Application of Robust Control to the Flutter in Long-Span Bridges

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The problem of flutter on a long-span, streamlined box girder bridge is revised in the view of the active control application. Particular aspects such as uncertainties in the aerodynamic forces definition, the difficulty in the system identification and their interdependent relationship with the control design are pointed out. In face of these problems, a control method based on the H^∞ robust control theory is proposed. As the system is prameterized by a single factor, a control designed to admit this factor as uncertainty is proved to be simple. Some practical considerations on the arrangement of control are discussed and numerical examples on a simple 2D model are conducted to verify the effectiveness.
Key Words: cable-stayed bridge, flutter responses, active control, robust control.

1. Introduction

The design of very long span bridge has always been an excitation and a challenge for the structural engineering. Recently, with the rapid advances of technology, the span length of suspension type bridges has been pushed up to 2000-3000m and options of bridge with span as long as 5000m are seriously considered. One of many difficulties involved in such an enterprise is the dynamic aspect, since this structure becomes extremely flexible and easily excited. In a windy environment, a very flexible structure is vulnerable which is a serious concern since the wind flow could trigger the flutter, a phenomenon that in the past experiences, has led to disastrous consequences. As such, the dynamic characteristic and flutter resistance has become an increasingly important factor in the selection and design of the bridge deck. Experiments and studies have shown that the aerodynamic stability can be improved by utilizing a high torsional stiffness, truss-type deck or by carefully designing the streamlined form of its section. While such measures have been proved successful so far, they might fall short in the future for even-longer span bridges. Specially, when the requirements for other aspects such as the dead weight, the drag forces, etc., become more stringent, the choices of these measures could be much more restrictive. In such situation, the traditional view of structure as a passive, static and massive one may need to be thoroughly revised where active control could have some role and becomes an integral part. By itself, the idea is attractive since it starts a new generation of structures which are adaptive and more responsive to the changing environment.

Conceptually, active structure and active control by them-self are well known in other disciplines like mechanics, aeronautics, or electronics. In the field of civil engineering, however, these ideas have been only recently drawing serious attention. Although theoretical background is the same, their practical application requires some special considerations due to many particularities of civil engineering structures such as large masses, long service life, seriousness of the failure, uncertainties, etc. In the case of flutter control, exceptional attention should be emphasized on the later issue since the flutter is closely related to the stability problem of a system whose description is often heavily dependent on the experimental data. Thus, control method proposed to solve this problem should address both the system stability and structural uncertainties. Among many control algorithms devised for various purposes, the robust control theory based on the H^∞ norm seems to amply response to this requirement. The frequency domain approach of this control method makes it even more appealing and compatible with most of the tools developed in wind engineering. Another particularity of vibrating structure in an air flow is the dependency of the system definition on the mean wind speed. To monitor continuously and on line this parameter in a turbulent wind is both impractical and unreliable. By using the robust control theory as it will be shown next, this difficulty can be nicely circumvented. It will turn out that all the uncertain factors involved in the system definition are gathered in a single parameter called reduced frequency. By designing a control to be independent of this variable, it could be expected to be robust against all kind of uncertainties.

The application of active devices to the bridge flutter control was examined by Kobayashi, H. et al [1]. The stabilizing action is achieved through the aerodynamic forces on a small computer-controlled appendage attached to the deck. The control law, by the direct measurement feedback, aims at increasing the damping of the system. Basing on the similar setup, Wilde, K. et al. [2] used the concept of rational functions approximation to reduce the mechanico-aerodynamic system to the standard form where the optimal control can be applied. In reference [3] the idea of robust control was presented and carried out on a scheme of a static state feedback controller. However, as it is shown in their formulation that the aerodynamic forces depend on some time lag terms, a dynamic measurement feedback scheme should intuitively have a better performance. It also enables the incorporation of frequency-dependent weighting function in the generalized plant making the control more adaptable to the complex model of real bridge. Thus, in this study, a more sophisticated control scheme with the dynamic output feedback will be explored.

2. Flutter Problem in View of Control Application

The problem of flutter can be easily understood by a simple 2D model of the bridge deck as shown in the Figure 1. With the non-dimensional variable chosen as $\mathbf{r}=\{h/b,\alpha\}^T$, the equation describing the motion of the system can be expressed in the following form:

$$\begin{bmatrix} mb^2 & 0 \\ 0 & I_\alpha \end{bmatrix}\ddot{\mathbf{r}} + \begin{bmatrix} k_h b^2 & 0 \\ 0 & k_\alpha \end{bmatrix}\mathbf{r} + \begin{Bmatrix} f_L b \\ f_M \end{Bmatrix} + \mathbf{w} = 0 \qquad (1a)$$

Here, m, I_α are the mass and the moment of inertia with respect to the elastic center, k_h, k_α are stiffness coefficients corresponding to the heaving and pitching motion respectively, and b is the half width of the deck chord. The external forces on the system are divided into the motion-independent disturbance $\mathbf{w}$ and the aerodynamic forces: lift f_L and moment f_M. It is also noted that the first equation is multiplied with the dimension b so that all the disturbances will have the same unit of moment. In the compact form, the equation can be writen as:

$$\mathbf{M}.\ddot{\mathbf{r}} + \mathbf{K}.\mathbf{r} + \mathbf{f}_w + \mathbf{w} = 0 \qquad (1b)$$

For a harmonic motion of frequency ω, according to Scanlan [4], the aerodynamic forces can be expressed in function of the states, air density ρ, mean wind speed U and a set of experimentally determined coefficients H_i, A_i which are functions of a the reduced frequency k:

$$f_L b = 2\rho U^2 b^2 \left(kH_1^* \frac{\dot{h}}{U} + kH_2^* \frac{2b\dot{\alpha}}{U} + 2k^2 H_3^* \alpha + k^2 H_4^* \frac{h}{b} \right) \qquad (2a)$$

$$f_M = 4\rho U^2 b^2 \left(kA_1^* \frac{\dot{h}}{U} + kA_2^* \frac{2b\dot{\alpha}}{U} + 2k^2 A_3^* \alpha + k^2 A_4^* \frac{h}{b} \right) \qquad (2b)$$

where $$k = \frac{b\omega}{U} \qquad (3)$$

In frequency domain, if the relations among displacement, velocity and acceleration of harmonic motions are taken into account, the mean wind speed U can be eliminated among equations (2) and (3) and the aerodynamic forces are further reduced to:

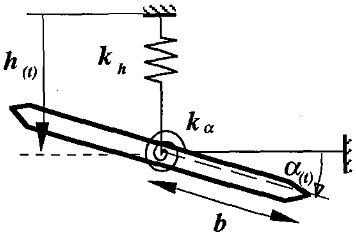


Fig. 1 Model of bridge deck.

$$\mathbf{f}_w = \omega \mathbf{F}_V \dot{\mathbf{r}} + \omega^2 \mathbf{F}_D \mathbf{r} = -[i\mathbf{F}_V + \mathbf{F}_D]\ddot{\mathbf{r}} = \mathbf{F}_W \ddot{\mathbf{r}} \qquad (4)$$

where :

$$\mathbf{F}_V = \rho b^4 \begin{bmatrix} 2H_1^* & 4H_2^* \\ 4A_1^* & 8A_2^* \end{bmatrix} \qquad (5a)$$

$$\mathbf{F}_D = \rho b^4 \begin{bmatrix} 2H_4^* & 4H_3^* \\ 4A_4^* & 8A_3^* \end{bmatrix} \qquad (5b)$$

In view of expression (2), the equation (1) now becomes a set of equations parameterized by the reduced frequency k:

$$\mathbf{M}_{(k)}.\ddot{\mathbf{r}} + \mathbf{K}.\mathbf{r} = \mathbf{w} \qquad \text{where: } \mathbf{M}_{(k)} = \mathbf{M} + \mathbf{F}_w \qquad (6)$$

The equation (6) describing the system is defined for values k, which relating the mean wind speed U and the frequency ω. By convention, the flutter occurs when the damping of the system becomes negative, i.e. the real part of any system eigenvalue turns positive. Physically, this means that the vibration is self-excited and the system becomes unstable. Thus, the calculation of flutter involves the determination of the wind speed at which the poles of the system cross over to the right half of the complex plane. There are different methods for this purpose, and in this study, an iterative scheme based on a step-by-step increase of U is proposed.

1) At step i let ω_i are known. The wind speed is increased from U_i to U_{i+1}, and an estimation of ω_{i+1} is made by extrapolation of ω_i, ω_{i-1},...
2) The reduced frequency k is calculated from (3) and the equations (6) are formed.
3) Eigenvalue problem can be solved, preferably by some iteration methods since an approximated estimation is available.
4) The newly calculated eigenvalue is compared with the assumed one, if the error is small enough go to the next step, if not, a better estimation of ω_{i+1} is made and the procedure goes back to the step 2.
5) The steps 2-4 will be repeated for every mode.

The above described scheme is based on a reasonable assumption that the dynamic characteristics of the system do not change radically with a small increase in wind speed. The procedure is therefore like tracking the movement of the system poles in the complex plane when U varies. For starting the loop, a zero wind condition can be assumed and the poles of system without aerodynamic forces can be easily computed and used as the first values of ω. Once the maximum wind speed is attained, the real part of the eigenvalues plotted against U will give a picture of the dynamic behavior of the system. If a feedback control force $\mathbf{u}$ is applied, it may be more convenient to work with the state variable $\mathbf{x} = \{\dot{\mathbf{r}}, \mathbf{r}\}^T$. The system then will become:

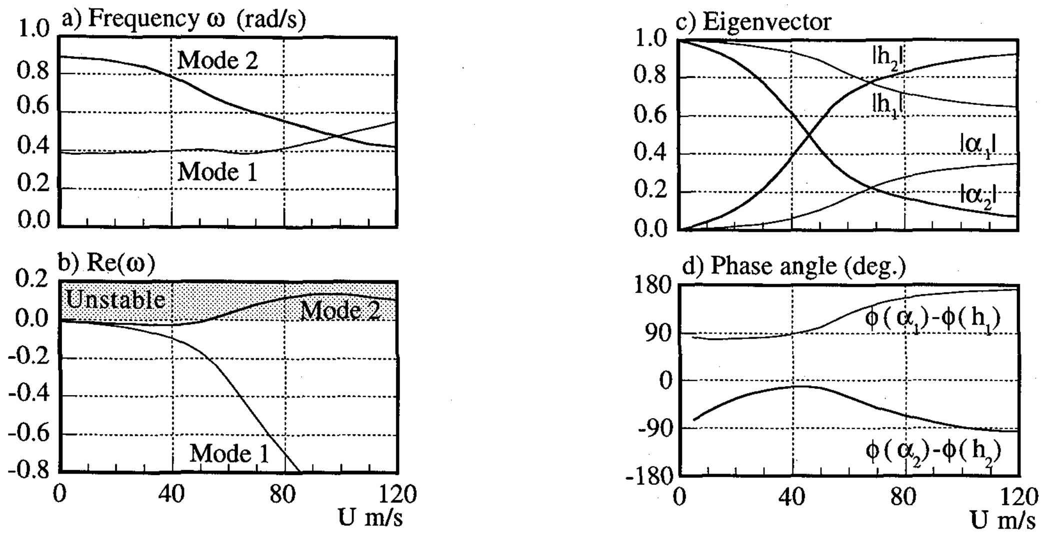


Fig 2. Flutter behavior of the 2D model.

$$\overline{\mathbf{M}}\dot{\mathbf{x}}+\overline{\mathbf{K}}.\mathbf{x}+\mathbf{B}_1.\mathbf{w}+\mathbf{B}_2\mathbf{u}=0 \;;\quad \mathbf{y}=\mathbf{C}\mathbf{x}$$
$$\dot{\mathbf{q}}=\mathbf{A}_c\mathbf{q}+\mathbf{B}_c\mathbf{y} \;;\quad \mathbf{u}=\mathbf{C}_c\mathbf{q}+\mathbf{D}_c\mathbf{y} \qquad (7a,b)$$

where :

$$\overline{\mathbf{M}}=\begin{bmatrix}\mathbf{M}_{(k)} & 0\\ 0 & \mathbf{I}\end{bmatrix} \;;\quad \overline{\mathbf{K}}=\begin{bmatrix}0 & \mathbf{K}\\ -\mathbf{I} & 0\end{bmatrix} \qquad (8)$$

The variable $\mathbf{y}$ is the measurement, and the second equation describes the controller with $\mathbf{q}$ as the internal state variable and other matrices have the appropriate dimension. The two coupling systems: structure-controller can be merged:

$$\begin{bmatrix}\overline{\mathbf{M}} & 0\\ 0 & \mathbf{I}\end{bmatrix}\begin{Bmatrix}\dot{\mathbf{x}}\\ \dot{\mathbf{q}}\end{Bmatrix}+\begin{bmatrix}\overline{\mathbf{K}}+\mathbf{B}_2\mathbf{D}_c\mathbf{C} & \mathbf{B}_2\mathbf{C}_c\\ -\mathbf{B}_c\mathbf{C} & -\mathbf{A}_c\end{bmatrix}\begin{Bmatrix}\mathbf{x}\\ \mathbf{q}\end{Bmatrix}=\begin{Bmatrix}-\mathbf{B}_1\mathbf{w}\\ 0\end{Bmatrix} \qquad (9)$$

The stability of this system again can be predicted by the same procedure described previously. In the Fig. 2, the results of flutter analysis by the mentioned method for the 2D model of Fig. 1 are shown. The stability of the system is indicated by the real part of eigenvalue. In this case, the vibration becomes self-excited for U>50m/s. The vibration mode shows that the initial pitching dominant mode is the unstable one, however, as the wind speed rises, both modes become heaving dominant (Fig. 2c). The main difference here is the phase angle between the 2 motions. In comparison with the heaving, pitching motion of unstable mode is 90° lag while the difference in stable mode is 180°. This observation seems to point out that control based on the natural modes at zero wind main not be appropriate.

3. Robust Control and Flutter Suppression

In the general case, a dynamic system with a feedback control $\mathbf{u}$ can be expressed as

$$\dot{\mathbf{x}}=\mathbf{A}.\mathbf{x}+\mathbf{B}_1.\mathbf{w}+\mathbf{B}_2\mathbf{u}$$
$$\mathbf{y}=\mathbf{C}_1.\mathbf{x}+\mathbf{D}_{12}.\mathbf{u} \qquad (10)$$
$$\mathbf{z}=\mathbf{C}_2.\mathbf{x}+\mathbf{D}_{21}.\mathbf{w}$$

Where $\mathbf{x}$ is the state variable, $\mathbf{y}$ is the controlled output, $\mathbf{z}$ is the measured variables, $\mathbf{w}$ is the disturbances. Matrix $\mathbf{D}_{12}$ is for including the control forces magnitude in the output and matrix $\mathbf{D}_{21}$ is to include the noise on sensors. Providing that the matrix $\mathbf{M}_{(k)}$ is not degenerated, the equation (7) can always be reduced to this form. In the frequency domain, equation (10) can be equivalently represented by the transfer function $\mathbf{G}$ from input to output. If the input is $[\mathbf{w}\ \mathbf{u}]^T$ and output $[\mathbf{y}\ \mathbf{z}]^T$, then $\mathbf{G}$ will be:

$$\mathbf{G}=\mathbf{C}[(\mathbf{I}.s-\mathbf{A})]^{-1}\mathbf{B}+\mathbf{D} \qquad (11)$$

$$\mathbf{B}=[\mathbf{B}_1\quad \mathbf{B}_2];$$
$$\text{with : }\ \mathbf{C}=\begin{bmatrix}\mathbf{C}_1\\ \mathbf{C}_2\end{bmatrix};\quad \mathbf{D}=\begin{bmatrix}0 & \mathbf{D}_{12}\\ \mathbf{D}_{21} & 0\end{bmatrix} \qquad (12)$$

The control structure $\mathbf{Q}$ is represented by:

$$\mathbf{Q}_{(s)}:=\begin{Bmatrix}\dot{\mathbf{q}}=\mathbf{A}_c.\mathbf{q}+\mathbf{B}_c.\mathbf{z}\\ \mathbf{u}=\mathbf{C}_c.\mathbf{q}+\mathbf{D}_c.\mathbf{z}\end{Bmatrix} \qquad (13)$$

The design task is the determination of $\mathbf{A}_c,\mathbf{B}_c,\mathbf{C}_c,\mathbf{D}_c$ according to certain performances criteria. For the H^∞ control, the "performance" is reducing the "size" of the close loop transfer function $\mathbf{G}$ measured by the H^∞ norm below certain level and guaranteeing the stability of the system:

$$\left\|\mathbf{G}_{Q(s)}\right\|_\infty=\sup_{\omega\in\Re}\rho\left[\mathbf{G}_{K(s)}(i\omega)\right]<\gamma$$
$$\sigma(\mathbf{A}+\mathbf{B}.\mathbf{Q})\subset\left[C^-:=\{s\in C|\mathrm{Re}\ s<0\}\right] \qquad (14)$$

The control design procedure is seemed to be straight forward: first, the state equation for the flutter is derived and then, a static state-feedback control law can be determined from the equation (7) and the stability of the system is guaranteed by equation (14). However, a careful inspection reveals that the problem is more complicate. The state equation as in equation (7) is dependent on the reduced frequency k which by itself relating the wind speed with the frequency of vibration. Thus, the complete identification of the system requires the knowledge of the mean wind speed and the frequency of the system. Continuous monitoring of the mean wind speed for the feedback might pose some difficulties due to interaction of the structure with the air flows and their turbulent characteristic. In the Fig. 3, the contour map of frequency responses measured by the singular value of the transfer function is plotted against

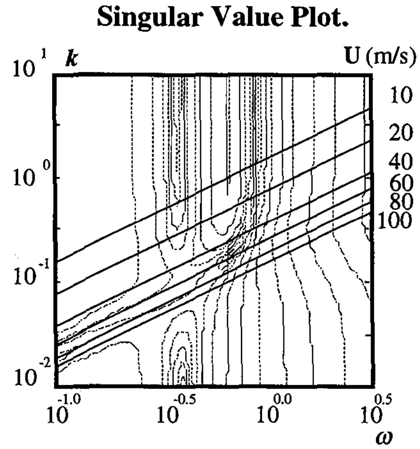


Fig. 3 Contour plot of responses in the k-ω plane.

ω and k. The actual system corresponding to certain U is defined at more than one k. For a real control system which works at a fixed wind speed at a time, this means that there some induced uncertainties by taking k as a constant. From the traditional conservative standpoint of the civil engineering, the solution is then to devise a unified control which can render an acceptable performance for all possible situations. As all the mentioned uncertainties are induced into the plant through k, a control designed to be robust against the variation of this parameter should work reasonably well in all situations. Consequently, this control is expected to be more tolerant to any inaccuracy in the flutter derivative coefficients whose determination is often based on the uncertainties-prone data of experiments.

For a nominal plant $\mathbf{G}_o$ corresponding to a chosen value k_o, the whole family of other plants on the neighborhood will be defined as $\mathbf{G}_i=\mathbf{G}_o+\Delta\mathbf{G}$. The size of the disturbance $\Delta\mathbf{G}$ can be again measured by the H^∞ norm and is bounded by a function $r(j\omega)$ as:

$$\|\Delta\mathbf{G}\|_\infty = \rho_{\max}[\Delta\mathbf{G}(j\omega)] \le |r(j\omega)| \quad \text{for all } \omega \qquad (15)$$

According to Francis and Doyle (1987) [5] the control $\mathbf{Q}$ stabilizes all $\mathbf{G}_i$ if it stabilizes $\mathbf{G}_o$ and:

$$\left\| r\mathbf{Q}(\mathbf{I}-\mathbf{GQ})^{-1} \right\|_\infty \le 1 \qquad (16)$$

Hence, once $r(j\omega)$ is determined, the design procedure is straight-forward. In the general case where number of uncertainty parameter is high, determining the upper bound of $\|\Delta\mathbf{G}\|_\infty$ as a multi-variables function is not so easy and obvious. Fortunately, in the flutter problem, as it is pointed out in the previous section, the system can be determined in function of one parameter k, therefore, numerical technique can be used to evaluate the norm of $\Delta\mathbf{G}$ and determine the function $r(j\omega)$. It can be also

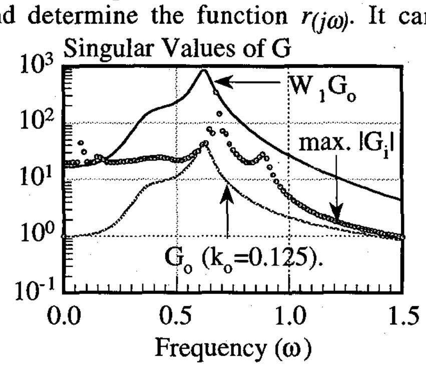


Fig. 4 Nominal plant with weighted function.

observed that the range of k can be limited according to the interest of design. The most convenient way to cover all $\mathbf{G}_i$ is by scaling up the nominal chosen plant $\mathbf{G}_o$ as shown in the Fig 4.

The fact that the active control by minimizing the H_∞ norm can be made more robust with any upper bound frequency-dependent function of uncertainties opens up many more possibilities for the design. For a nominal plant $\mathbf{G}_o$ chosen, some frequency weighted function W_i can be added to scaled up the outputs. The resulting generalized plant with all the weight W_i embedded will have the norm that meets with the criteria established in equation (15). One of such generalized plant is shown in the figure 5. The inputs include the disturbances and the noise in the sensors. The outputs are the structural state and the control forces scaled up by the frequency weighted functions W_1,W_2. The frequency response of the system scaled up by W_1 functions is shown in the figure 4. The control objective then becomes:

$$\left\| \begin{matrix} \mathbf{W}_2(\mathbf{I}-\mathbf{GQ})^{-1} \\ \mathbf{W}_1\mathbf{Q}(\mathbf{I}-\mathbf{GQ})^{-1} \end{matrix} \right\|_\infty < \gamma \qquad (17a,b)$$

$$\text{with: } \begin{cases} \rho_{\max}[\mathbf{W}_1] \ge \rho_{\max}[\Delta\mathbf{G}_i] \\ \Delta\mathbf{G}_i = \mathbf{G}_i - \mathbf{G}_o \end{cases}$$

In this case, the nominal plant $\mathbf{G}_o$ is decided by choosing a value of $k=k_o$, then singular values of other plants $\mathbf{G}_i$ determined by k_i will be analyzed for a decision on weighting functions. The transfer function in expression (17) can be again written in the standard form of a dynamic system as in equation (10). Evidently, all the vectors and matrices are now representing the generalized plant. The existence of a control law depends on the two Riccati equations:

$$\begin{aligned} &\mathbf{A}'\mathbf{X}+\mathbf{XA}+\mathbf{X}(\gamma^{-2}\mathbf{B}_1\mathbf{B}_1'-\mathbf{B}_2\mathbf{B}_2')\mathbf{X}+\mathbf{C}_1'\mathbf{C}_1=\mathbf{0} \\ &\mathbf{AY}+\mathbf{YA}'+\mathbf{Y}(\gamma^{-2}\mathbf{C}_1'\mathbf{C}_1-\mathbf{C}_2'\mathbf{C}_2)\mathbf{Y}+\mathbf{B}_1\mathbf{B}_1'=\mathbf{0} \end{aligned} \qquad (18)$$

Providing that the solutions of both (18) exist and are semi-definite, it will be possible to construct dynamic controllers as in (7b) which makes the H^∞ norm of the close loop transfer function less than the attenuation factor γ [5]. One of such controller is:

$$\begin{aligned} &\mathbf{D}_c=\mathbf{0} \\ &\mathbf{C}_c=-\mathbf{B}_2'\mathbf{X} \\ &\mathbf{B}_c=(\mathbf{I}-\gamma^{-2}\mathbf{YX})^{-1}\mathbf{YC}_2' \\ &\mathbf{A}_c=\mathbf{A}+(\gamma^{-2}\mathbf{B}_1\mathbf{B}_1'-\mathbf{B}_2\mathbf{B}_2')\mathbf{X}-\mathbf{B}_c\mathbf{C}_2 \end{aligned} \qquad (19)$$

The controller described is only a sub-optimal one. To determine the optimal H^∞, an iterative scheme by successively decreasing the value of γ until a marginally conceivable controller is found. Once the control is designed, the close loop system can be formed as in the

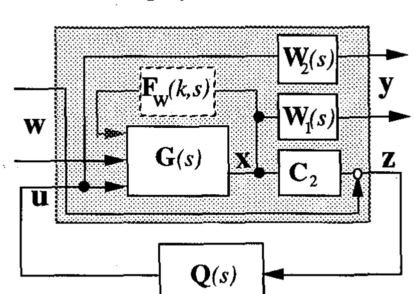


Fig.5 The generalized plant with weighting function.

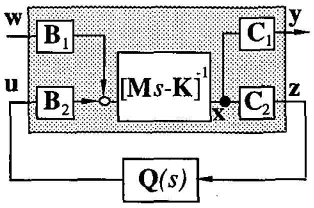


Fig. 6 Close-loop of structure-control system.

equation (9) and analyzed by the method of flutter prediction described in the preceding section.

The control problem adressed so far has been based on the system stability and small size of transfer function, which does not take into account the intensities of the disturbances. It is obvious that magnitude of the control forces should be determined in diret relation with the later as a steady response of the system. In an equilibrium status, the level of vibration once stabilized is a direct function of the motion independent disturbances. The controller as a feedback has the system output as their input and therefore, a relationship could be derived. Consequently, the control forces hold an indirect relation with the motion-independent disturbances and their determination is only possible in function of the later. However, since both are inputs to the system, a direct relation between them is not straightforward. One way to estimate the ratio of control force intensity and the disturbance is obtained by relating both through the state response.

Considering the close loop system, the magnitude of control forces as an internal signal should be uniquely determined by **w**. To quantitatively assess the control forces, the transfer function between **w** and **u** can be determined and the norm of this function can be a good index of control expense and control performance. For the system as shown in the Fig. 6, the close loop transfer function from **w** to **y** is:

$$\mathbf{y}=\mathbf{C}_1\left[(\overline{\mathbf{M}}.s+\overline{\mathbf{K}})-\mathbf{B}_2\mathbf{Q}_{(s)}\mathbf{C}_2\right]^{-1}\mathbf{B}_1.\mathbf{w} \qquad (20)$$

On the other hand, if the control forces are considered as external disturbances, the open loop of the system could be:

$$\mathbf{y}=\mathbf{C}_1\left[(\overline{\mathbf{M}}.s+\overline{\mathbf{K}})\right]^{-1}\left[\mathbf{B}_1.\mathbf{w}+\mathbf{B}_2.\mathbf{u}\right] \qquad (21)$$

If the observed output y is identical to the state **x**, i.e., $\mathbf{C}_1$=**I**, then by comparing (20) with (21) it leads to:

$$\begin{aligned}\mathbf{B}_2.\mathbf{u}&=\left\{(\overline{\mathbf{M}}.s+\overline{\mathbf{K}})\left[(\overline{\mathbf{M}}.s+\overline{\mathbf{K}})-\mathbf{B}_2\mathbf{Q}_{(s)}\mathbf{C}_2\right]^{-1}-\mathbf{I}\right\}\mathbf{B}_1.\mathbf{w}\\&=\mathbf{T}_{(s)}\mathbf{B}_1.\mathbf{w}\end{aligned} \qquad (22)$$

The "size" of **T**(s) measured by some appropriated norm can serve as an index of the magnitude of control forces for the disturbances of intensity equal unit. The control law **Q**(s) determined in equation (7b) is the transfer function from **z** to **u** and can be explicitly expressed as:

$$\mathbf{Q}_{(s)}:=\mathbf{C}_c\left[\mathbf{I}.s-\mathbf{A}_c\right]^{-1}\mathbf{B}_c+\mathbf{D}_c \qquad (23)$$

By substituting **Q**(s) into equation (22) the transfer function **T** and their frequency response can be determined. However it should be pointed out that this measure of **T** as the norm of a transfer function usually represents the worst case and at such overestimates value of **u**.

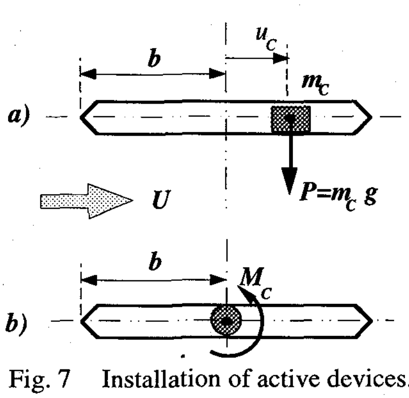
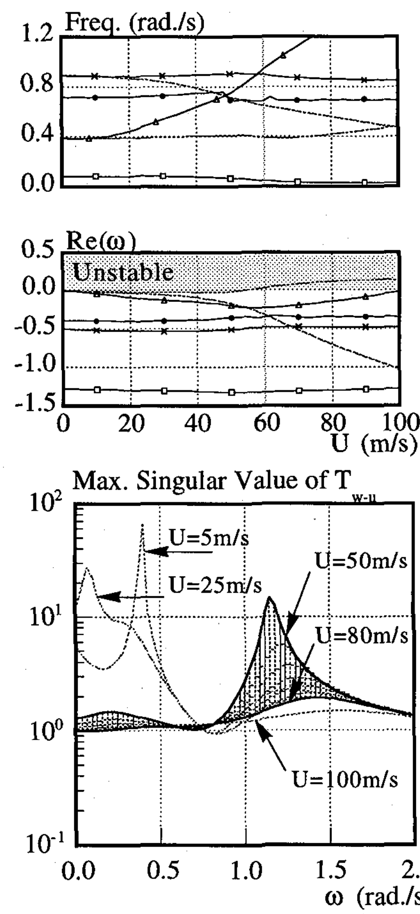


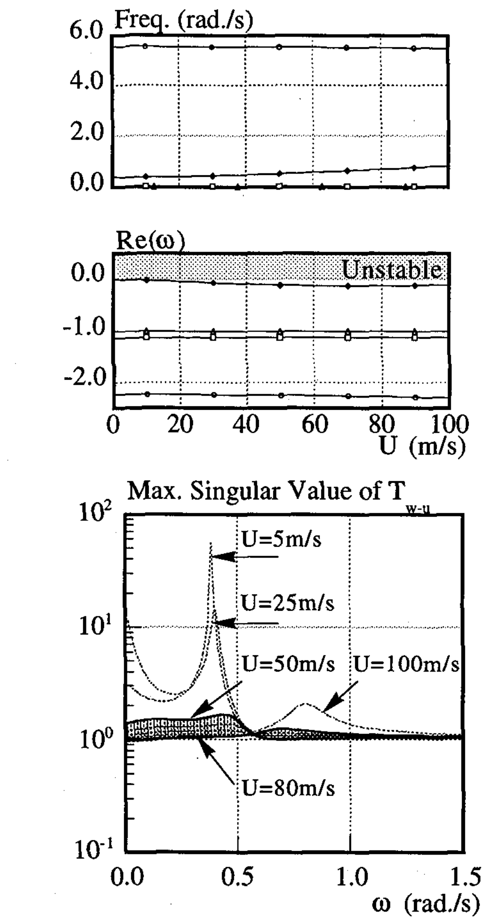
Fig. 7 Installation of active devices.

4. Numerical Examples

To investigate the effect of this control method, a 2D model of the bridge section as shown in Fig. 1 is used. The main parameters of model are: m=2.95×10^3 kgf-m/s^2; b=15m; I_α=5.88×10^5 kgf-m^3/s^2; k_h=447 kgf.m/m; k_α=4.68×10^5 kg-f/m. The deck is considered as a flat plate and therefore, their aerodynamic coefficients are computed from the Theodosen function. The natural frequencies of this structure is very low, 0.892 rad/s for the pitching and 0.389 rad/s for the heaving motions. The wind speed at which flutter occurs is very low, just over 50m/s. The control will be based only on the twisting moment, which can be generated by a rotating cylinder or by an eccentric weight as suggested in the

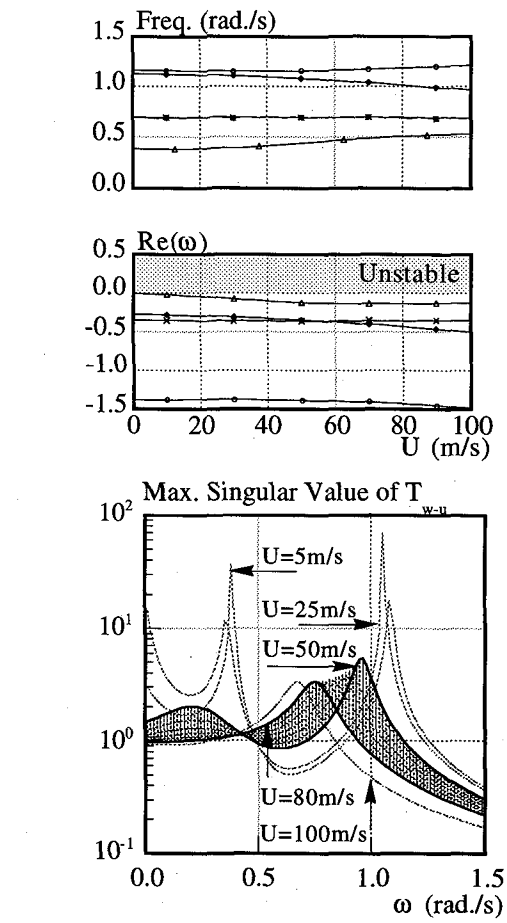


a) Eigenvalues of the close loop b) Frequency responses of T(s) function from **w** to **u**.
(dash line is no-control response).
Fig 8 . Control designed with k_o=0.3.



a) Eigenvalues of the close-loop b) Frequency response of **T**(s) function from **w** to **u**

Fig. 9 Control designed with k_0=0.125, and high γ.



a) Eigenvalues of the close loop b) Frequency responses of **T**(s) function from **w** to **u**.

Fig. 10 Control designed with k_0=0.125, and small γ.

figure 7. From the installation point of view, the application of active moment seems to be simpler than the active forces. It could be indicated that at the low wind speed, the pitching and heaving motions are uncoupled. Consequently, the active moment should have a very weak controllability over the heaving motion. As the main objective is the flutter, this observation is not the limitation.

First, appropriate weighting functions should be designed. For that purpose, the singular values of transfer function G_i are plotted against k and ω (Fig. 3). It can be observed that the structure could become more unstable near the origin and small k. However, as the whole plane is divided by the constant-wind lines, not all of these points have to be considered. Specifically, all points below the maximum design wind speed line can be considered as unreal and discarded. This may suggest that the range of design wind speed can be prefixed beforehand and the weighting functions are determined accordingly. For this example, a Umax=100 m/s is assumed for the plot in Fig. 4. Here W_1 is chosen to penalized the observed output in the low bandwidth where the structure modes are expected. By contrast, the function W_2 is applied to the controller output therefore, it penalizes on the high frequency responses of actuator. Such a strategy can make the controller energy more concentrated on the interested frequency range and at the same time, reduces the sensitivity where the noise could be disturbing. The details of designing these weighting functions can be seen in reference [6].

Once the weighting functions are resolved, a generalized plant is build as shown in the Fig. 4. It should be noted that there are still many choices on the selection of the outputs to regulated among the state of the system. The implementation of this control as a compensator is applied to the system of equation and the model is subjected to a flutter analysis with the results presented as the variation of the real and imaginary part of the system eigenvalues in function of the wind speed. To compare the intensity of control forces, the frequency response of the transfer function $\mathbf{T}_{w-u}$ as defined in the equation (22) is also shown at different values of wind speed U (5-100m/s), however, the range from 50m/s to 80m/s, where an active control is most likely needed is clearly shaded. Three examples are considered, the model 1 is calculated for a nominal k_0=0.3 (Fig.8) which corresponding to a rather low wind speed. Other two examples are calculated for k_0=0.125 with different attenuation factor γ and therefore, different degree of robustness. Model 2 (Fig.9) has a much higher robustness than model 3 (Fig.10). It can be noted that all models satisfy the requirement of stability with the flutter occurs in the model 1 at 100m/s wind meanwhile the other two models seem to be stable beyond this value. The choice of small k_0 could be more appropriated for the flutter suppression. In fact with

k_0=0.3, the stability requirement can only be met with a very robust design. The controller used here as a dynamic feedback or compensator, has its own internal poles which increase considerably the total number of the poles in the final system (for simplicity, poles in the figures that seem constant or irrelevant are omitted).

The frequency responses of transfer function **T**(s) can be used to both investigate the dynamic behavior of the system and assess the "economy" aspect of the control design, i.e., the forces required. In general, when the controller becomes more robust, the frequency response becomes flat like an all-pass filter, however, in all cases there is a pick at $\omega \approx 0.4$ at low wind speed. This is due to the limitation in the controllability at small U. As only moment is used, the heaving mode which has the natural frequency equal 0.389 can not be controlled when it is not coupled with the pitching mode. However, as the main target it the improvement of flutter resistance that always occurs at the high wind, this point should not jeopardize the control performance. In fact, at a higher wind speed, controller seems to be more effective. This observation does not necessarily mean that the control intensity will become less since this is expressed for the unit disturbances forces which often depend on the frequency and wind speed as well. Judging by overall consideration, the control as seen in model 3 that is well behaved at the flutter threatening wind speed while makes a minimum change in the system frequency seem to be preferable.

5. Concluding Remarks

The results of this study have shown that robust control methods can be effectively applied to improve the flutter resistance of the bridge deck. It also provides a simple and convenient way to tangle the problem of uncertainties that has always been a great concern in the application of active control to the civil engineering structures in general. The inclusion of frequency weighted function in the generalized plant makes this control method more reliable in the application on the real, complex and multidegree of freedom structure system. By targeting fixed frequency bandwidth, the control allows the use of method of model reduction without the danger of spill-over. Through examples in this paper, it is shown that it is possible to design the control with only active moment. The intensity of the control forces, can be related to the external disturbances by the transfer function norm. Through the examples, as the sizes of this norm measured by the singular value are shown to be less than 10 folds, it is possible to design the controller with forces about 3 times the order of the motion-independent disturbances. This observation can be encouraging since in the natural windy environment the initial disturbances are usually small.

The application of this control method to a more complex and real 3D structure model in affiliation with some suitable model reduction scheme will be the topic of future research.

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