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Study of Amplification of Vertical Ground Motion Using Inverse-Analysis to Determine Dynamic Soil Parameters

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This paper focuses on the amplification of vertical ground motion during an earthquake. A strong vertical acceleration was observed at the epicentral region during the 1995 Hyogoken Nambu earthquake, and it seemed to be strongly related to the heavy damage to structures. The amplification of vertical ground motion is evaluated by using the multiple reflection theory. Dynamic soil parameters ν, E, h which are used in the analysis, are determined by inverse-analysis using the downhill simplex optimum method. The sum of the squared differences between the Fourier amplitude spectra of observed and calculated accelerations is minimized in the inverse-analysis. A fairly good result is obtained using the vertical array observation data in Port-island in Kobe City during the 1995 Hyogoken Nambu earthquake.

Key Words: Inverse-analysis, soil dynamics, ground motion, P-wave

1. Introduction

Until recently, most studies of earthquake ground motions have been limited to the horizontal directions only. For design purposes, the vertical ground motions have been mostly ignored or have been simplistically treated as being some fraction of the horizontal ground motions because vertical ground motions were usually less than horizontal ground motions. However, recent earthquakes such as the Hyogoken Nambu and Northridge earthquakes have produced vertical ground motions that have been significantly greater than the corresponding horizontal ground motions. These occurrences have stimulated the current studies to further an understanding of ground motions in general. Such studies are also motivated by the increasing importance of vertical ground motions for structural design purposes. With more architecturally unique and structurally complicated buildings, dynamic analyses including the vertical ground motions will become more common. In the design of aseismatic structures, it is believed that the vertival ground motions will be very important to consider as it may strongly amplify the damages to the structures.

As is known that the earthquake response of ground motions can be only analyzed when dynamic soil parameters are given. The results of such studies are found to be closely dependent on the parameters chosen. For horizontal ground motions, the shear stiffness characteristics of soil under dynamic loads have been evaluated by laboratory or field tests[1)~3)]. The damping characteristics were primarily determined using laboratory techniques, but the studies of dynamic soil properties such as Poisson ratio ν, Young's modulus E and normal damping constant h_v which govern the vertical ground amplifications can be seldom found.

With the development of dense vertical array observations of actual earthquakes, recent research has been directed towards the inverse analyses to determine dynamic soil parameters because neither laboratory test nor field probe could reproduce the dynamic soil properties under actual earthquakes . Ota et al.[4] used the multiple reflection theory of S-wave and optimum method to determine the velocity of S-wave and Q value for the level layered ground based on the observed earthquake data at surface and in GL-50 m. Honda et al.[5] used the finite element method and simplex optimum method to determine some empirical coefficient which connected the relation of the velocity of S-wave and SPT N-value, and shear damping constant for both level layered ground and embankment ground. Tsujihara et al. [6)-8)] and Sawada et al. [9)] minimized the least squared differences between the observed and calculated ground frequency response functions to determine ground shear velocity Vs and Q value.

What the above researchers achived has contribute to the study of the site reponses for horizontal ground motions. To analyze the earthquake ground motions in general this paper focuses on the amplification of vertical ground motion during an earthquake by using the inverse-analysis to determine dynamic soil parameters.

This paper applied the multiple reflection theory[10),11)] to both S-wave and P-wave propagations and used the downhill simplex optimum method[12),13)] to firstly determine ground shear modulus G and shear damping constant h_h by minimizing the sum of squared differences between the Fourier amplitude spectra of observed and calculated horizontal accelerations, and then to determine ground Poisson ratio ν, Young's modulus E and normal damping constant h_v by minimizing the sum of squared differences between the Fourier amplitude spectra of observed and calculated vertical accelerations. For design purposes, the equivalent linear method is usually used to represent the non-linearity of ground behavior when to calculate the response of horizontal ground motion. According to such method it is thought that the non-linearity of ground behavior is represented in both horizontal and vertical directions by making the dynamic soil parameters approach to their real values in convergent calculations of the objective functions. In order to examine the reliabilities of determined dynamic soil parameters, a kind of vertical array observation in which at least three different layers are measured is needed. Fortunately the vertical array observations in Port-island of Kobe during the Hyogoken Nambu earthquake can meet the demand.

2. Response Analyses of Vertical Ground Motions

2.1. Frequency Response Function

The multiple reflection theory of one dimensional wave propagation is introduced here to analyze the site responses of horizontal and vertical ground motions.

Let $\xi_i(Z_i,t)$ be the displacement in i-th layer for layered ground, then hamornic solution for i-th layer can be introduced as

$$\xi_i(Z_i,t) = \left(A_i e^{ip_iZ_i} + B_i e^{-ip_iZ_i}\right)e^{i\omega t} \quad i=1,2,\cdots,n \qquad (1)$$

The acceleration in upper side of an objective layer (r-th layer) is

$$\xi_r(0,t) = -\omega^2\left(A_r + B_r\right)e^{i\omega t} \qquad (2)$$

The acceleration in upper side of an reference layer (s-th layer) is

$$\xi_s(0,t) = -\omega^2\left(A_s + B_s\right)e^{i\omega t} \qquad (3)$$

The ratio of these two accelerations represents the frequency response function of objective layer (r-th layer) to reference layer (s-th layer). Meanwhile the frequency response function of shear strain in the middle of objective layer (r-th layer) to the acceleration of reference layer (s-th layer) can be represented as

$$\tilde{Z}_{r/s}(\omega) = -\frac{ip_r\left(A_r e^{ip_rH_r/2} - B_r e^{-ip_rH_r/2}\right)}{\omega^2\left(A_s + B_s\right)} \qquad (4)$$

where $p = \omega\sqrt{\rho/G^*}$ for S-wave

and $p = \omega\sqrt{\rho/E^*}$ for P-wave

G^*, E^* :complex shear and Young's moduli; H_r :depth of the objective layer.

2.2. To Determine Dynamic Soil Parameters Using Inverse-analyses

2.2.1. Minimization of Objective Function

The accelerations observed in the bedrock and at surface or at the other layers in ground are needed to form an objective function for the optimization during inverse-analyses. If minimization is applied directly to the sum of squared differences between the observed and calculated accelerations in the bedrock and at surface or at the other layers in the ground, it is possible that the error is accumulated highly because of the time lag of acceleration sampling. This paper minimizes the sum of squared differences between the Fourier amplitude spectra of observed and calculated accelerations. To let the objective function converge, the Fourier amplitude spectra between observed and calculated accelerations were smoothed with Parzen window. J_h, J_v are the objective functions minimized to determine G, h_h and ν, E, h_v, respectively.

$$J_h = \sum_{i=1}^{N_f/2-1}(c_{h,i} - C_{h,i})^2 / \sum_{i=1}^{N_f/2-1} C_{h,i}^{\ 2} \tag{5}$$

$$J_v = \sum_{i=1}^{N_f/2-1}(c_{v,i} - C_{v,i})^2 / \sum_{i=1}^{N_f/2-1} C_{v,i}^{\ 2} \tag{6}$$

where $C_{h,i}, C_{v,i}$:the Fourier amplitude spectra of observed horizontal and vertical accelerations, respectively, in i-th frequency; $c_{h,i}, c_{v,i}$:the Fourier amplitude spectra of calculated horizontal and vertical accelerations corresponding to above $C_{h,i}, C_{v,i}$; N_f:sampling number.

2.3. Dynamic Soil Parameters

To analyze the amplification of vertical ground motion, dynamic soil parameters such as Poisson ratio ν, Young's modulus E and normal damping constant h_v must be known. In order to get the stable solutions for such parameters, this paper firstly conducts the inverse-analysis to get the dynamic shear modulus G and shear damping constant h_h using horizontal accelerations obtained from vertical array observations, and then obtains ν, E, h_v on the basis of G and vertical accelerations through the inverse-analysis. If we directly evaluate shear velocity Vs and h_h at each layer, it is likely for the objective functions not to converge due to a large unknown number. Therefore the following relation[5] was chosen from several empirical relations available to evaluate shear modulus for this study.

$$V_s = aN^b \tag{7}$$

$$G = \rho V_s^2 / g \tag{8}$$

where N :SPT N-value; a,b : coefficients representing soil properties; ρ :soil density; g :gravity acceleration. According to Imai[14] and Toki[11] b can be fixed at 0.341 for soils with different properties such as sand and silt. Therefore, as long as a is evaluated we can use the above relations to determine shear modulus G. It can be observed from convergent calculations of the objective function that the value of damping constant h_h affects the objective function much less than the value of shear modulus G does. Much work has been done by Tsujihara et al.[7] to show the similar observation in this point. Therefore, to decrease unknown number for convergence we assume h_h is the same in all layers.

To evaluate Poisson ratio ν, Young's modulus E and normal damping constant h_v, the following relation is adopted.

$$E = 2(1+\nu)G \tag{9}$$

Then we assume ν and h_v are the same in all layers.

3. Application to Level Ground

The above method can be applied to a level ground. Of many vertical array observations those observed in Port-island of Kobe during the Hyogoken Nambu earthquake were chosen for this study. Fig.1 shows the ground geological conditions and SPT N-values in the observation site.

3.1. Observed Accelerations and Ground Properties

The accelerations were observed at surface, GL-16m, GL-32m and GL-83m. They were recorded for 6 minutes including one main quake and 5 after quakes. The recorded accelerations of main quake of 40 seconds were base-line-revised and then applied to this study. The sampling step of data was 0.01 second. Fig.2 shows the recorded accelerations at surface, GL-32m and GL-83m and their corresponding Fourier amplitude spectra. The ground properties were shown in Table 1 which was opened by The Bureau of Development of Kobe City via Kansai Earthquake Observation Research Association.

3.2. To Determine Shear Modulus G and Damping Constant h_h

The observed horizontal accelerations at surface and GL-83m site were used as input data in convergent calculation. The objective function J_h got stable smoothly after 30 iterations. Fig. 3 shows the parameters a and h_h in the process of convergent calculations. The values of a and h_h were stable at 76.4 and 0.436, respectively. Compared with

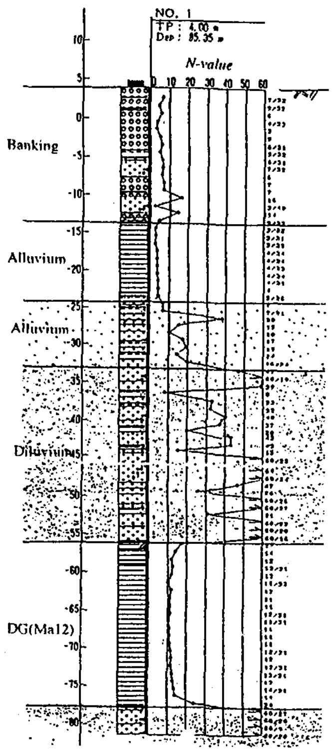

Fig.1 Ground geological conditions and PST N-values

Table 1. Soil-properties in Port-island

Depth (m)	Soil type	Unit Weight (tf/m^3)	Mean N-value
0-3.0	Fill,sandy gravel	1.7	5.2
2.0-5.0	Fill,sandy gravel	1.7	5.2
5.0-12.6	Fill,sandy gravel	1.7	6.5
12.6-19.0	Alluvial Sandy gravel	1.7	6.5
19.0-27.0	Alluvial clay	1.6	3.5
27.0-33.0	Sand	1.7	13.5
33.0-50.0	Diluvial sandy gravel	1.8	36.5
50.0-61.0	Sand	1.8	61.9
61.0-79.0	Diluvial clay	1.7	11.7
79.0-85.0	Diluvial sandy gravel	2.0	68.0

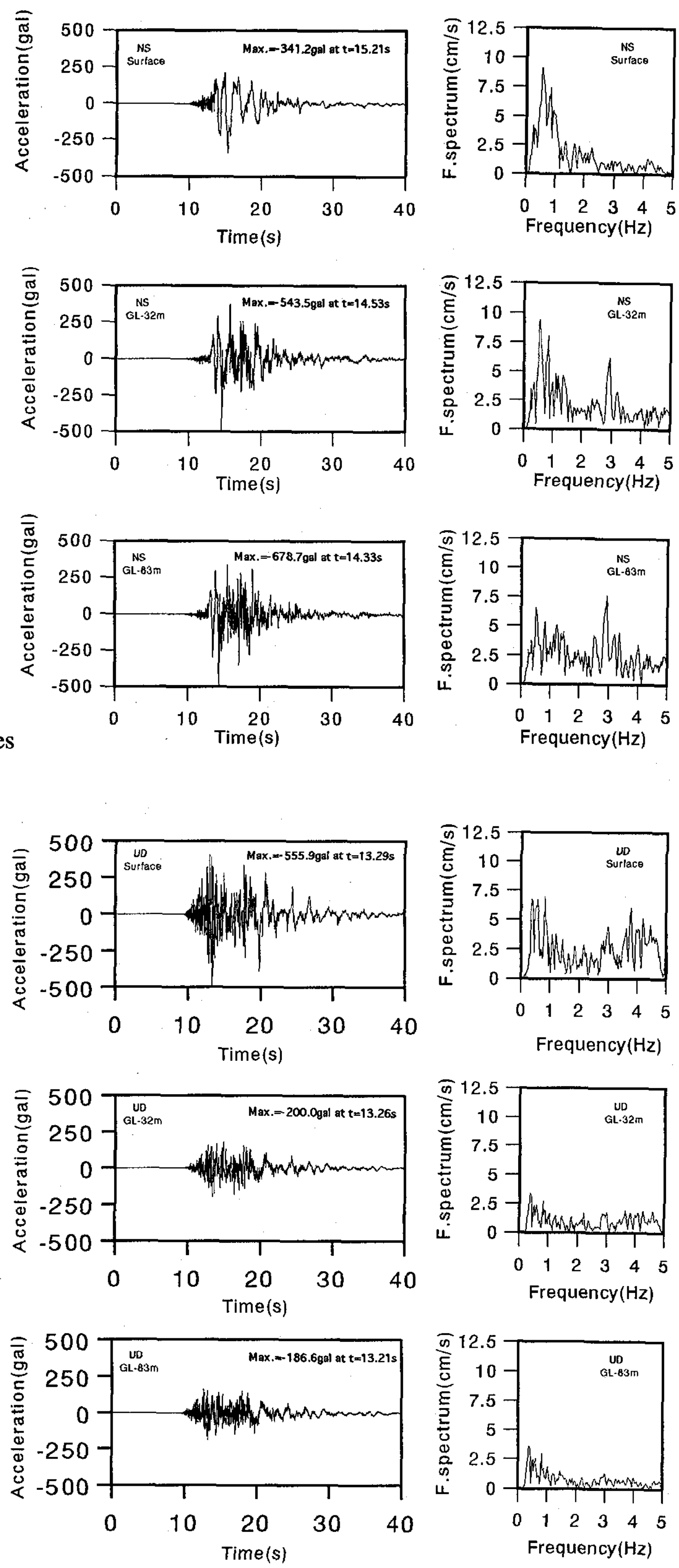

Fig.2 Array observed accelerations and their Fourier spectra in Port-island During the 1995 Hyogoken Nambu Earthquake

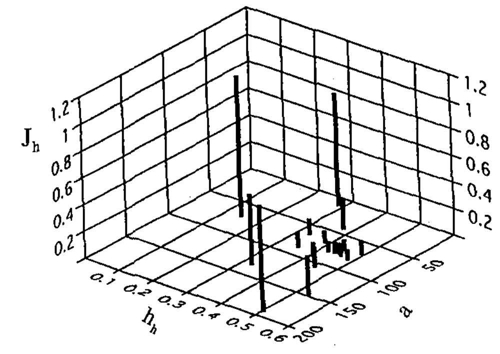

Fig.3 Convergent calculation to determine a and h_h

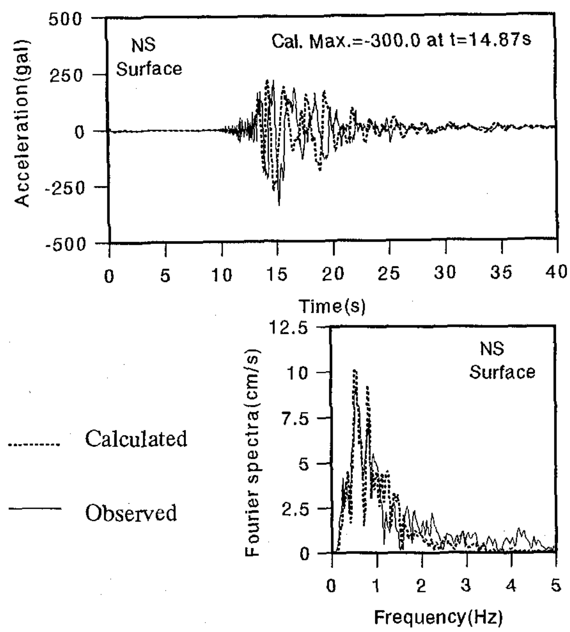

Fig.4 Observed and calculated horizontal accelerations and their Fourier spectra at surface

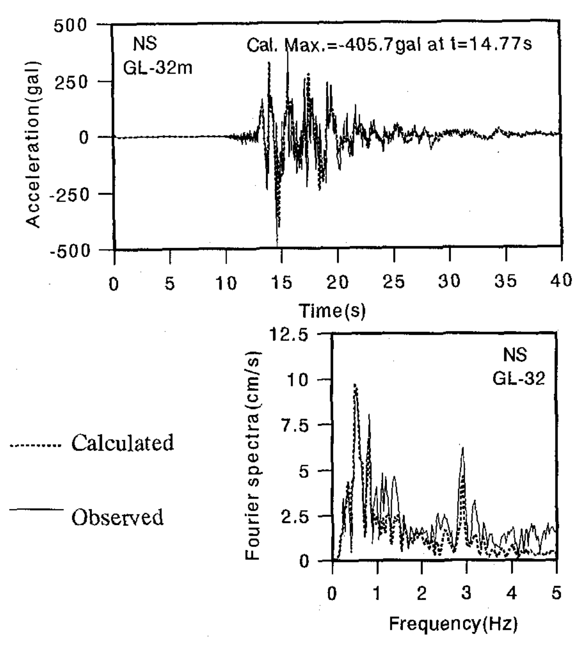

Fig.5 Observed and calculated horizontal accelerations and their Fourier spectra at GL-32m site

regressional results[11] a was a little low. It is because the regressional results were evaluated by small vibration probe. The ground of Port-island was strongly disturbed by the actual earthquake, it can be found from Fig. 2 that the propagation of accelerations was much damped from GL-83m to the surface. Fig. 4 shows the comparison of calculated and observed accelerations and their corresponding Fourier amplitude spectra at surface in convergent status by using the dynamic soil parameters obtained through the inverse-analysis. The calculated and observed results were found to be in good agreements. Furthermore, to examine the reliabilities of shear modulus and damping constant obtained here, these two parameters were used to calculate the response at GL-32m site by normal analysis. The calculated and observed accelerations and their corresponding Fourier amplitude spectra are shown in Fig. 5. We could find that the calculated acceleration is in good shape with the observed one. Corresponding with observed maximum acceleration at time 14.53 second the calculated acceleration gets maximum at 14.77 second. Meanwhile, both calculated and observed Fourier amplitude spectra get predominant roughly in the same frequencies. The calculated maximum acceleration is lower than that of the observed. It may be due to the uncontinuity of ground from GL-83m to GL-32m and from GL-32m to the surface under earthquake loading. The dynamic soil parameters inversely calculated here were not initial parameters but ones under strong ground motions.

3.3. To Determine Poisson Ratio ν , Young's Modulus E and Damping Constant h_v

After shear modulus G is determined the relation between Young's modulus E and G could be connected by Poisson ratio ν as shown in Eq. 9. The observed vertical accelerations at surface and GL-83m site were used as input data in convergent calculation. The objective function J_v got stable smoothly after 32 iterations. Fig. 6 shows the parameters ν and h_v in the process of convergent calculations. The values of ν and h_v were stable at 0.42 and 0.06, respectively. Fig. 7 indicates the comparison of observed and calculated accelerations in convergent status and their Fourier amplitude spectra at surface. Corresponding with the observed maximum acceleration at time 13.29 second the calculated acceleration gets maximum at 13.30 second. It is in good agreement. The acceleration wave shapes are also in good agreement though the maximum values are

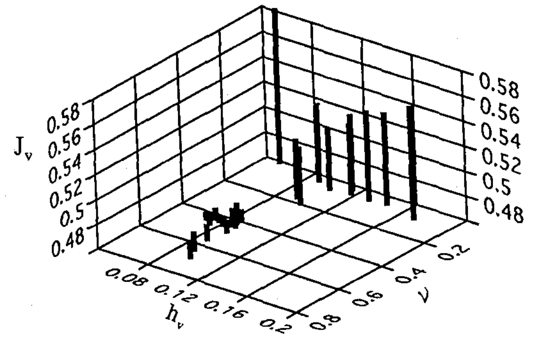

Fig.6 Convergent calculation to determine ν and h_v

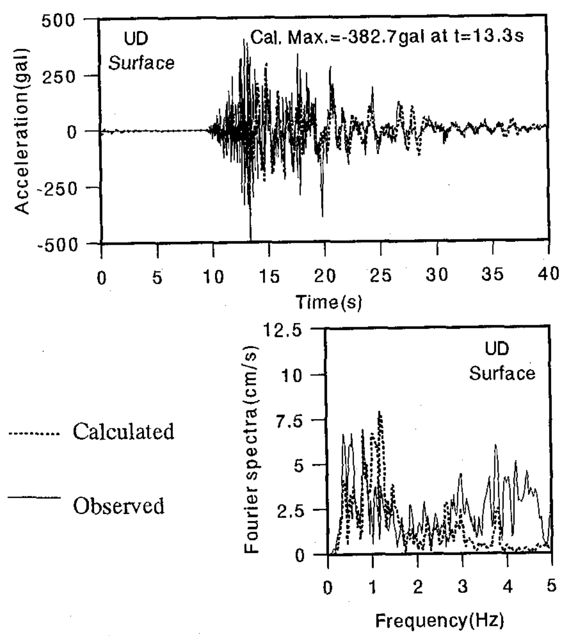

Fig. 7 Observed and calculated vertical accelerations and their Fourier spectra at surface

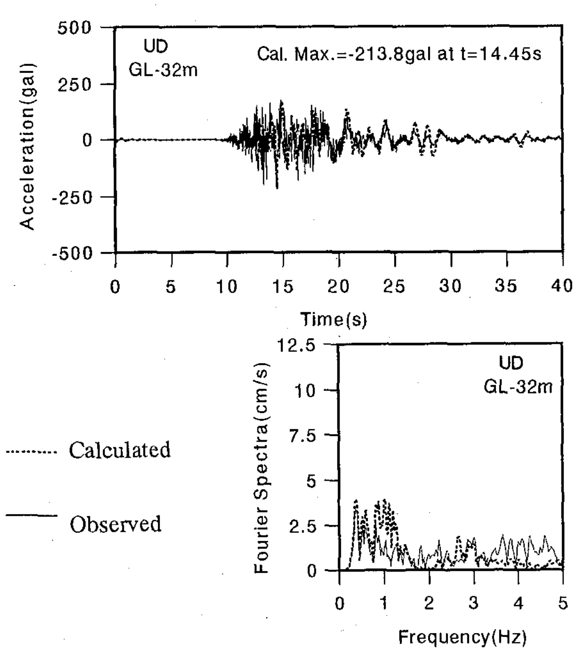

Fig.8 Observed and calculated vertical accelerations and their Fourier spectra at GL-32m site

different. The predominant frequencies of calculated Fourier amplitude spectrum lag a little in comparison with those of the observed. But totally they are in agreement. To examine the reliabilities of Poisson ratio ν , Young's modulus E and normal damping constant h_v obtained through the inverse-analysis, these parameters were used to calculate the vertical amplification of strong ground motion at GL-32m site by normal analysis. The calculated and observeded accelerations and their corresponding Fourier amplitude spectra are shown in Fig. 8. The observed maximum acceleration is 200.0 gal at time 13.26 second and the calculated maximum acceleration is 213.8 gal at time 14.45 second. They are in good agreement also in wave shapes. The predominant frequencies of calculated Fourier amplitude spectrum are distributed roughly from 0.3 Hz to 1.3 Hz, those of the observed are distributed from 0.3 Hz to 0.8 Hz. Totally they are in agreement.

4. Conclusion

This paper applied the multiple reflection theory into analyzing the amplification of vertical ground motion by using the downhill simplex optimum method to inversely calculate dynamic soil parameters. A vertical array observation of actual earthquake is utilized to determine Poisson ratio ν , Young's modulus E and normal damping constant h_v by considering the relation between shear modulus G and Young's modulus E. Five dynamic soil parameters G,h_h,ν,E,h_v are inversely analyzed by minimizing the sum of squared differences between the Fourier amplitude spectra of observed and calculated accelerations with the vertical array observations. The non-linearity of ground is thought to be represented by the equivalent linear method as the dynamic soil parameters approach to their real values in convergent calculations. The actual vertical array observation data in Port-island during the 1995 Hyogoken Nambu earthquake is applied as an example. The calculated responses of vertical strong ground motions are found to be in good agreement with the observed ones. The method proposed in this paper can be applied to the other place where SPT N-value and vertical array observation data have been obtained. A further work is expected to clarify the characteristics of non-linearity of ground behavior in vertical direction so that the amplification of vertical ground motion and its effect on the damage to structures can be analyzed in the area where a vertical array observation cannot be obtained.

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