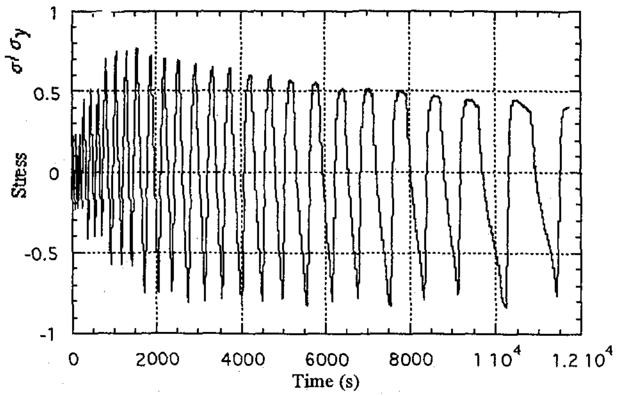
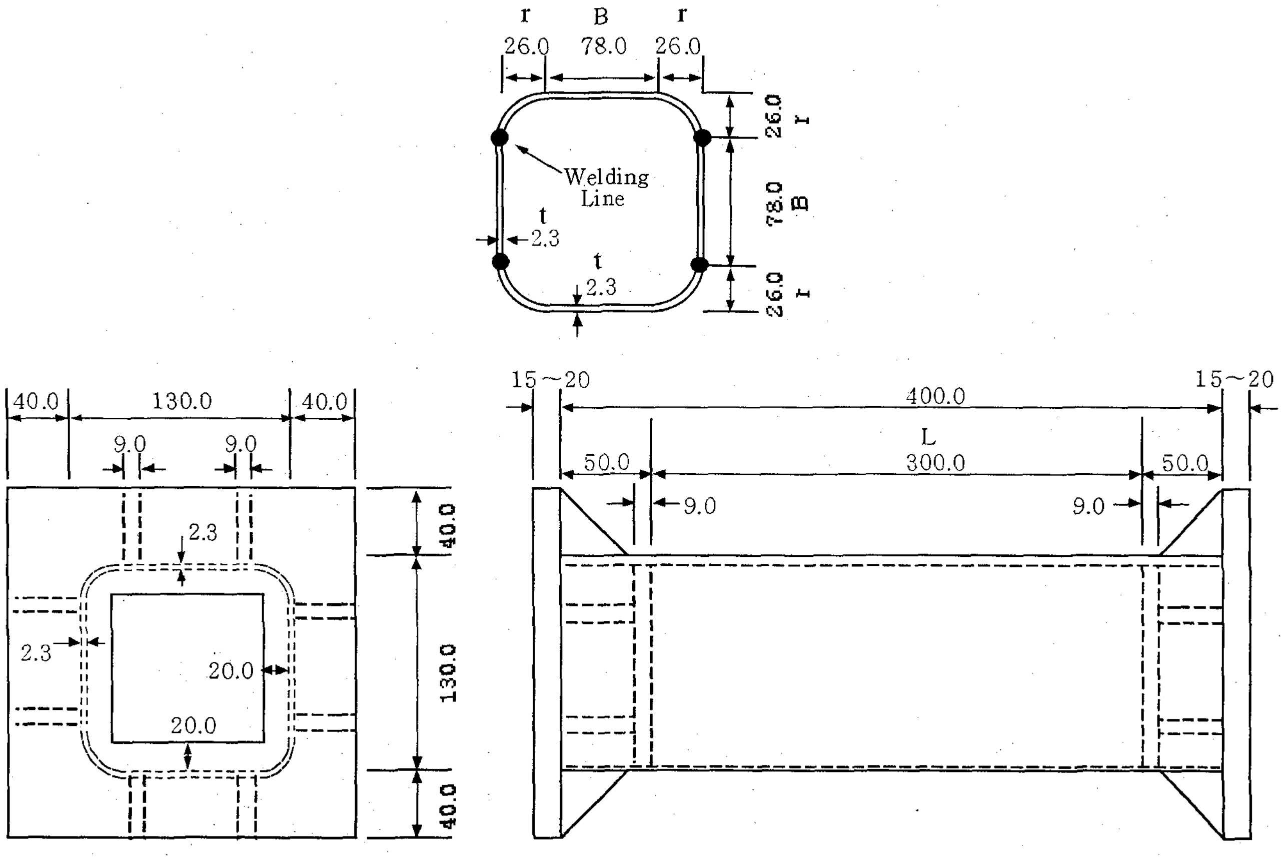



**Fig. 3** Time Series of Stress




(Unit : mm)

**Fig. 4** Specimen Used for Experiment

## 2. Experiment of Thin Steel Plates and Momentary Deformation Modulus

A cyclic compressive-tensile experiment has been conducted to obtain hysteretic data for verifying validity of the proposing prediction method of hysteresis in relatively conspicuous nonlinear domain[2].

Axial stress - strain relation, namely, a hysteretic curve obtained by the compressive-tensile experiment on a specimen is shown in **Fig. 1**. The loading is conducted in the way of the displacement control as shown in **Fig. 2**. As a response, the time series of stress shown in **Fig. 3**

**Table 1** Design Measurements of Specimen

| PLATE B (mm) | CORNER r (mm) | THICKNESS t (mm) | LENGTH L (mm) | AREA B×t (cm²) |
|---|---|---|---|---|
| 78.0 | 26.0 | 2.3 | 300.0 | 10.77 |

**Table 3** Actual Measurements and Yield Points

| CORNER r (mm) | LENGTH L (mm) | AREA B×t (cm²) | Py (tonf) |
|---|---|---|---|
| 26.62 | 299.2 | 10.75 | 37.37 |

**Table 2** Actual Measurements of Specimen

| PLATE 1 | | PLATE 2 | | PLATE 3 | | PLATE 4 | |
|---|---|---|---|---|---|---|---|
| $B_1$ (mm) | $t_1$ (mm) | $B_2$ (mm) | $t_2$ (mm) | $B_3$ (mm) | $t_3$ (mm) | $B_4$ (mm) | $t_4$ (mm) |
| 78.2 | 2.321 | 75.8 | 2.247 | 77.0 | 2.402 | 75.2 | 2.256 |

has been obtained. The shape of the specimen is a round-cornered box type as depicted in **Fig. 4**. The steel type of the specimen is SS400. It is noted that the compressive side is regarded as that of positiveness and the tensile side is considered as that of negativeness. Incidentally, an average of four values measured at four corners of the specimen is axial displacement. And the axial stress and strain are nominal values. The four values measured at four corners of the specimen are expected to be almost same. However, there may be possibility that the four values are different a little bit from each other. That is why the average of them is defined as axial displacement in this study. Besides, design measurements of the specimen adopted for this compressive-tensile experiment are tabulated in **Table 1**.

In the displacement control, the average axial displacement of the specimen is controlled. The controlled displacement $u$ is determined by

$$u = \frac{u_1 + u_2 + u_3 + u_4}{4} \qquad (1)$$

where $u_1$, $u_2$, $u_3$, $u_4$ are axial displacement in compressive - tensile direction and are measured at four corners of the specimen. The sensitivity of measurement is 500 micro/mm. The axial displacement, that is, averaged values of the four values are output through a bridge-terminal of the experimental system as voltage.

The relation between the nominal strain $\varepsilon$ and controlled displacement is expressed by

$$\varepsilon = \frac{u}{L} \qquad (2)$$

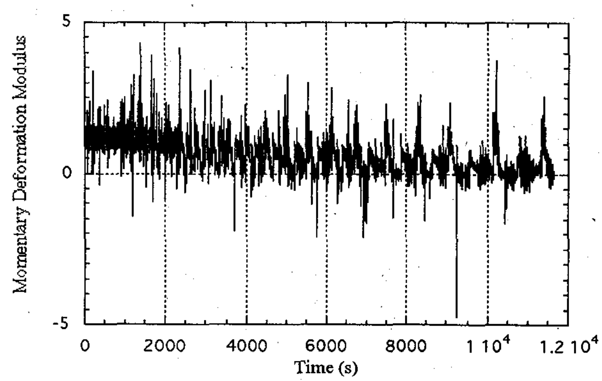



**Fig. 5** Time Series of Momentary Deformation Modulus

where $L$ is length of the specimen in the compressive - tensile direction. On the other hand, stress is determined from vertical load measured by a load cell inside the actuator.

As can be seen from **Tables 2** and **3**, the specimen used for obtaining the hysteretic data has unusually large generalized width-thickness ratio. In practice, this kind of plates are not used as the segments subjected to the seismic load. However, in this study the principal objective is to apply a prediction method of chaotic time series to the prediction of hysteretic curves of thin steel plates. It is not the primary objective to investigate the dynamic characteristics as the earthquake-resistant segments of the plate elements. Hence, it may be said that using data related to this specimen has a significance.

When the specimen is subjected to the repetitive

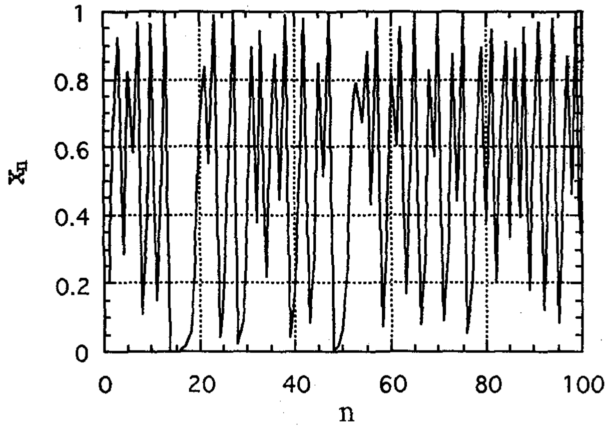



**Fig. 6** Logistic Progression

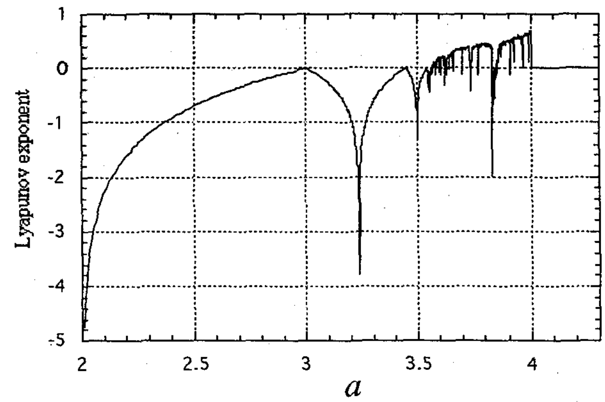



**Fig. 8** Lyapunov Exponents for Logistic Mapping

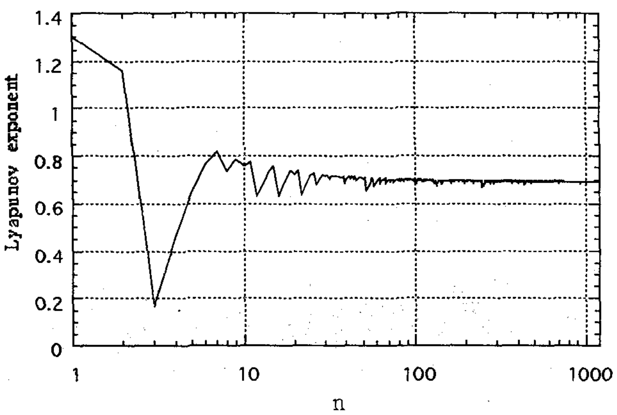



**Fig. 7** Lyapunov Spectrum for Logistic Progression

load whose pattern is shown in **Fig. 2** and the strain amplitude is small at the first stage, the bearing strength increases stably on both the compressive and tensile sides as depicted in **Fig. 1**. However, the local buckling occurs as the deformation out of the plane develops when subjected to the large compressive thrust and the repetitive number of loading becomes large and the strain amplitude is made greater.

For that reason, if the buckling did not occur, the bearing strength would show the similar or same change on both the compressive and tensile sides.

Although the stresses in both the compressive and tensile sides, in initial loading stage, grow on with progress of the loading cycle, the peak of the strength in the compressive side is attained earlier than that in the tensile side. Afterwards, the strength in the compressive side goes on reducing under the influence of the deformation out of the plane and the local instability, whereas the peak stress in the tensile side becomes larger successively after the peak of the strength has been attained in the compressive side, and in due time, the collapse comes into existence.

According to the experimental result shown in **Fig. 1**, the peak points shift remarkably after the specimen has been subjected to the considerably great tensile strain. But it is not evident whether the great tensile strain causes the remarkable shift of the peak points. As another studies, some experiments are expected so as to investigate the problem in detail.

A deformation modulus at each moment can be determined from the hysteretic data of the constitutive relation. The deformation modulus at each moment is called "Momentary Deformation Modulus" in this paper. The Momentary Deformation Modulus $e_M(t)$ is stiffness of a specimen at each moment, which is defined as

$$e_M(t) = \frac{\sigma(t+1) - \sigma(t)}{\varepsilon(t+1) - \varepsilon(t)} \qquad (3)$$

in which $\sigma(t)$, $\varepsilon(t)$, $\sigma(t+1)$, $\varepsilon(t+1)$ are axial stress and strain at points of time $t$ and $t+1$ respectively.

**Fig. 5** shows a time series of the Momentary Deformation Modulus which is determined from the hysteretic data of the thin steel plates. This time series is thought to have important information of dynamic properties of the thin steel plates. This time series whose behavior looks apparently complicated is directly taken into consideration, in this paper, for prediction of hysteretic curves in more nonlinear domain.

In **Fig. 5**, the time series of stress and strain have a little phase difference each other at many points of time. That is why the time series of the Momentary Deformation Modulus yields behavior of high frequency. The more stiffness of the specimen subjected to large strain is reduced, the more components of the negative Momentary Deformation Modulus increase. Conversely, increase of points of the minus Momentary Deformation

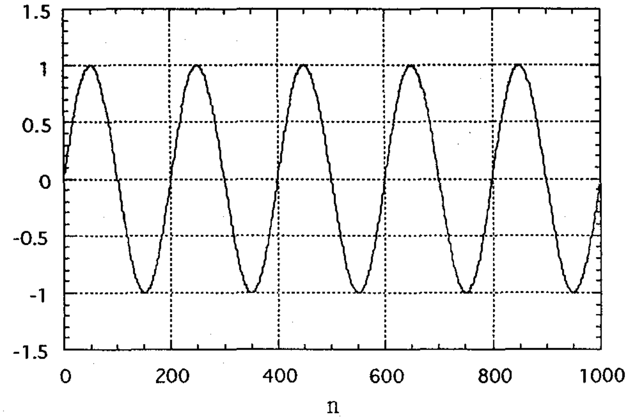



**Fig. 9** Sine Function

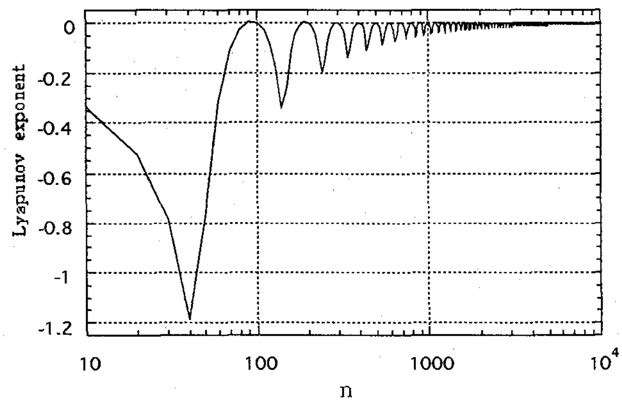



**Fig. 10** Lyapunov Spectrum for Sine Function

Modulus physically means reduction of stiffness of the specimen in the loading axial direction or occurrence of buckling.

**3. Discrimination of Chaos via Lyapunov Exponents**

Here, we consider the spectrum of Lyapunov exponents, which has proven to be the most useful dynamical diagnostic for chaotic systems[3),4)]. Lyapunov exponents are the average exponential rates of divergence or convergence of nearby orbits in phase space.

In order to find the Lyapunov exponents from one-dimensional data $e_M(t)$, the data is first embedded into a reconstructed multidimensional state space. Embedding means that $m$ - dimensional location vectors are formed at each moment with the one-dimensional data as

$$\vec{E}_M(t) = \left\{ e_M(t), e_M(t+\tau), e_M(t+2\tau), \cdots, e_M(t+(m-1)\tau) \right\}^T \tag{4}$$

where $\tau$ expresses a constant time delay. The Lyapunov

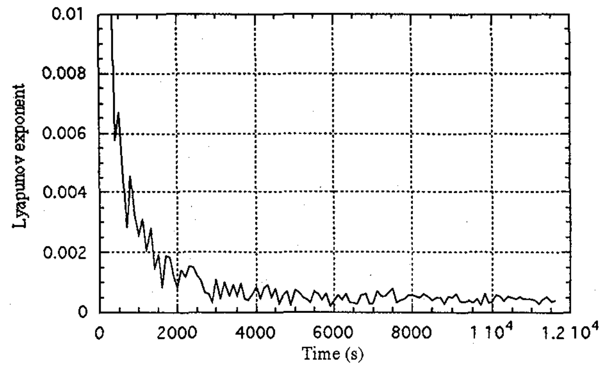



**Fig. 11** Lyapunov Spectrum for Momentary Deformation Modulus

exponents are determined by definition:

$$\chi = \frac{1}{t_N - t_0} \sum_{k=1}^{N} \ln \frac{L'(t_k)}{L(t_{k-1})} \tag{5}$$

where $N$ means the number of total steps and $L(t_{k-1})$ is the distance between $\vec{E}_{M1}(t_{k-1})$ and $\vec{E}_{M2}(t_{k-1})$ on nearby orbits at time $t_{k-1}$ in the $(k-1)$-th step; the distance is expressed as

$$L(t_{k-1}) = \left| \vec{E}_{M2}(t_{k-1}) - \vec{E}_{M1}(t_{k-1}) \right| \tag{6}$$

$L'(t_k)$ is the distance between $\vec{E}_{M1}(t_k)$ and $\vec{E}_{M2}(t_k)$ at time $t_k$ in the $k$ -th step; the distance is expressed by similar equation to Eq. (6) :

$$L'(t_k) = \left| \vec{E}_{M2}(t_k) - \vec{E}_{M1}(t_k) \right| \tag{7}$$

In general, when the spectrum of Lyapunov exponents converges into a positive real number, the system is defined to be chaotic. **Fig. 6** shows the evolution of the logistic progression. The logistic progression is known to be a typical chaotic progression, and produced by the following formula [5),6)]:

$$x_{n+1} = a x_n (1 - x_n) \tag{8}$$

where $a$ is a real coefficient, and deterministic chaos is produced by Eq. (8) when the following expression holds:

$$3.57 \leq a \leq 4 \tag{9}$$

Eq. (8) expresses the relation between $n$ -th term and $(n+1)$ -th term of a progression, namely, Eq. (8) is a recurrence formula. By giving a value of coefficient $a$ and an initial value $x_0$, Eq. (8) produces a progression,

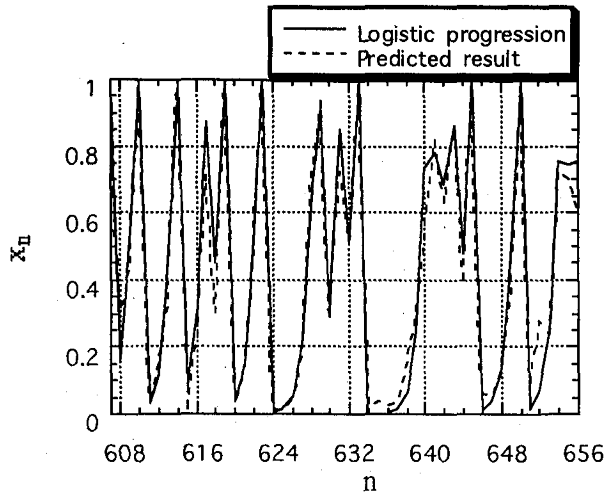



**Fig. 12** Prediction of Logistic Progression

which is called the logistic progression.

In case of **Fig. 6**, $a$ is set to 4.0 and the initial value $x_0$ is equal to 0.2. $n$ is a number of a datum in a progression. **Fig. 7** shows the spectrum of Lyapunov exponents in case of this logistic progression. In this case, the spectrum of the Lyapunov exponents converges into 0.693 ($\fallingdotseq \ln 2$). Hence, it is confirmed that the evolution of the logistic progression is chaotic.

**Fig. 8** depicts Lyapunov exponents for the logistic Mapping. As shown in this figure, positive values of the Lyapunov exponents are within the range of Eq. (9), which means that Eq. (8) produces chaotic progression under the condition on coefficient $a$, that is, Eq. (9).

On the other hand, we have found the Lyapunov spectrum in also case of the harmonic sine wave shown in **Fig. 9** so as to verify validity of the discrimination method of chaos by Lyapunov exponents. As can be seen from **Fig. 10**, the spectrum of Lyapunov exponents converges into the negative number; it follows that the behavior of the sine wave is discriminated as not chaos.

**Fig. 11** depicts the convergence of the Lyapunov spectrum in case of using the Momentary Deformation Modulus as the time series. The spectrum converges into almost 0.0005. Accordingly, the evolution of the Momentary Deformation Modulus behaves chaotically.

## 4. Prediction of Momentary Deformation Modulus and Hysteretic Curve[4), 5), 6), 7), 8)]

The total number of the Momentary Deformation Modulus determined from the hysteretic data is 11653. In this time series, almost the first half, that is, 6000 points are assumed to be known quantities, and the evolution of

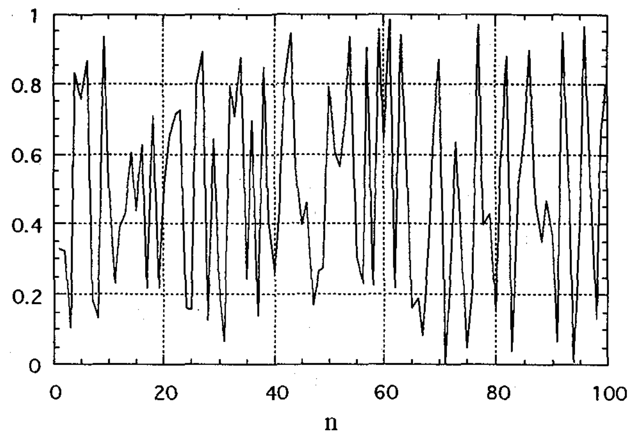



**Fig. 13** Uniform Random Numbers

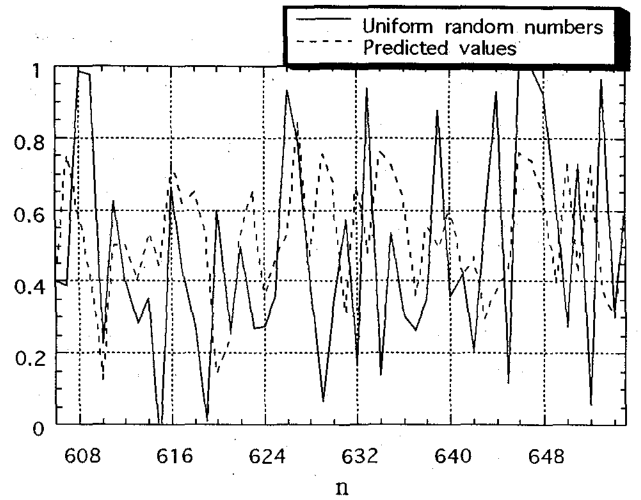



**Fig. 14** Prediction of Uniform Random Numbers

the latter time series of the Momentary Deformation Modulus is predicted by means of the Local Fuzzy Reconstruction Method which is one of prediction methods of a chaotic time series.

The following presuppositions are necessary for appropriate prediction using the prediction method of a chaotic time series:

(1) The Data assumed to be known has nonlinear hysteretic stress - strain relation at least.
(2) The behavior of the time series of the Momentary Deformation Modulus is deterministic chaos; If the behavior is not deterministic chaos but random, short time prediction is impossible as well as long term prediction.

Hereafter, the following definition is used:

$e_M(t_s)$ present datum of the momentary

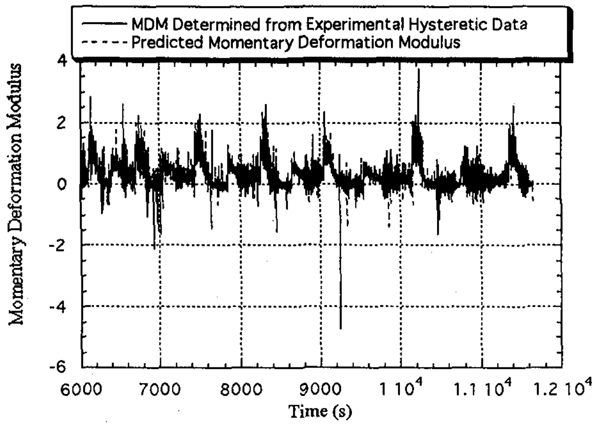



**Fig. 15** Prediction of Momentary Deformation Modulus

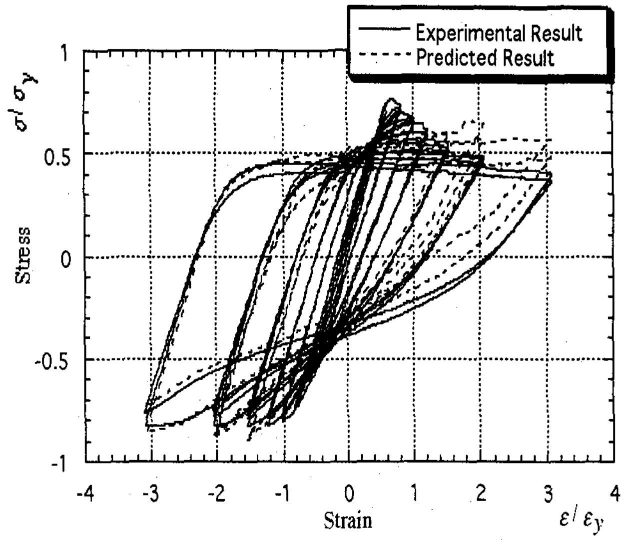



**Fig. 16** Prediction of Hysteretic Curve

deformation modulus; the latter time series of this modulus should be predicted

$e_M(t_s+s)$ momentary deformation modulus after $s$ steps from $e_M(t_s)$ location

$\vec{E}_M(t_s)$ vector which includes $e_M(t_s)$ in its components

$\vec{E}_M(t_s+s)$ location vector after $s$ steps from $\vec{E}_M(t_s)$; the vector has $e_M(t_s+s)$ as its components

$\vec{e}_M(t)$ nearby vector in a local part including an end point of $\vec{E}_M(t_s)$

$\vec{e}_M(t+s)$ vector after $s$ steps from $\vec{e}_M(t)$

In the first stage of prediction, one dimensional data such as the Momentary deformation modulus is embedded into multidimensional state space. Embedding is to make many location vectors expressed by Eq. (4) whose components have a same time lag each other.

All of the known Momentary Deformation Modulus determined from the experimental data of hysteresis are embedded into a reconstructed multidimensional state space. In the space, an orbit from $\vec{E}_M(t_s)$ to $\vec{E}_M(t_s+s)$ is inferred based on trajectories from $\vec{e}_M(t)$ to $\vec{e}_M(t+s)$.

Fuzzy reasoning is applied to inference of the orbit from $\vec{E}_M(t_s)$ to $\vec{E}_M(t_s+s)$. When the evolution of the time series of the Momentary Deformation Modulus follows deterministic chaos, the trajectories from $\vec{e}_M(t)$ to $\vec{e}_M(t+s)$ can be expressed by deterministic rules. The rules are defined in terms of fuzzy inference rules.

If $\vec{E}_M(t_s)$ is input to the fuzzy inference rules, $\vec{E}_M(t_s+s)$ is derived by the fuzzy reasoning and accordingly $e_M(t_s+s)$ is obtained because $\vec{E}_M(t_s+s)$ has $e_M(t_s+s)$ as its component.

**Fig. 12** shows a comparison between the logistic progression and the predicted result. In this case, the correlation coefficient is 0.982. That is to say, the typical chaotic progression can be predicted with very good accuracy.

A comparison between the uniform random numbers shown in **Fig. 13** and the predicted result is shown in **Fig. 14.** The random numbers do not follow the deterministic rules. That is why the correlation coefficient in this case is -0.188. Hence, the indeterministic random numbers whose behavior is not deterministic chaos can not be predicted with good accuracy, that is in the same way as other methods.

**Fig. 15** shows the predicted result in case of the Momentary Deformation Modulus. The correlation coefficient is 0.679 in the predicted latter part of the time series. If the Momentary Deformation Modulus is converted to the constitutive relation, the prediction accuracy becomes better, namely, the correlation coefficient results in 0.987.

From the analytical results described above, it may be said that the more time series and progressions are the deterministic chaos, the better they are accurately predicted by the prediction method of a chaotic time series.

The constitutive relation is derived from converting the predicted Momentary Deformation Modulus to the hysteresis between stress and strain by the following equations:

$$d(\sigma/\sigma_y) = e_M(t)d(\varepsilon/\varepsilon_y) \quad (10)$$

$$d(\sigma/\sigma_y) = e_M(t_s+s)d(\varepsilon/\varepsilon_y) \quad (11)$$

$$(\varepsilon/\varepsilon_y)_{new} = (\varepsilon/\varepsilon_y)_{old} + d(\varepsilon/\varepsilon_y) \quad (12)$$

$$(\sigma/\sigma_y)_{new} = (\sigma/\sigma_y)_{old} + d(\sigma/\sigma_y) \quad (13)$$

The experimental result and the predicted hysteretic curve are compared in **Fig. 16**. The total experimental data of hysteresis is 11654 points and almost the first half, namely, 6000 points have been assumed to be known, and the hysteretic curve in more nonlinear strain level is predicted as a result. In this case, the correlation coefficient in the predicted part is 0.987 as described previously.

## 5. Concluding Remarks

When the progressions and the time series whose behavior is discriminated as chaotic are embedded into a reconstructed multidimensional phase space, the set of the location vectors is obtained; the progressions and the time series follow the deterministic rules. The dynamics of the location vectors can be predicted by the fuzzy reasoning. Consequently, the evolution of the progressions and the time series are predicted with good accuracy; the typical deterministic chaos series such as the logistic progression can be predicted by the method with very good accuracy and also the time series of the Momentary Deformation Modulus of the thin steel plates can be predicted with good agreement between the predicted time series and the time series determined from the experimental data.

The conclusions are summarized as follows.

(1) Using the spectrum of Lyapunov exponents as the diagnostic for chaotic systems results in chaos of the Momentary Deformation Modulus of the thin steel plates.
(2) Application of the local fuzzy reconstruction method as one of prediction methods of chaotic time series leads to good accuracy in prediction of the evolution of the progressions and the time series whose behavior is deterministic chaos.
(3) The proposed method permits prediction of the dynamical hysteretic behavior of the thin steel plates from relatively linear strain level to more nonlinear strain level.

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