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Localization of Vibration in Nonlinear Two–Degree–of–Freedom System

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The mode localization phenomenon in the nonlinear two–degree–of–freedom system is studied in this work. The perturbation method is employed to obtained the solutions of the nonlinear system for the free vibration case and harmonically forced vibration case. The results from both free vibration analysis and forced vibration analysis show that when the mode localization takes place, the vibration amplitudes of each subos-cillator are not the same. The steady state solutions are triggered from non-localized mode to localized mode at some criteria of initial conditions and structural parameters. The frequency response curve show the bifurcation solutions at the region where jump phenomenon exists.

Key Words : *nonlinear dynamics, mode localization*

1. Introduction

The periodic system which consists of identical subsystems repeated along its coordinates may exhibit a phenomenon known as mode localization when there are variations in their periodicity. Hodge [1] firstly introduced this phenomenon from a solid state physics into an acoustical research interest. Following by a number of researchers [2),3),4),5], the phenomenon has been extensively studied in many aspects because of its distinct and important characteristics. It has been found that the presence of small quantity of imperfections in their periodicity can lead to drastic changes in the dynamics of the system and the vibration modes become localized. When this phenomenon takes place, the energy injected into the system may not propagate along the coordinates but it is confined near the sources of external disturbance.

In addition to the localization phenomenon in linear systems due to structural irregularities, Vakakis [6),7] showed that the structural nonlinearity can be the cause of "*nonlinear mode localization*". It was found that the origin of nonlinear mode localization in the nonlinear periodic system is the amplitude dependence of the response frequency of the nonlinear system.

In engineering applications, the periodic structures are frequently encountered. One of the simplest example of periodic structures is the two identical oscillators connected together by a coupling spring. This simple and frequently used model, for example the model of bridge towers [8], may be suspected to have localized mode in the existence of mistuning frequencies. Moreover, in some cases, the system may vibrate with large amplitudes so that the geometrical nonlinearity becomes more significant. The effect of nonlinearity can cause the assumed system with identical natural frequency subsystems (tuned system) becomes the system with different natural frequencies subsystems (mistuned system).

As it has been pointed out that the ignorance of frequency mistuning in periodic structures may lead to completely erroneous results, it is particularly important to understand the localization phenomenon and to study the possibility of the occurrence of this phenomenon. Therefore the attempt of this paper is to study the mode localization in the nonlinear two–degree–of–freedom system. The primary objective is to investigate the causes and results of mode localization comprehensively by using the simple models. The discussions on time domain response of both free vibration and forced vibration which were not mentioned in previous work [6),7)] will be done in this paper in order to support the interpretation of the phěnomenon. This work also reveals the phenomenon in harmonically forced vibration case in which the frequency response curves are bifurcated in localized mode.

The two-degree-of-freedom systems consisting of two subsystems with geometrical nonlinear property are examined for their vibration confinement phenomenon. The free vibration analysis and the harmonically forced vibration analysis will be performed by using the multiple scales method [9)]. The advantage of this perturbation method over the direct integration of the equations of motion is that it intermediately provides understanding the cause of this phenomenon.

The organization of this paper is separated into two main parts. Section 2 is the study of free vibration. The undamped system is selected to study the steady state response. The numerical examples for several initial conditions are shown, in comparison with that of the linear system. The criteria for occurrence of localization are discussed by considering the initial conditions and structural parameters. In section 3, the damped system of harmonically forced vibration is studied. In addition to the response in time domain, the frequency response which show the bifurcation is obtained for localized mode.

2. Free Vibration of the Nonlinear Two-Degree-of-Freedom System

2.1 Formulation

The system studied in this section consists of two identical single-degree-of-freedom suboscillators connected together by means of a weak spring having stiffness k_c, as shown in Fig. 1a. Each suboscillator has mass m and is attached to rigid support by a massless spring having a linear stiffness, k_s, and a nonlinear hardening type stiffness of cubic order, $\hat{\alpha}$. The linear natural frequency of each oscillator is $\omega = (k_s/m)^{\frac{1}{2}}$. The equations of motion are, then

$$\ddot{x}_i + \omega^2 x_i + \frac{\hat{\alpha}}{m} x_i^3 + \frac{k_c}{m}(x_i - x_{(i+1)}) = 0, \quad i = 1, 2 \tag{1}$$

where $x_{2+1} \equiv x_1$ and $(\dot{\cdot})$ denotes differentiation with respect to time, t. For a weak nonlinear system, i.e. $\hat{\alpha}/m$ is small compared with ω^2, the solutions of the above nonlinear equations can be obtained by perturbing the response of the corresponding linear system. The method of multiple scales is employed to the problem in order to obtain a uniformly valid, first order approximation to the dynamic response of the system.

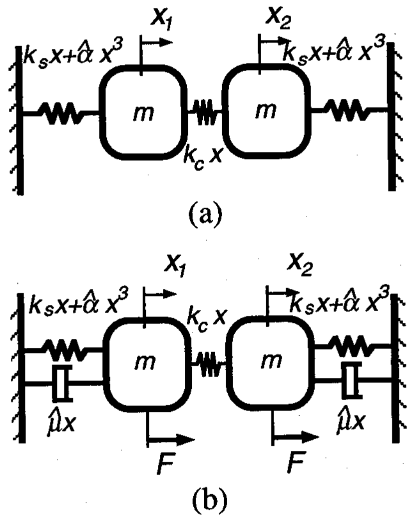

Fig. 1 The system configuration. a) Free vibration case, b) Harmonically forced vibration case.

To the limit of weakly nonlinear and weakly coupling cases, the nonlinear term and coupling term can be scaled as

$$\frac{\hat{\alpha}}{k_s} = \epsilon\alpha \quad \text{and} \quad \frac{k_c}{k_s} = \epsilon k, \tag{2}$$

where ϵ is a small dimensionless parameter. Therefore the equations of motion in Eq. 1 become

$$\ddot{x}_i + \omega^2 x_i + \epsilon\omega^2\alpha x_i^3 + \epsilon\omega^2 k(x_i - x_{(i+1)}) = 0, \ i = 1, 2. \tag{3}$$

2.2 Steady state solutions

The solutions of Eq. 3 are approximated in the following form by neglecting the higher order terms of ϵ.

$$x_i(t;\epsilon) \doteq x_{i0}(T_0, T_1) + \epsilon x_{i1}(T_0, T_1), \ i = 1, 2 \tag{4}$$

where $T_n = \epsilon^n t$ $(n = 0, 1)$ represent different time scales. Thus, instead of determining x_i as a function of t, we determine x_t as a function of T_0, T_1 as Eq. 4. We change the independent variable in the original Eq. 3 from t to T_0, T_1. Using the chain rule, we have

$$\frac{d}{dt} = D_0 + \epsilon D_1 \tag{5a}$$

$$\frac{d^2}{dt^2} = D_0^2 + 2\epsilon D_0 D_1, \tag{5b}$$

where $D_n = \partial/\partial T_n\ (n = 0, 1)$. Substituting Eq. 4 and Eq. 5 into Eq. 3 and equating the coefficients of like powers of ϵ, one obtains

Order ϵ^0

$$D_0^2 x_{i0} + \omega^2 x_{i0} = 0 \qquad (6)$$

Order ϵ^1

$$D_0^2 x_{i1} + \omega^2 x_{i1} =$$
$$-2D_0D_1x_{i0} - \omega^2\alpha x_{i0}^3 - \omega^2 k(x_{i0} - x_{(i+1)0}). \quad (7)$$

The solutions of Eq. 6 are

$$x_{i0} = a_i(T_1)\cos(\omega T_0 + \beta_i(T_1)), \qquad (8)$$

The first order approximate solutions of Eq. 1 are in the harmonic form as in Eq. 8 where a_i and β_i represent amplitude and phase of the motion, respectively. The unknown a_i and β_i are obtained by substituting Eq. 8 into Eq. 7, eliminating "*secular terms*" and grouping the sine and cosine terms.

$$a_i' = \frac{\omega k}{2} a_{i+1}\sin(\beta_{i+1} - \beta_i) \qquad (9)$$

$$\beta_i' = \frac{3}{8}\omega\alpha a_i^2 + \frac{\omega k}{2}(1 - \frac{a_{i+1}}{a_i}\cos(\beta_{i+1} - \beta_i)), (10)$$

$(\cdot)'$ denotes differentiation with respect to T_1.

Considering Eq. 9, it can be shown that the summation of a_i^2 is constant, $a_i^2 + a_{i+1}^2 = c$, which implies that the energy during vibration is conserved.

In general, at the steady state of free vibration of the nonlinear multi–degree–of–freedom system, the amplitudes of each suboscillators are not constant because vibration energy transfers between the coupling subsystems during their oscillations. For this reason, the commonly assumed conditions at steady state as $a_i' = 0$ may not be employed properly to this problem. However, the constant steady state amplitudes can be considered for the limit case of a system with zero coupling stiffness. For the case of $k = 0$, the system becomes uncoupled and Eq. 9 yields $a_i' = 0$, implies constant steady state amplitude. Integrating Eq. 10 results

$$\beta_i = \epsilon\frac{3}{8}\omega\alpha a_i^2 t + \beta_{i0}$$
$$= \beta_i^* t + \beta_{i0}. \qquad (11)$$

The first term of Eq. 11 is a function of time, consequently it modifies the frequency of the system from linear value, ω, to $\omega + \beta_i^*$. Then the frequency difference between two nonlinear single–degree–of–freedom system may be written as

$$\beta_{i+1}^* - \beta_i^* = \Delta\omega = \epsilon\frac{3}{8}\omega\alpha(a_{i+1}^2 - a_i^2). \quad (12)$$

From Eq. 12, it can be seen that two identical nonlinear single–degree–of–freedom systems may contain different vibration frequencies for different vibration amplitudes. After connecting two nonlinear single–degree–of–freedom system together, each suboscillators may remain vibrating with frequencies close to their uncoupled frequencies, if the coupling is small enough. Consequently, the weakly coupled nonlinear two–degree–of–freedom system may consist of two suboscillators which have different frequencies. It has been studied that, in the limit of small coupling of a periodic system, the system with frequency detuning may show mode localization phenomenon. The localization phenomenon in nonlinear system will be examined numerically in the following examples.

2.3 Numerical examples and discussions

The preceding study of nonlinear mode localization is applied to numerical examples in this section.

Firstly, Eq. 9 and Eq. 10 are integrated numerically to obtain a_i and β_i. The response x_i can be subsequently calculated from Eq. ??. The parameters of the system are selected as $k_s = 1.0$, $\hat{\alpha} = 0.033, k_c = 0.01, \epsilon = 0.001$ and $\omega = 1.0$. Moreover, the analysis of linear system ($\hat{\alpha} = 0$) is performed together for the comparison. The initial conditions are considered for two example cases; for the first case, $x_1(0) = 1.0, \dot{x}_1(0) = 1.0$, $x_2(0) = 1.0$ and $\dot{x}_2(0) = 2.0$, and for the second case, all the initial values are kept the same as the first case except $\dot{x}_2(0) = 3.0$.

The response amplitude a_i and phase βi are shown in Figs. 2– 4. The linear responses for two cases of initial conditions are similar so that only the results of the second case are shown in Fig. 2. The linear responses show non-localized vibration mode of two oscillators. The energy is exchanged freely between two oscillators during vibration so that their amplitudes vary within the same range. The responses of nonlinear system are shown in Fig. 3 for the first set of initial conditions and in Fig. 4 for the second one. In Fig. 3, the system possesses non-localized vibration mode and the energy is exchanged be-

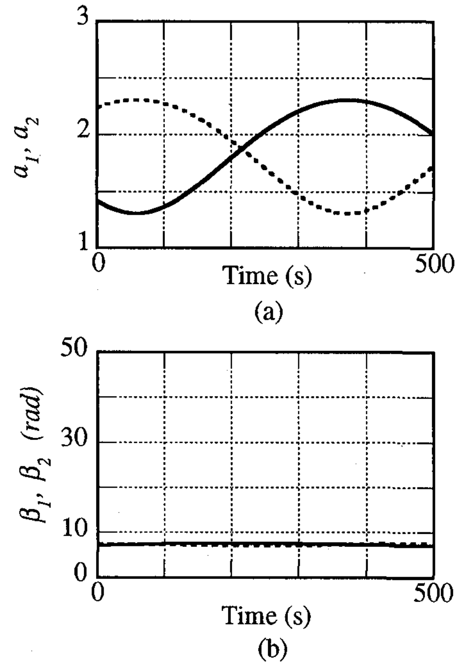

Fig. 2 Free vibration response of the linear 2DOF system with the second case of initial conditions. a) a_i, b) β_i: ———, 1-st suboscillator; ...., 2-nd suboscillator.

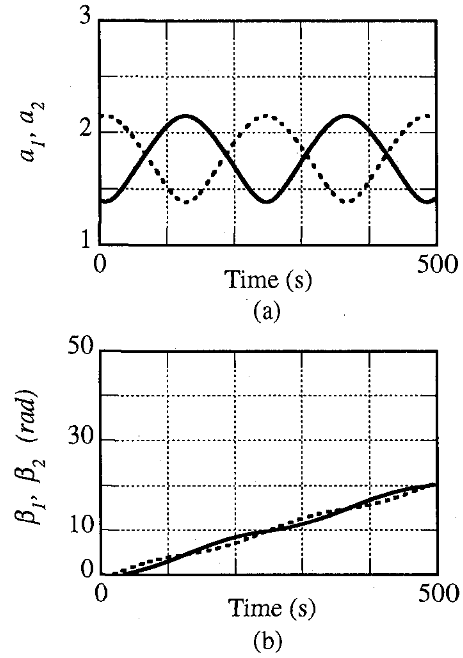

Fig. 3 Free vibration response of the nonlinear 2DOF system with the first case of initial conditions. a) a_i, b) β_i: ———, 1-st suboscillator; ...., 2-nd suboscillator.

tween two oscillators during vibration. Unlike the linear system, the average values of β_i of the nonlinear system do not remain constant, nonzero gradient, but increase with time, implying the increase of frequency of nonlinear vibration. The localization phenomena are shown in Fig. 4a where one of the oscillator vibrates with larger amplitude than the other. Fig. 4b shows that the gradient of β_i with respect to time is different between two oscillators. The difference of gradient means that two oscillators contain different vibration frequencies. The different frequencies lead a perfect periodic system to be a detuned system and the mode localization takes place. These numerical results are corresponding to the discussions in the preceding section.

In Fig. 5 and Fig. 6, the possibilities of occurrence of localized mode are shown for various cases of initial conditions. The abscissas are the initial difference between frequency of two suboscillators, $\Delta\omega_0$, calculated from initial conditions. The ratio of average value of amplitude, $\bar{a}_2/\bar{a}_1$ and the difference of average value of frequency , $\Delta\bar{\omega}$, at steady state are shown in Fig. 5 and Fig. 6, respectively. For small $\Delta\omega_0$, or small difference of initial state of two suboscillators, the localization mode does not exist as can be observed from Fig. 5 where both subos-

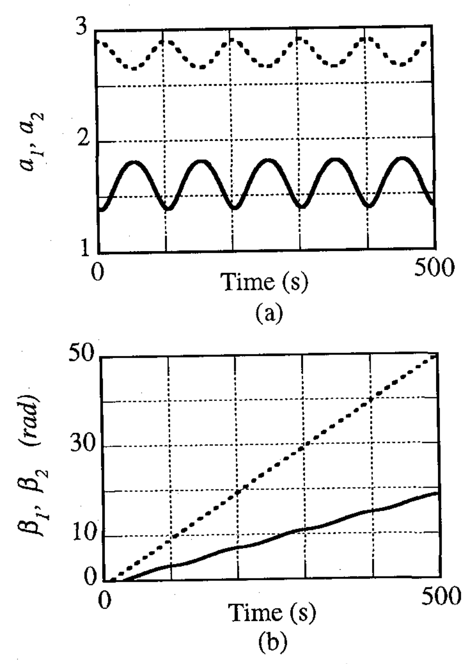

Fig. 4 Free vibration response of the nonlinear 2DOF system with the second case of initial conditions. a) a_i, b) β_i: ———, 1-st suboscillator; ...., 2-nd suboscillator.

cillators have the same average steady state amplitudes, $\bar{a}_2/\bar{a}_1 = 1$. In this case there is no difference of average value of frequency at steady state, $\Delta\bar{\omega} = 0$ as shown in Fig. 6. When $\Delta\omega_0$ increases to an onset value, $\Delta\omega_{0cr}$, the steady state solutions transform from non-localized vi-

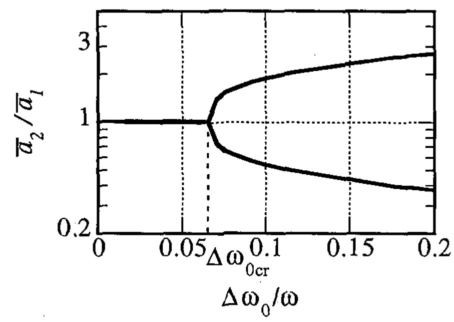

Fig. 5 Initial state frequency difference–ratio of average steady state amplitude relations

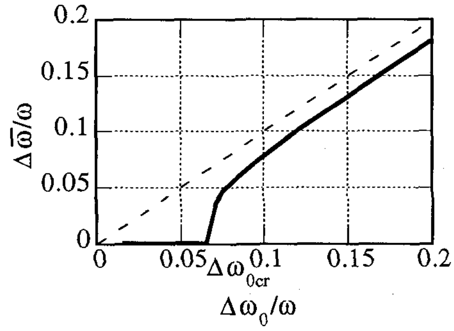

Fig. 6 Initial state frequency difference–average steady state frequency relations

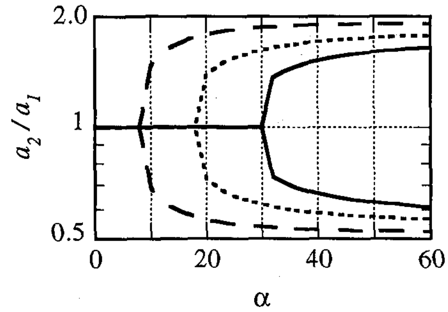

Fig. 7 System parameters–ratio of average steady state amplitude relations : — — -, $k = 5$; ...., $k = 10$; ———, $k = 15$.

bration modes into localized modes. $\bar{a}_2/\bar{a}_1$ bifurcates from one and $\Delta\bar{\omega}$ increases from zero at point $\Delta\omega_{0cr}$.

The effects of nonlinearity, α, and coupling stiffness, k, on the degree of localization are shown in Fig. 7. The degree of localization is presented in term of the ratio of an average value of amplitude at steady state, $\bar{a}_2/\bar{a}_1$, obtained from several combinations of nonlinearity and coupling stiffness for the initial conditions of the second case. For the system with small nonlinearity, the localized mode does not occur because the system contains small values of frequency detuning quantity. At sufficiently large amount of nonlinearity, the localized mode is observed. Fig. 7 also shows that the systems with weakly coupling or small value of k are more likely to have mode localization phenomenon. As a result, the strong nonlinear mode localization is occurred in the systems with small coupling stiffness and sufficient large nonlinearity. Due to the fact that the nonlinearity is the cause of detuning frequencies, the relation found in Fig. 7 is equivalent to the theory of localization of linear system with detuning frequencies, where the strong localization occur when the ratio of coupling stiffness to detuning frequencies decreases [3]. The results presented in this section show the possibility of the additional localized solutions which are not found in the linear systems. It should be noted that, for the multiple solutions of nonlinear system, the stable solutions are conformed to the initial conditions. As a result, either localized solutions or non-localized mode solutions may occur, depending on the initial conditions.

3. Forced Vibration of the Nonlinear Two–Degree–of–Freedom System

3.1 Formulation

In this section, the harmonically forced vibration of the system in Fig. 1b is considered. In order to obtain the bounded responses of forced vibration, we add the dashpot with linear damping coefficient, $\hat{\mu}$, at the supports of both oscillators. The system is excited by harmonic forces having constant amplitude, F, frequency, Ω and no phase difference between the applied forces of two suboscillators. The equations of motion are,

$$\ddot{x}_i + \omega^2 x_i + \frac{\hat{\alpha}}{m}x_i^3 + \frac{k_c}{m}(x_i - x_{(i+1)}) + \frac{\hat{\mu}}{m}\dot{x}_i = \frac{F}{m}\cos(\Omega t), \quad i = 1, 2. \tag{13}$$

The nonlinear term and coupling term are scaled as Eq. 2 and we assume that the damping and the excitation terms are small with the same order as the nonlinearity and the coupling. The problem considered here is the primary resonance of the system and the forcing frequency is set very close to the linear natural frequency of structure, $\Omega \approx \omega$. To describe the closeness of Ω to ω, we introduce a detuning parameter, σ, which can be used as a parameter instead of using Ω. Then we write

$$\frac{\hat{\mu}}{k_s} = \epsilon\mu, \quad \frac{F}{k_s} = \epsilon f \quad \text{and} \quad \Omega = \omega + \epsilon\sigma. \tag{14}$$

The equations of motion, therefore, become

$$\ddot{x}_i + \omega^2 x_i + \epsilon\omega^2 \alpha x_i^3 + \epsilon\omega^2 k(x_i - x_{(i+1)}) + \epsilon\omega^2 \mu\dot{x}_i = \epsilon\omega^2 f\cos((\omega+\epsilon\sigma)t), \quad i = 1,2. \quad (15)$$

3.2 Solutions

By following the same procedures as explained in the free vibration part, we come up with the set of equations for a_i and β_i of the response in the form of Eq. 8 as

$$a_i' = -\frac{1}{2}\omega^2\mu a_i + \frac{\omega k}{2}a_{i+1}\sin(\beta_{i+1}-\beta_i) + \frac{\omega f}{2}\sin(\sigma T_1 - \beta_i) \quad (16)$$

$$\beta_i' = \frac{3}{8}\omega\alpha a_i^2 + \frac{\omega k}{2}(1 - \frac{a_{i+1}}{a_i}\cos(\beta_{i+1}-\beta_i)) - \frac{\omega f}{2a_i}\cos(\sigma T_1 - \beta_i). \quad (17)$$

Eq. 16 and Eq. 17 can be integrated numerically to obtain the response in time-domain. Furthermore, the relation between response amplitude, a_i, and excitation frequency parameter, σ, can be evaluated by considering the steady state solutions of Eq. 16 and Eq. 17. The system should be transformed into an autonomous form in which T_1 does not appear explicitly. One of the alternates can be done by letting

$$\gamma_1 = \sigma T_1 - \beta_1 \quad (18)$$

$$\gamma_2 = \beta_2 - \beta_1. \quad (19)$$

Substituting Eqs. 18 and 19 into Eqs. 16 and 17 and using the conditions for steady state solutions; $a_i' = 0$ and $\gamma_i' = 0$, result

$$-\frac{1}{2}\omega^2\mu a_1 + \frac{\omega k}{2}a_2\sin\gamma_2 + \frac{\omega f}{2}\sin\gamma_1 = 0 \quad (20)$$

$$-\frac{1}{2}\omega^2\mu a_2 - \frac{\omega k}{2}a_1\sin\gamma_2 + \frac{\omega f}{2}\sin(\gamma_1 - \gamma_2) = 0 \quad (21)$$

$$-\frac{3}{8}\omega\alpha a_1^2 - \frac{\omega k}{2}(1 - \frac{a_2}{a_1}\cos\gamma_2) + \frac{\omega f}{2a_1}\cos\gamma_1 + \sigma = 0 \quad (22)$$

$$-\frac{3}{8}\omega\alpha(a_2^2 - a_1^2) + \frac{\omega k}{2}(\frac{a_1}{a_2} - \frac{a_2}{a_1})\cos\gamma_2 + \frac{\omega f}{2}(\frac{1}{a_2}\cos(\gamma_1-\gamma_2) - \frac{1}{a_1}\cos\gamma_1) = 0. \quad (23)$$

The set of equations above can be solved numerically yielding the frequency-response relations. It should be noted that the stability analysis should be also performed to check for the stable solutions.

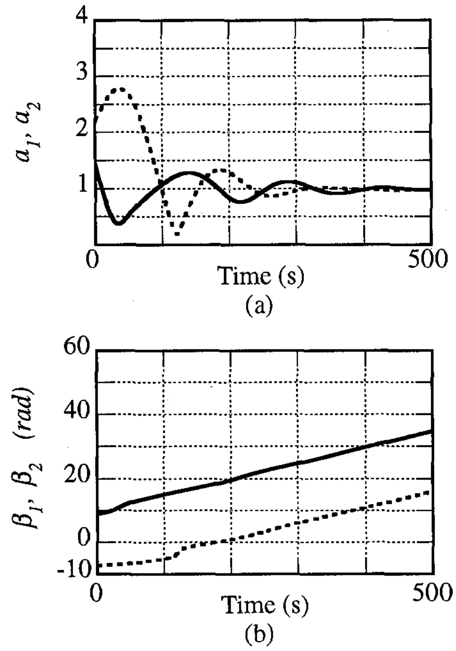

Fig. 8 Forced vibration response of the linear 2DOF system a) a_i, b) β_i: ———, 1-st suboscillator; - - - - - - -, 2-nd suboscillator.

3.3 Numerical examples and discussion

The system parameters of numerical examples are selected as $\hat{\alpha} = 0.01$, $k_c = 0.01$, $\hat{\mu} = 0.02$ (1 % damping ratio), $k_s = 1.0$, $\omega = 1.0$ and $f = 0.1$.

The responses in time-domain obtained by integrating Eq. 16 and Eq. 17 are shown in Fig. 8 and Fig. 9 for linear system ($\hat{\alpha} = 0$) and nonlinear system, respectively. The initial conditions are $x_1(0) = -1.0$, $\dot{x}_1(0) = -1.0$, $x_2(0) = 1.0$ and $\dot{x}_2(0) = 2.0$, and the detuning parameter is taken as $\epsilon\sigma = 0.05$. It is shown in Fig. 8a on the responses of linear system that both suboscillators have constant and equal steady state amplitudes. For the nonlinear system in Fig. 9a, one of the suboscillators vibrates with larger steady state amplitude and the responses are localized modes. Fig. 8b and 9b show similar results of β_i where two curves are in parallel. Therefore, the gradient at steady state is identical for both suboscillators. It can be interpreted from the identical constant β_i that the steady state vibration frequency is the same for both suboscillators, which is the frequency of external excitation, Ω.

Fig. 10a shows the frequency-response curves and Fig. 10b is the enlargement of the area where many possible solutions occur. The abscissa is

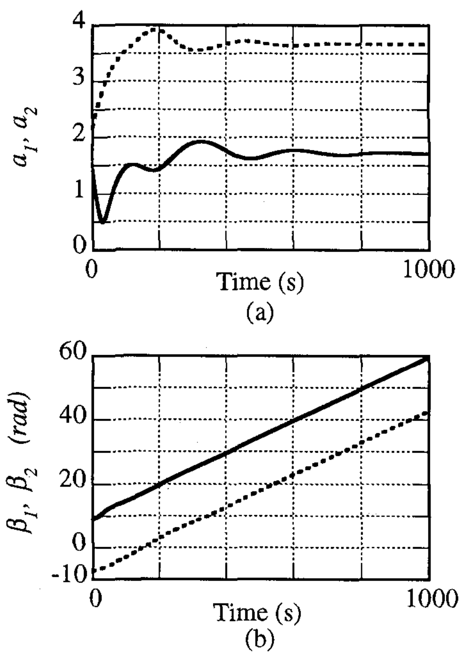

Fig. 9 Forced vibration response of the nonlinear 2DOF system a) a_i, b) β_i: ———, 1-st suboscillator; ...., 2-nd suboscillator.

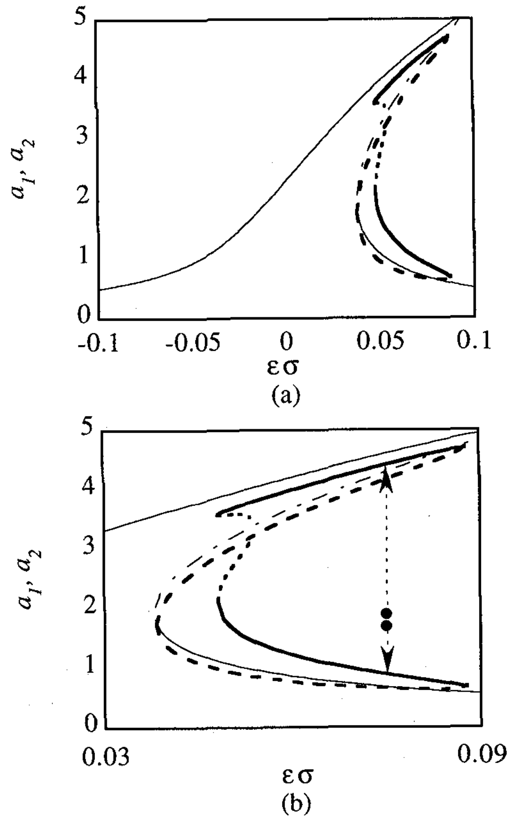

Fig. 10 Frequency–response curve:
———, non-localized stable solutions;
———, localized stable solutions;
– - – - -, non-localized unstable solutions;
– – – – –, localized unstable solutions set 1;
........., localized unstable solutions set 2.

the excitation frequency written in term of the detuning parameter, σ, and the ordinate is the steady state amplitude. The nonlinearity bends the frequency-response curve and shows jump phenomena. In this case, the responses of the two–degree–of–freedom system consist of two sets of stable solution. The first set contains solutions of the non-localized mode where both suboscillators vibrate with the same steady state amplitudes so that the responses of both suboscillators can be shown in the same single line as a thin-solid line. The second set of solutions is the mode of vibration in which the two suboscillators have different steady state response amplitudes as shown in the thick lines. This mode is the localized one occurred near the possible region of jump phenomena. It is seen that one oscillator vibrates with large amplitude close to the upper curve while the other oscillator vibrates with much smaller amplitude close to lower curve of the frequency response. The unstable solutions are shown in dash thick lines. Note that in case of localized modes, there are two sets of unstable solutions. The relation is more clearly depicted in Fig. 11 where the relation between a_1 and a_2 is given for the possible solutions. The non-localized mode is shown in the straight line of $a_1 = a_2$. The other solutions of localization are shown in the thick lines where $a_1 \neq a_2$. It is noted that, in this case, there is no difference of vibration frequencies between two suboscillators, as shown in Fig. 9b. Consequently, the cause of localized mode is not the frequency difference, as in free vibration case, but it can be considered as a result of different initial states in a weakly coupling system. When the coupling stiffness decreases, which is not shown in Fig. 10, the discrepancy between localized mode and non-localized mode is reduced, i.e. the thick lines move close to the thin lines. It can be explained by considering the limit case of zero coupling in which each suboscillator becomes an independent single–degree–of–freedom system. For a single–degree–of–freedom nonlinear system, the steady state response at jump phenomena can be either the upper curve or the lower one depends on its initial conditions. When the two single–degree–of–freedom systems are connected together and for certain initial conditions, one of them has steady state solutions close to the up-

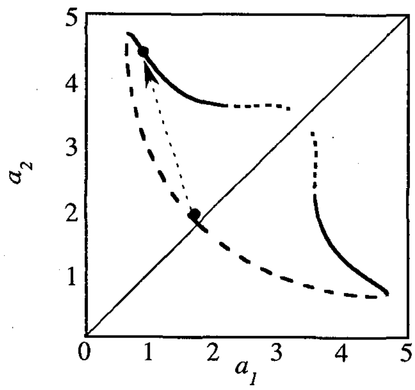

Fig. 11 Relation between steady state amplitude of two suboscillators:
——, non-localized solutions;
——, localized stable solutions;
- - - - -, localized unstable solutions set 1;
·········, localized unstable solutions set 2.

per curve while the other solution is close to the lower curve. The steady state solutions may remain in this mode if the coupling stiffness is small enough.

An example of occurrence of mode localization is also shown in Fig. 10b and Fig. 11. When the system starts form different initial conditions, for this case $a_1 = 1.7$, $a_2 = 1.9$, $\beta_1 = -0.8\pi$, $\beta_2 = 0$ and $\epsilon\sigma = 0.075$, the steady state responses are the localized vibration shown as the arrow head points. However, for some other combinations of initial conditions, the mode localization may not occur.

Although the phenomenon has shown some important characteristics which are needed to be considered carefully in the structures with periodicity, it has not been studied for civil engineering problems. However, some civil engineering structures also have the periodicity configuration with weakly coupling stiffness, for example, the multispan cables in transmission lines system. The possibility of occurrence and its effects to such a system are under investigation by the authors.

4. Summary and conclusions

The free vibration and harmonically forced vibration of the nonlinear two–degree–of–freedom system were studied in view of localization phenomena. The nonlinear solutions were obtained by the method of multiple scales.

In free vibration case, there is an onset initial condition in which the steady state solutions are triggered from non-localized mode to localized mode. The steady state amplitudes at localized mode are different for both suboscillators, which results from the difference in vibration frequency at steady state condition. The strong localization was observed for the system with small coupling stiffness and sufficiently large amount of nonlinearity.

The responses of forced vibration also exhibit localization phenomenon for certain initial conditions and in a specific forcing frequency range. The frequency–responses relation shows that the cause of localized mode results from different initial states for the system with weak coupling stiffness.

The condition for the occurrence of nonlinear mode localization is generally dependent on three main factors. Those are the closeness of natural frequencies, the order of nonlinearity and the difference of the initial conditions among subsystems.

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