A Fractional Derivative Hysteretic Model for Viscoelastic Damper

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A fractional-derivative 5-parameter hysteretic model is proposed to simulate the frequency dependent behavior of a viscoelastic (VE) damper. The development of the model is based on experimentally observed dynamic characteristics of VE dampers. A new viscoelastic damper has been fabricated, which is simple and robust in construction. The proposed model is validated by dynamic tests on this new damper and a good agreement between predicted and experimental results is obtained. Numerical algorithms for the solution of the force-displacement relationship in both the frequency domain as well as in the time domain are presented. Furthermore using the Kelvin fractional derivative model, response analysis of viscoelastically damped single-degree-of-freedom system (SDOF) has been carried out in the time domain as well as in the frequency domain. Finally, the problems associated with memory parameter of VE material and obtaining the true poles for response analysis have also been addressed.

Key Words : *Fractional derivative, Frequency-dependent behavior, Num. algorithm*

1. Introduction

Viscoelastic dampers are used for energy dissipation. These dampers are employed in tall buildings to suppress wind and earthquake induced vibration. Nowadays very reliable and durable viscoelastic materials are available, some of the VE material even have temperature independent mechanical properties. This makes the VE damper very attractive for vibration control.

Since energy dissipation in such dampers is due to deformation of the VE material, this mechanism of energy dissipation minimizes the mechanical wear and tear of the damper. This makes the VE damper almost maintenance free as compared to conventional oil dampers. Keeping these points in mind a new VE damper has been fabricated, which is simple and robust in construction as compared to other VE dampers, used by other reseachers, such as Markis[8] and Kasai[4]. The device configuration, presented in this paper exploits the energy dissipation capability of semi-solid type viscoelastic material, in an improved manner than the configuration studied by Markis[8] and Kasai[4]. This new damper can be used not only for retrofitting of bridges against earthquake, but also as a discrete energy dissipation device for any structure.
Viscoelastic materials usually consist of polymers or glassy substances and have the property of dissipating energy in the form of heat when subjected to deformations. Damping arises from the relaxation and recovery of the polymer network after it has been deformed [13) 7)].

It has been recognized that the mechanical properties of viscoelastic material depend strongly on the excitation frequency. Therefore the mechanical properties of the damper are dependent on the natural frequency of the structure as well as on the frequency content of the excitation. This frequency dependent property of the viscoelastic damper provides an interface between the external excitation and the dynamic system. Furthermore, the frequency dependent stiffness and damping adds an extra parameter to calculate the damped natural frequency of the structure [16)].

At present, viscoelastic models popularly applied in structural analyses are to relate time-dependent stresses to time-dependent strains through a series of time derivatives acting on the stress and strain fields [11]. The major drawback of this approach is that a large number of derivative terms, acting on stress and strain, are required to model the frequency dependent stiffness and damping properties for many viscoelastic materials. This complicates the process of performing a least square fit of the model to the data.

The robustness of passive and active control schemes depend on the degree of uncertainty involved in the estimation of structural parameters. This warrants the need of accurate mechanical models for discrete energy dissipation devices. To construct a model for the frequency dependent mechanical property of viscoelastic material, a fractional derivative model (FDM) has an advantage over the commonly used linear integer models. The attractive features of fractional derivative models are as follows:

- Fractional derivative model has its foundation in accepted molecular theories governing the mechanical behavior of viscoelastic materials.
- The model satisfies the second law of thermodynamics and predicts the stress-strain hysteresis loops for VE materials accurately.
- This viscoelastic model uses few parameters, thereby leading itself to straight forward and accurate least-square fits to measured mechanical properties [1].

The above mentioned features motivate the use of FDM for the response analysis of viscoelastically damped structures.

The objectives of this study are as follows:

- To develop an accurate model for the new damper configuration based on the experimental data.
- To solve the problems associated with the prediction of the force displacement relationship in the frequency domain as well as in the time domain.
- To address the problem associated with the response of a viscoelastically damped single-degree-of-freedom system in the frequency domain as well as time domain.

This paper has been organised in the sequence of the above stated objectives.

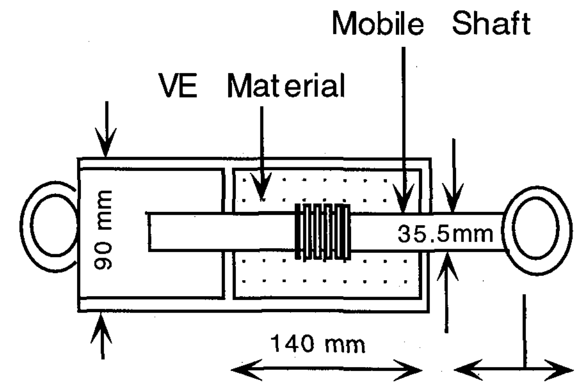

Fig. 1 Viscoelastic Damper

2. Fractional Derivative Model

In this study, a five-parameter(G, b, c, α, β) fractional derivative model has been used to express the time dependent stress $\tau(t)$-strain $\gamma(t)$ relationship which is as follows:

$$\tau(t) + bD^{\beta}\tau(t) = G\gamma(t) + cD^{\alpha}\gamma(t) \qquad (1)$$

where G, b, c, α and β are empirical parameters.

Definition

Fractional differintegration is an operator that generalizes the differentiation or integation to non-integal order. A commonly used definition by Liouville is as follows[5]:

$$_aD_t^q[f(t)] \equiv \frac{d^q f(t)}{[d(t-a)]^q} \equiv \frac{1}{\Gamma(n-q)}\frac{d^n}{dt^n}[\int_a^t \frac{f(\tau)}{(t-\tau)^{q-n+1}}d\tau] \qquad (2)$$

where Γ is a Gamma function.
In the theory of viscoelasticity, we are concerned with the influence of the entire response history on the current response. The lower limit (a) should be $-\infty$. However, if we assume $f(t) = 0$, $t < 0$, then $_{-\infty}D^q{}_t$ is identical to $_0D_t^q$. Hence a is usually taken as zero.

An analogy to understand what is a fractional derivative is as follows [10].
For elastic material:

$$\tau = k\gamma(t) = kD^0\gamma(t) \qquad (3)$$

For purely viscous material:

$$\tau = k\frac{d\gamma(t)}{dt} = kD^1\gamma(t) \quad (4)$$

Eq.(3) and Eq.(4) are the well known Hooke's law of elasticity and Newton's law of viscosity, respectively. Therefore, a visco-elastic material which has a property in between elastic and viscous material could be represented by D^α, where α lies between 0 and 1, which is a fraction.

The application of fractional derivative to VE material was first proposed by Gemant [6] way back in 1934. Recent researches [1] show that the fractional derivative of order 0.5 does arise naturally in the shear stress-strain relation of polymeric solids with no crosslinking, with some restrictions. Similarly, a related molecular theory taking into account the intermolecular hydrodynamic forces is developed by Zimm [17]. These theoretical findings provide a link between the microscopic behaviour of real materials. The attractive feature of the fractional derivative operator is the ability to vary the degree of its frequency dependence through the choice of α .

3. Outline of Experiments

3.1 Viscoelastic Damper

The construction of the VE damper is shown in Fig.(1). This damper allows uniaxial shearing of VE material. The surface of the moveable shaft has been roughened by threadings. This arrangement is enough to provide an effective bond between the shaft and the VE material. This roughness is confined to the middle third of the shaft's length. The rest part is smooth thus allowing the shaft to move freely in and out of the housing. The height of the thread is 3mm. The other important dimensions of the damper are shown in Fig.(1). Semisolid type VE material has been filled in the main housing. The shear deformation of the VE material trapped between the roughened shaft and the housing provides energy dissipation.

A schematic representation of the testing arrangement is shown in Fig.(2). The mobile shaft of the VE damper is attached to the actuator, and the other end of the damper is attached to a load cell (mounted on rigid support). Dynamic tests on the viscous damper were conducted by imposing sinusoidal motions of specified amplitude (1 to 4 mm) and frequency (0.1 Hz. to 5 hz.)

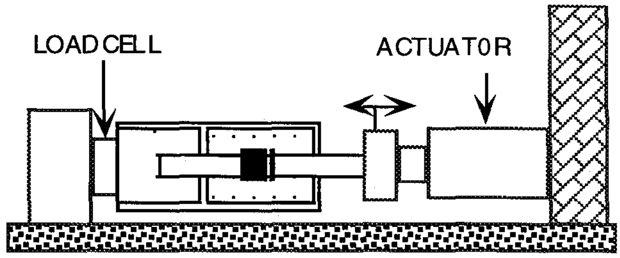

Fig. 2 Experimental Setup

to the horizontally mobile shaft of the damper and by measuring the force needed to maintain the motion.

3.2 Observed Mechanical Properties

The variation of mechanical properties with the excitation frequency is shown in Fig.(3) and Fig.(4). Fig.(3) shows that the damping decreases with frequency. The hysteresis plots from Fig.(7) to Fig.(9) show that the area of loop decreases with increase in the frequency. This indicates the reduction in energy-dissipation capacity, with the increase in the frequency. The stiffening of the damper with increase in the frequency is shown is Fig.(4). Similiar trends are observed in force responses shown in Fig.(10) to Fig.(12).

It should be noted that the reaction force is the force needed to maintain the motion, whereas the force imposed by the actuator differs from the reaction force by the inertia force of the moving part. This effect becomes significant with higher frequencies.

4. Development of Mathematical Model for Viscoelastic damper

Processing of Experimental Data

The recorded force-displacement loops had an almost precise elliptical shape (see Fig.(7) to Fig.(9)). These loops were used in obtaining the frequency dependent properties of the damper. The procedure is as follows:

Under steady-state conditions, the force and displacement are

$$u = U_0 \sin(\omega t) \quad (5)$$

$$P = P_0 \sin(\omega t + \delta) \quad (6)$$

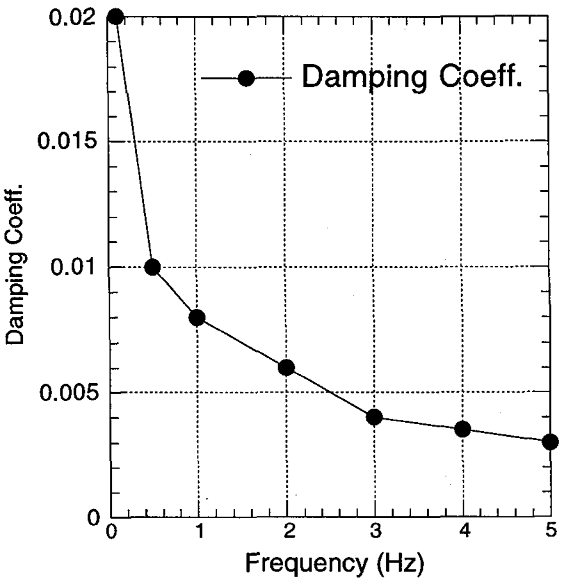

Fig. 3 Variation of Damping with Frequency

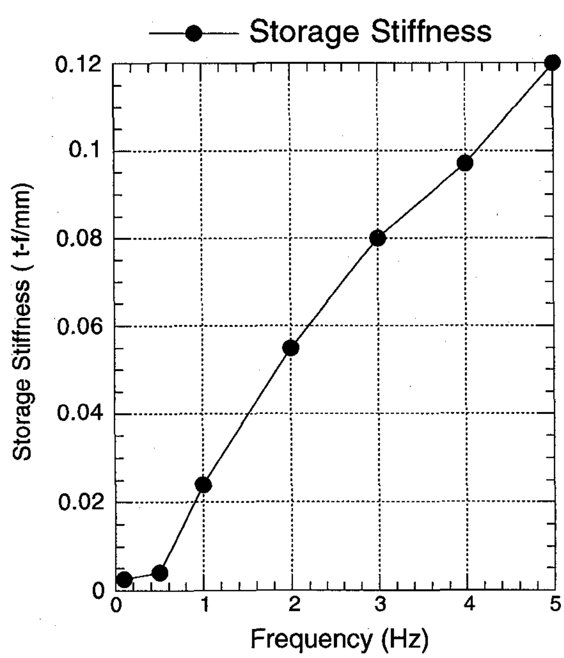

Fig. 4 Variation of Storage Stiffness with Frequency

where P_0 is the recorded amplitude of the force, U_0 is the recorded amplitude of displcement, ω is the frequency of motion and δ is the phase difference. The energy dissipated in a cycle of steady-state motion is

$$W_d = \oint P du = \pi \sin \delta P_0 U_0 \tag{7}$$

Physically W_d is the area of hysteresis in one cycle. Furthermore, Eq.(6) may be written as

$$P = K_1 U_0 \sin \omega t + K_2 U_0 \cos \omega t \tag{8}$$

where

$$K_1 = \frac{P_0}{U_0} \cos \delta, K_2 = \frac{P_0}{U_0} \sin \delta \tag{9}$$

K_1 and K_2 are the storage stiffness and loss stiffness of the damper respectively. The quantity $P_0/U_0 = K_0$ represents the elastic stiffness. It should be noted that the two parts of Eq.(8) represent the in-phase and 90^o out of phase parts of the force respectively. Accordingly, using Eq.(5) and its time derivative (velocity),

$$P = K_1 u + \frac{K_2}{\omega} \dot{u} \tag{10}$$

The quantity K_2/ω is the damping coefficient of the damper,

$$C = \frac{K_2}{\omega} \tag{11}$$

Returning to Eq.(7) and using Eq.(9)

$$K_2 = \frac{W_d}{\pi U_0^2} \tag{12}$$

To get a better understanding of energy dissipation capacity of the VE damper from structural engineering point of view, the variation of damping with the frequency is plotted in Fig.(3). These damping coefficent are derived from loss stiffness as per Eq.(11). Using Eq.(5) to Eq.(12) to extract the frequency-dependent properties of the damper from the measured quantities P_0, U_0 and W_d. The experimental values of storage stiffness and loss stiffness are plotted in Fig.(4) and Fig.(6) (shown by dotted points) respectively. The strong dependency of mechanical properties of the damper on the frequency is evident.

Fractional Derivative Modeling

The mathematical model of the VE damper is written in a form analogous to that of the stress-strain relationship of the viscoelastic material. This is based on the assumption that the VE material is primarily subjected to shearing action while the shaft moves in horizontal direction. The force-displacement relationship in the horizontal motion is expressed as

$$P(t) + bD^{\beta}P(t) = ku(t) + cD^{\alpha}u(t) \tag{13}$$

In the above equation P and u are force and displacement respectively.

Taking the Fourier Transform of Eq.(13) and assuming a zero initial condition. The force-displacement relationship becomes:

$$P(\omega) + b(i\omega)^{\beta} P(\omega) = ku(\omega) + c(i\omega)^{\alpha} u(\omega) \quad (14)$$

From Eq.(14), the complex stiffness can be deduced as

$$K^{\star} = \frac{k + c(i\omega)^{\alpha}}{1 + b(i\omega)^{\beta}} \quad (15)$$

Substituting $i^{\alpha} = \cos(0.5\pi\alpha) + i\sin(0.5\pi\alpha)$ into Eq.(15), then separating real and imaginary parts of complex stiffness we get

Storage Stiffness:

$$K_1 = \frac{k(1 + c\omega^{\alpha}\cos(0.5\pi\alpha) + b\omega^{\beta}\cos(0.5\pi\beta))}{(1 + b\omega^{\beta}\cos(0.5\pi\beta))^2 + (b\omega^{\beta}\sin(0.5\pi\beta))^2} + \frac{kcb\omega^{\alpha+\beta}\cos 0.5\pi(\alpha-\beta)}{(1 + b\omega^{\beta}\cos(0.5\pi\beta))^2 + (b\omega^{\beta}\sin(0.5\pi\beta))^2} \quad (16)$$

Loss Stiffness:

$$K_2 = \frac{k(1 + c\omega^{\alpha}\sin(0.5\pi\alpha) - b\omega^{\beta}\sin(0.5\pi\beta))}{(1 + b\omega^{\beta}\cos(0.5\pi\beta))^2 + (b\omega^{\beta}\sin(0.5\pi\beta))^2} + \frac{kcb\omega^{\alpha+\beta}\sin 0.5\pi(\alpha-\beta)}{(1 + b\omega^{\beta}\cos(0.5\pi\beta))^2 + (b\omega^{\beta}\sin(0.5\pi\beta))^2} \quad (17)$$

Parameter Estimation

To estimate the parameters involved in the expression of complex stiffness, a nonlinear least square fit has been used on the observed experimental values of the storage and loss stiffnesses. A powerful alogrithm for least square fit, known as "Leverberg Marquart" alogrithm has been used herein, which takes into account the partial derivative of the involved parameters. These fitted plots of storage stiffness and loss stiffness is shown in Fig.(5) and Fig.(6) respectively. This model fits very well, for the frequency range of 0.1 Hz. to 5 Hz. to the experimental results.

The values of the estimated parameters are: $k = 0.01$, $c = 0.0193$, $\alpha = 1.0$, $b = 0.48$ and $\beta = 0.6$. Hence complex stiffness of the damper takes the form

$$K^{\star} = \frac{0.01 + 0.0193(i\omega)}{1 + 0.48(i\omega)^{0.6}} \quad (18)$$

The force-displacement relationship in terms of time derivative becomes

$$P(t) + 0.48D^{0.6}P(t) = 0.01u(t) + 0.0193Du(t) \quad (19)$$

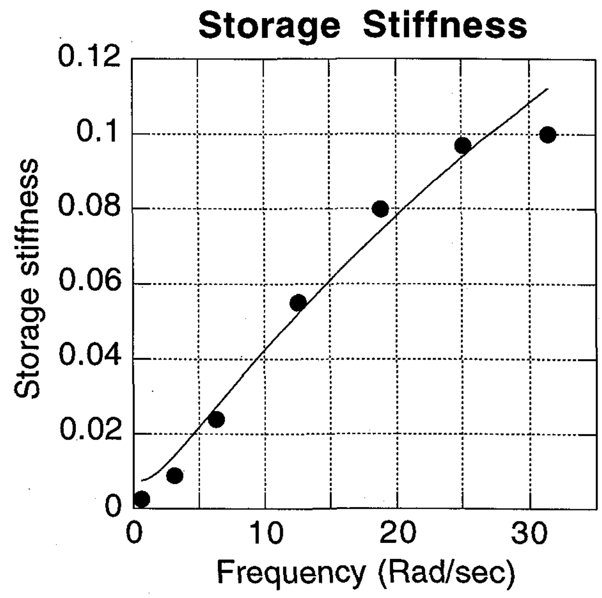

Fig. 5 Fitted Storage Stiffness of VE Damper

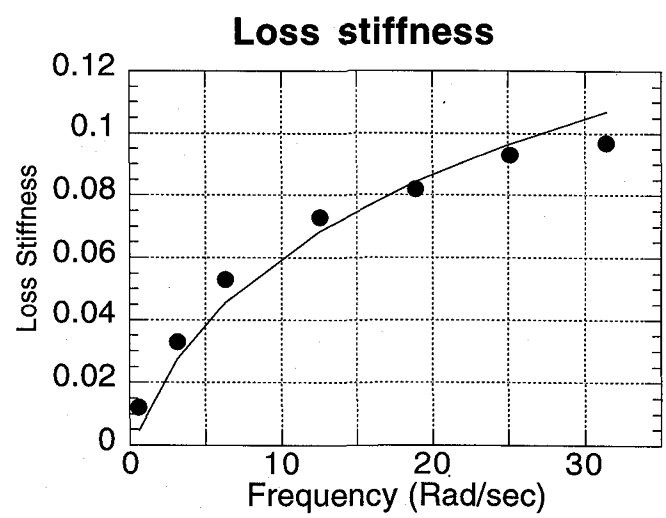

Fig. 6 Fitted Loss Stiffness of VE Damper

5. Prediction of Force-Displacement Relationship

In this section, the analytical schemes, to solve the force-displacement relationship developed in section(4) have been detailed, for the analysis in the frequency domain as well as in the time domain.

5.1 Laplace or Frequency Domain

Generally, viscoelastic stress analysis problems become more complicated due to the involvement of the time derivative of variables. The time variable can be removed by employing the Laplace Transform. When a solution for the desired variable has been found in terms of the Laplace Transform variable s, the inverse Laplace transformation yields the desired solution in the time varible t for the time dependent behavior in the viscoelastic problem. This systematic method is known as the Elastic-Viscoelastic Analogy or the 'Correspondence Principle'[15]. A similiar approach can be used in the frequency domain.

Due to the above mentioned principles, the numerical schemes in the frequency domain are much more convenient to use. Again the complex stiffness given by Eq.(15) can be broken into two parts similiar to Eq.(9). Then Eq.(15) will take the form $K^\star = K_1(\omega) + iK_2(\omega)$ which represents the amplitude and phase angle of the steady-state force in the damper for a harmonic displacement input of unit amplitude. Accordingly, the time history of force is expressed as:

$$P(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} [K_1(\omega) + iK_2(\omega)]\,\bar{u}(\omega)e^{i\omega t}d\omega \tag{20}$$

where $\bar{u}(\omega)$ represents the Fourier Transform of the imposed motion. The computation of the force is thus obtained by the Discrete Fourier Transform (DFT) approach in combination with Fast Fourier Transform (FFT) algorithms (Veletsos and Ventura) [14].

5.2 Time Domain

Rewriting the force-displacement relationship of Eq.(19)

$$0.48D^{0.6}P(t) = 0.01u(t) + 0.0193Du(t) - P(t) \tag{21}$$

In evaluating $D^\beta P(t)$, $P(t)$ is needed, which is unknown. Therefore, an iteration process is needed [8]. Moreover to evaluate the term $D^\beta P(t)$ numerically, Oldham [5] has given certain quadrature formulae. The quadrature scheme is disscussed in detail in the section(7.1.1) of this paper i.e. Response of VE damped SDOF system.

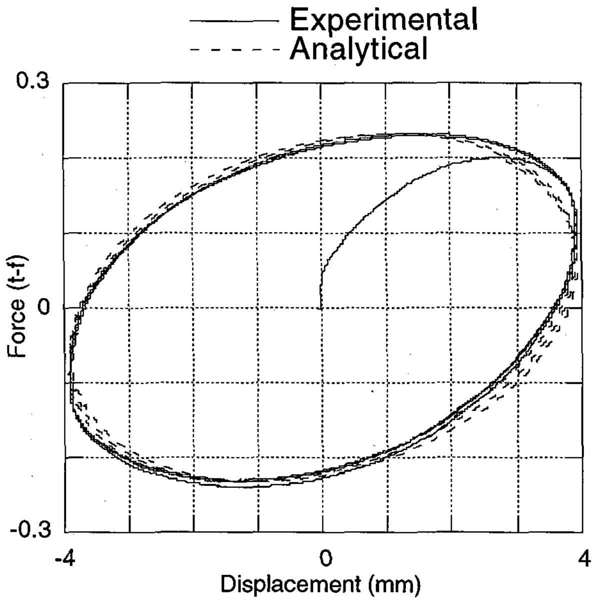

Fig. 7 Hyteresis loop 1 Hz

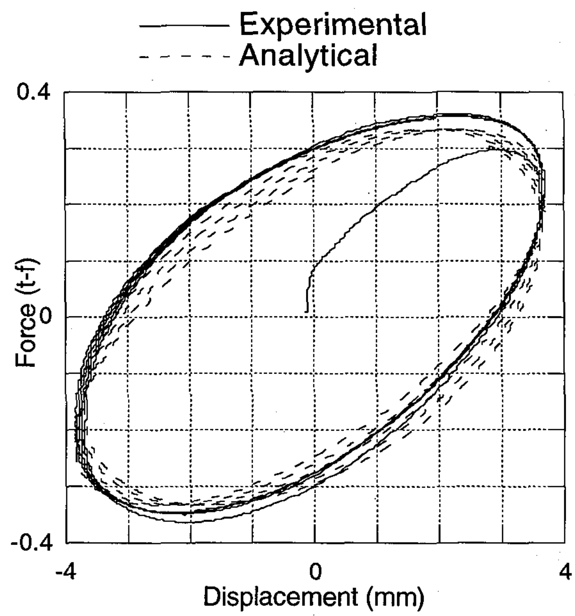

Fig. 8 Hyteresis loop 2 Hz

6. Comparison of Results

The model fits very well to the experimentally observed values of storage stiffness and loss stiffness as shown in Fig.(5) and Fig.(6) respectively. Moreover Fig.(7) to Fig.(9) demonstrate good agreement between the analytical prediction and recorded force-displacement loops. Fig.(10) to Fig.(12) also show good agreement for the force response in the tests on the VE damper. The

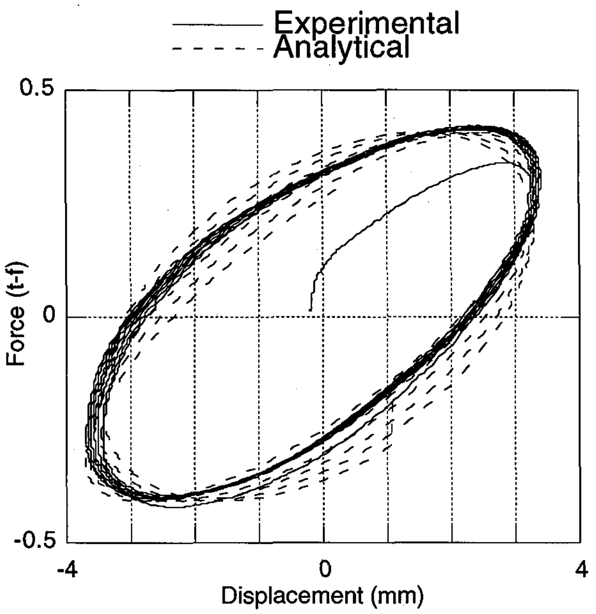

Fig. 9 Hyteresis loop 3 Hz

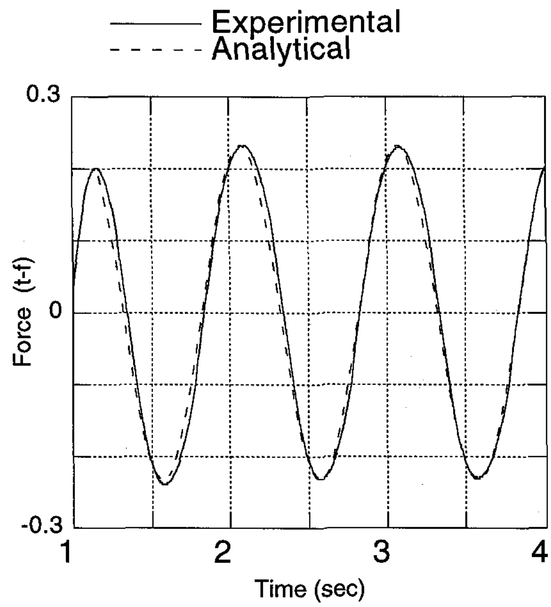

Fig. 10 Force Response 1 Hz

force-displacement relationship corresponding to 1 Hz, 2 Hz and 3 Hz has been analysied to predict experimental results. This analysis has been conducted in the frequency domain.

Looking at the hyteresis loop from Fig.(7) to Fig.(9), it is to be noted, that the major axis of these ellipitical loops rotates towards the force axis as the frequency of excitation increases from 1 Hz to 3 Hz. This indicates the stiffening effect of VE material with the increase in the frequency. It is clear from Fig.(7) to Fig.(9) that the area of hystersis loop decrease with the increase in the frequency, this indicates that the energy dissipation capacity of the VE material is decreased, with the increase in the frequency of excitation.

Therefore, it can be concluded that the proposed 5-parameter fractional derivative model not only fits the observed experimental values of storage stiffness and loss stiffness in a good way but also accurately predicts the hyteresis of the damper used herein.

7. Response of VE Damped Single Degree of Freedom System

In this section of this paper the salient features of fractional derivative approach in calculating the response of viscoelastically damped system has been discussed with the help of a numerical solution. The numerical analysis has been carried out in the time domain as well as in the frequency domain. The problems associated with numerical solution and their remedies are discussed.

7.1 Numerical Scheme

Adopting the well known Kelvin's model, the constitutive relationship can be expressed as:

$$\sigma(t) = GD^0\epsilon(t) + bD^\alpha\epsilon(t) \tag{22}$$

where G, b,and α are the constitutive parameters. Now, similiar to the constitutive Eq.(22) the force-displacement relationship of the VE device can be expressed as $f(t) = kx(t) + bD^\alpha x(t) = k^*x(t)$; which renders $k^* = k + bD^\alpha$. Let the equation of motion of a SDOF system be $m\ddot{x}+k^*x(t) = F(t)$. This equation in a derivative form becomes:

$$mD^2x + bD^\alpha x + kx = F(t) \tag{23}$$

Taking the Laplace Transform of Eq.(23)

$$(ms^2 + bs^\alpha + k)X(s) = F(s) \tag{24}$$

The numerical analysis is carried out on SDOF system defined by Eq.(23) for sinusoidal excitation($\sin \Omega t$). Numerical values taken for analysis are: mass(m) = 1.0, k = 1.0, b = 0.1, α=0.5, h = 0.5 sec, Ω=1.0 rad/s.

(1) Time Domain

To facilitate the incorporation of nonlinearity a numerical step-by-step solution technique has

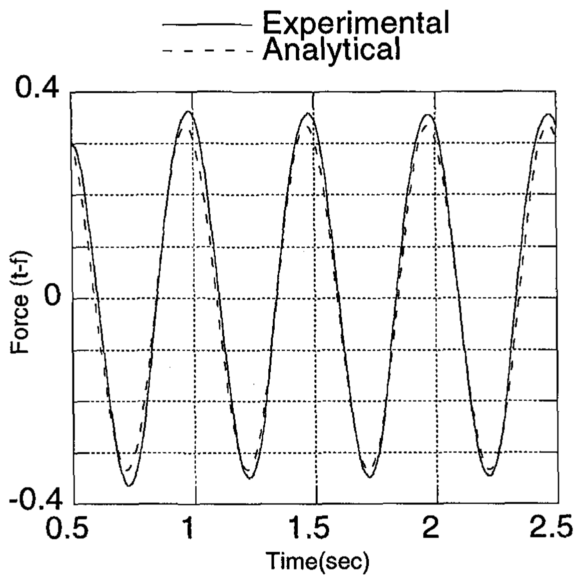

Fig. 11 Force Response 2Hz

to be developed. The acceleration can be approximated by a central difference as follows:

$$\ddot{x}(t) = \frac{1}{h^2}[x(t+h) - 2x(t) + x(t-h)] \quad (25)$$

where h is the time step size used in the numerical scheme. Let x_n be the numerical solution of $x(t = nh)$ at the n^{th} step. Then the numerical representation is

$$\ddot{x}_n = \frac{1}{h^2}[x_{n+1} - 2x_n + x_{n-1}] \quad (26)$$

The typical term in Eq.(23) is $D^\alpha x$ which is evaluated by the quadrature formulae given by Oldham [5]. Now $D^\alpha x$ can be expressed in a quadrature form as:

$$D^\alpha x_n = \frac{1}{h^\alpha}\sum_{j=0}^{n} w_j x_j, \quad 0 \le \alpha < 1 \quad (27)$$

where w_0, w_{n-j} and w_n are weights [5], n = total no. of time steps ; h = time step size. It is to be noted that the subscript j is zero at the n^{th} step and takes the value n at the initial step of calculation. Therefore, for middle terms the value of j should be used accordingly.

Memory Parameter

It is obvious, that the evaluation of $D^\alpha x$ given by Eq.(27) at a particular step, recalls all the previous steps, which is commonly known as the memory characteristic of the VE material. Now, if the time step size is small, or the structure has many dampers, the computational time becomes large. To overcome this problem an algorithm proposed by Koh [3], which facilitate as to how many previous step are to be recalled as

$$D^\alpha x_n = \frac{1}{h^\alpha}\sum_{j=0}^{N} w_j x_{n-N+j}, \quad 0 \le \alpha < 1 \quad (28)$$

where at n^{th} step; N = no. of previous steps to be recalled; w_0, w_{n-j} and w_n are weights [4]. Different researchers have their own rule of thumb to estimate the number of previous step to be recalled [4]. We have used the no. of step to be recalled(N) by assigning the relative error and using a formula:

$$N \approx \frac{\alpha^2}{2\epsilon_r} \quad (29)$$

where ϵ_r is a relative error [5]. So for α=0.5 and 1% relative error, renders N =12. Now the modified weights after incorporating N are as follows:

$$w_0 = \frac{1}{\Gamma(2-\alpha)}[(N-1)^{1-\alpha} - N^{1-\alpha} + (1-\alpha)N^{-\alpha}]$$

$$w_n = \frac{1}{\Gamma(2-\alpha)}$$

$$w_{n-j} = \frac{1}{\Gamma(2-\alpha)}[(j+1)^{1-\alpha} - 2j^{1-\alpha} + (j-1)^{1-\alpha}]$$

$$1 \le j \le N-1$$

Substituting the values of Eq.(26) and Eq.(28) into Eq.(23),

$$\frac{m}{h^2}(x_{n+1} - 2x_n + x_{n-1}) + \frac{b}{h^\alpha}\sum_{j=0}^{N} w_j x_j + kx_n = f(nh) \quad (30)$$

This renders a multistep numerical scheme

$$\bar{w}_{n+1}x_{n+1} = f(nh) - \sum_{j=0}^{N} \bar{w}_j x_j \quad (31)$$

Fig.(13) and Fig.(14) show the transient and steady state response of the VE damped system respectively. Results obtained by the time domain analysis are in good agreement with the laplace domain analysis as shown in Fig.(13) and Fig.(14).

(2) Laplace Domain

From Eq.(24) the transfer function of the system H(s)=$(ms^2 + bs^\alpha + k)^{-1}$, since $X(s) = H(s)F(s)$, taking the inverse laplace transform of $X(s)$ will give the time history of response $X(t)$. The Laplace domain solution of the system requires solving the characteristic equation

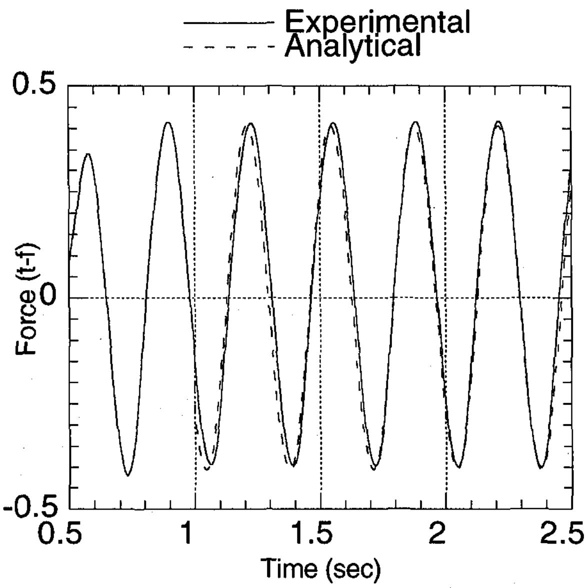

Fig. 12 Force Response 3Hz

expressed as

$$s^2 + 0.1s^{0.5} + 1 = 0 \qquad (32)$$

Since Eq.(32) involves a fractional power to s, for mathematical simplicity, it is to be converted into an integer. Therefore Eq.(32) is written as

$$s^4 + 2s^2 - 0.01s + 1 = 0 \qquad (33)$$

True System Characteristics

It should to be noted that in writing Eq.(33), the order of the system has been increased and thus the solution will involve some "false poles" related to the added higher order of s due to the transformation from Eq.(32) to Eq.(33). One of the conventional method to filter out the false poles is as follows. When the poles of the system are potted on the complex plane,then the imaginary axis represents a region of mere stablity, the left half of the plane is the region of asymptotic stablity. Therefore, the poles with the negative real part should be the stable poles; but in the course of mathematical manipulation sometimes many more poles satisfy this criteria, all of which are not true representatives of the system. It is the designer's skill to seperate the false poles and choose the correct ones. One of such method is to estimate the undamped natural frequency of the system without VE dampers, then calculating the stiffness and damping of the VE damper corresponding to the undamped natural frequencies of the system. Now one can get an approximation of damped natural frequencies of the system by adding the stiffness and damping of the VE damper to the system without VE damper. The poles, out of the filtered ones (by the conventional methods)[9], which are close to this approximation of damped natural frequencies will be the right poles. The poles for the current problem is obtained as

$$s = -0.0353443 \pm 1.03537i \qquad (34)$$

Now, the inverse Laplace transform of Eq.(24), with the poles given by Eq.(34) gives the time history of displacement response under sinusoidal excitation starting from $t = 0$ with the zero initial condition as

$$X(t) = 9.489e^{-0.0353t}\cos(1.035t + 0.768) - 9.824\cos(t + 0.8031252) \qquad (35)$$

The response is plotted in Fig.(13) and Fig.(14) along with the solution obtained by the time domain analysis.

8. Conclusion

It has been concluded that the five parameter fractional derivative model accurately models the mechanical properties of the viscoelastic damper. The new viscoelastic damper configuration works very well to exploit the energy dissipation capacity of the VE material. The problems associated with the development and calibration of the mathematical model for VE damper are addressed extensively. Moreover the numerical scheme in the time domain and in the frequency domain has been disscussed. It is found that the frequency domain analysis is more convenient than the time domain analysis for prediction of the force-displacement relationship of the damper. The problems associated with the response of viscoelastically damped single-degree-of-freedom-system in the time domain, such as the memory parameter, and in the frequency domain such as the eliminating the spurious poles are investigated and some remedies are proposed.

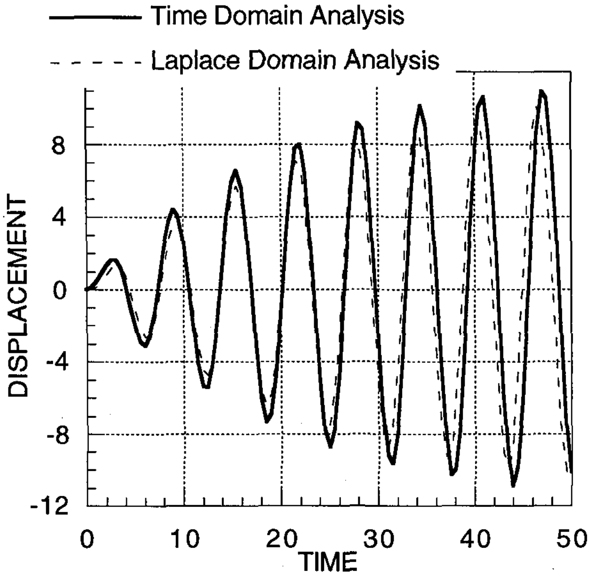

Fig. 13 Transient Response

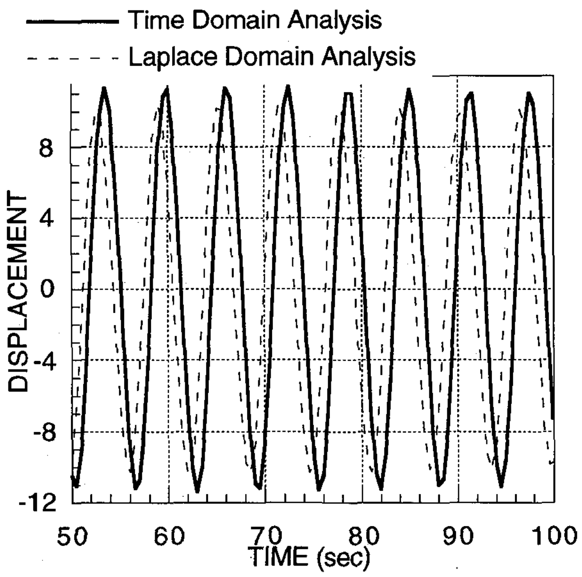

Fig. 14 Steady State Response

References

1) Bagley R.L., Torvik P.J; "Fractional Calculus in the Transient Analysis of Viscoelastically Damped Structures" AIAA Journal Vol.23 No.6, Sep.5, 1984.
2) Bagley R.L., Torvik P.J;"Fractional Calculus-A Different approach to the Analysis of Viscoelastically Damped Structures" AIAA Journal Vol.21 No.5, May 1983.
3) Koh C.G., Kelly J. "Fractional derivative representation of seismic response of base isolated models: Theory and Experiments". Report No. UCB/SEM 85/07, Berkeley.
4) Kasai K., Munshi.J., Lai M. "Viscoelastic Damper Hysteretic Model: Theory, Experiment and Application".
5) Oldham, K.B. and Spanier, J. (1974) The Fractional Calculus. Academic Press.
6) Gement, A.(1936),"A Method of Analyzing Experimental Results Obtained from Elasto-Viscous Bodies", Physics, 7, 311-317
7) Macosko, C.W (1994); "Rheology: Principles, measurements and applications". VCH Publishers Inc.
8) Markis, N., and Constantinou, M.C.(1990), "Viscous Dampers: Testing, Modeling and Application in vibration and seismic Isolation". Technical Report, NCEER-90-0028, Civil Engg. Dept; State University of New York, Buffalo, NY 14260.
9) Meirovitch, L. "Dynamics and Control of Structures", John Wiley & Sons pp. 68-73 .
10) Stiassnie, Michael(1979); Journal of Applied Math. Modelling, Vol.3 August 1979.
11) Schwahn, K.J., Reinsch, K.H. and Weber, F.M. (1988). "Description of the features of Viscous Dampers on the Basis of Equivalent Rheological Models, prented for pipework dampers". Proc., Pressure, Vessel and Piping Conference, Seismic Engineering, ASME, NewYork N.Y. Vol.127 Pg. 477-484
12) Press, W.H., Brain P. Flannery, Teukolsy, Saul A., Vetterly William T. "Numerical Recipes". 521-528. Cambridge University Press.
13) Nasif A.D., Jones D.I.G., and Henderson J.P. (1985). "Vibration Damping". John Wiley & Sons Inc.
14) Veletsos A.S., and Ventura C.E.(1985). "Dynamic analysis of structures by the DFT method." J. Struct. Engrg., ASCE, 111(2), 2625-2642.
15) Findley William N., Lai James S. and Onaran K.,(1975) "Creep and Relaxation of Nonlinear Viscoelastic Materials". Dover Publications, Inc., New York.
16) Zhang R.H., Soong T.T "Seismic Design of Viscoelastic Dampers for structural Applications". J. Struct. Engrg. ASCE Vol. 118 No.5 May 1992.
17) Zimm. B.H, J. Chem. Phys. 24(2), 269-278 (1956)

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