ULTIMATE STRENGTH OF AXIALLY COMPRESSED SHORT CYLINDRICAL STEEL SHELLS WITH INITIAL DEFLECTION

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This paper presents an evaluation of ultimate strength of imperfect cylindrical steel shells under axial compression. The ultimate strength is evaluated analytically using finite displacement method on the basis of the degenerated shell element. The parameter study consists of material (SM400 and SM570), slenderness ($R_t = 1.65(\sigma_Y/E)(R/t)$), and initial deflection-to-length of cylinder (w_o/L). A simple equation to calculate the ultimate strength of the cylinders with variable R_t, w_o/L, and R/t is derived. Using linear regression, the ultimate strength and R_t relationship formula is derived for the perfect cylinders, and using nonlinear regression the knockdown factor is suggested. The results are also compared with the formulae suggested by JSCE, Plantema, and ECCS.

Key Words: ultimate strength, cylindrical steel shells, initial deflection

1. Introduction

In designing tubular member of structures such as bridge pier, the ultimate strength of axial load is very important information to be known. A number of researches have evaluated the ultimate strength of the cylinders. The evaluation considered the effect of initial imperfection, radius-to-thickness and length-to-thickness ratios. These geometric parameters were used to predict the collapse load due to large deformation of structure, nonlinearity of material, or a combination of those. Several design curves and formulae concerning the above parameters were suggested by Samuelson and Eggwertz[8].

The analytical formula for buckling load of cylinders with general case of uniform axial pressure in elastic region was discussed by Timoshenko & Gere[10]. The study on effects of initial imperfection on buckling loads was done by Gelin[7]. He investigated the effect of a sinusoidal axisymmetric shape imperfection on the plastic buckling of relative long cylindrical shells. The ultimate loads of cylinders under axial compression were studied both analytically and experimentally by Batterman[1], and Sobel and Newman[9]. Moreover, Batterman presented the simple formula to calculate the plastic bifurcation buckling stress of the cylinders.

Bushnell[2,3] surveyed and discussed wide aspect of plastic buckling of various shells. Using BOSOR computer program he presented much valuable information about cylindrical shells. Yasukawa et al[11] investigated imperfect short cylinder with large diameter-to-thickness ratio under axial compression. They presented many curves; i.e., the effect of initial geometric imperfection, diameter-to-thickness ratio and length-to-thickness ratios on normalized ultimate loads. The ultimate strength of cylinders obtained analytically is also compared to others design criterion formulas, i.e., DnV criterion, Johnson-Ostenfeld criterion, and ECCS criterion.

The extreme minimum ultimate strength of cylinders under axial compression was investigated by Rizal and Nara[6]. At the basic length which the cylinders have the minimum ultimate strength, local buckling or collapse may occur. Considering this phenomenon, in this paper, those cylinders are used for analytical model.

Initial imperfections of cylinders are presented as initial deflection-to-length ratio rather than initial deflection-to-thickness or initial deflection-to-radius ratio. This index is more significance than the others because of two reasons. First, a gauge length is easier to evaluate the imperfection of cylinder in the axial direction rather than thickness or radius. Second, this index makes it possible to find a unique minimum ultimate strength at certain length of cylinder.

In this study, steel material is assumed to have three states; i.e., elastic, perfectly plastic and plastic one with strain hardening. Using finite displacement method, the ultimate strength of relative thick cylinders with mild steel and high strength steel material are evaluated.

The study that motivated by a current trend of using thick-walled steel tubular bridge piers is focused on load-displacement relationship, ultimate strength, effect of initial geometric imperfections, R_t parameter, and radius-to-thickness ratios. Several curves concerning those purposes are presented. The study on basic length of cylinders under axial compression with various parameters is aimed to obtain the minimum ultimate strength which can be used to formulate a simple equation to predict the ultimate strength of the cylinders.

2. Analytical Method

In order to predict the numerical results, finite displacement analysis in the basis of degenerated shell element is used. The element has eight nodes. Each node has five degrees of freedom. In this analytical method 2x2 reduced integration technique through element surface and eight equal layers approach through element thickness are used. Geometric nonlinearity due to large displacement and large rotation of structures are considered in this study. Nonlinear material is formulated following von Mises yield criterion and Prandtl-Reuss flow rule.

More detail explanation about the analytical method can be found in Refs. 4) and 5). In these papers, the validation of the analytical method especially for cylinders under axial load was examined.

3. Analytical Model

The numerical analysis of cylindrical steel shells under axial compression is carried out using a four-by-six mesh as shown in Fig. 1. The mesh is sufficient to analyzed the ultimate strength of the cylinder. The load-displacement relationship on post buckling behavior is still in good agreement[5].

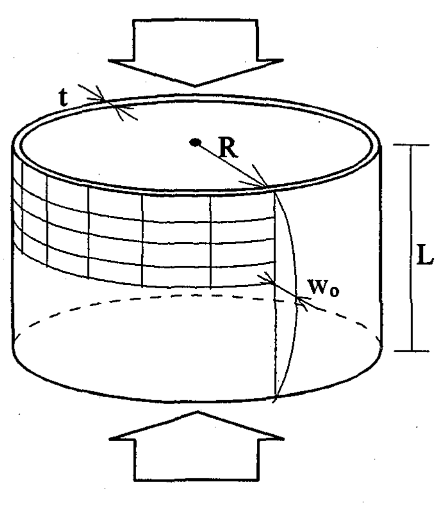

Fig. 1 Cylinder under axial compression

Due to symmetric pattern of the cylinder, only one-eighth of structures is analyzed. One half of axial direction is divided by four elements, and one quarter of circumferential direction is divided by six elements.

The cylinder has simple supported ends. The initial imperfection is introduced in the axial direction as a sinusoidal function with maximum amplitude equal to w_o.

The geometric parameters consist of slenderness R_t(= $1.65(\sigma_Y/E)(R/t)$) and maximum initial deflection-to-length ratios w_o/L. The thickness of cylinder is equal to 0.5 cm for SM400 and 1.0 cm for SM570. These geometric characteristics are listed in Tables 1 and 2.

Stress-strain relationship of steel material is assumed to have three clear states, i.e., elastic, perfectly plastic, and plastic with strain hardening. A power equation that consists of three constants, i.e., b, c, and n is adopted to express the strain hardening state in the following equation.

$$\sigma/\sigma_Y = b(c+\varepsilon/\varepsilon_Y)^n \tag{1}$$

in which σ_Y and ε_Y are yield normal stress and yield strain, respectively.

In this paper, two kinds of material, i.e., mild steel SM400 with yield strength equal to 2545 kgf/cm^2, and high strength steel SM570 with yield strength equal to 5191 kgf/cm^2 are studied. The characteristics of these materials are listed in Table 3, in which the Young's modulus is equals to 2.1×10^6 kgf/cm^2.

It should be noted here that the initial imperfection in the circumferential direction is not considered. It is because the present model is considered as a relatively stocky and short cylinder. Therefore, the influence of this initial imperfection on ultimate strength of the cylinder is much smaller than that of the initial imperfection in axial direction. Whereas, the residual stress is not considered due to a little data that are available.

Table 1 Geometric parameters of analytical models for steel SM400

w_o/L	t (cm)	R (cm) / $R_t=1.65(\sigma_Y/E)(R/t)$				
0.0	0.5	23.0	40.0	56.0	72.0	88.0
		0.092	0.160	0.224	0.288	0.352
1/500	0.5	23.0	40.0	56.0	72.0	88.0
		0.092	0.160	0.224	0.288	0.352
1/250	0.5	23.0	40.0	56.0	72.0	88.0
		0.092	0.160	0.224	0.288	0.352
1/100	0.5	23.0	40.0	56.0	72.0	88.0
		0.092	0.160	0.224	0.288	0.352

Table 2 Geometric parameters of analytical models for steel SM570

w_o/L	t (cm)	R (cm) / R_t=1.65(σ_Y/E)(R/t)				
0.0	1.0	23.0	39.0	55.0	71.0	87.0
		0.094	0.159	0.224	0.290	0.355
1/500	1.0	23.0	39.0	55.0	71.0	87.0
		0.094	0.159	0.224	0.290	0.355
1/250	1.0	23.0	39.0	55.0	71.0	87.0
		0.094	0.159	0.224	0.290	0.355
1/100	1.0	23.0	39.0	55.0	71.0	87.0
		0.094	0.159	0.224	0.290	0.355

Table 3 Mechanical properties of steel

Steel	σ_Y (kgf/cm^2)	σ_U (kgf/cm^2)	ε_H (10^{-2})	ε_U	b	c	n
SM400	2545	4255	1.121	0.300	0.5017	0	0.3100
SM570	5191	6053	0.916	0.216	0.8134	0	0.1576

4. Results and Discussion

Figs. 2-6 show load-displacement curves of the cylindrical shell with mild steel SM400 and Figs. 7-11 show load-displacement ones with high strength steel SM570. In the figures, solid lines show load-axial displacement curves and broken lines do load-deflection ones that have top horizontal abscissa and bottom horizontal abscissa, respectively. The ordinate of curve is normalized load to yield load P/P_Y ($P_Y=0.5\pi Rt\sigma_Y$), the top abscissa is normalized axial displacement to yield axial displacement u/u_Y ($u_Y=0.5L\sigma_Y/E$), and the bottom abscissa is normalized deflection to the length $(w_o+w)/L$.

The ultimate load or ultimate axial load is defined as the maximum load during monotonous axial compression without decrease in load.

It is shown in Ref. 6) that a cylinder with certain radius-to-thickness ratio has ultimate strength that varies in accordance to its length, and has extreme minimum ultimate strength at certain length. Therefore, the initial imperfection of cylinders is described using maximum initial deflection-to-length ratio (w_o/L). It was observed in this study that the cylinders with w_o/L parameter have an extreme minimum ultimate strength at certain length. For short cylinder, this parameter is also significant with respect to manufacturing rather than initial deflection-to-thickness ratio or initial deflection-to-radius ratio.

The minimum ultimate strength at certain length of cylinders is obtained using trial and error method. An Algorithm of the method used in this paper can be explained as follows:

1. Define length of cylinder (L_1^* and L_2^*).
2. Calculate the ultimate load of cylinder with L_1^* (P_{u1}^*) and L_2^* (P_{u2}^*) by using the finite displacement analysis.
3. Check P_{u1}^* and P_{u2}^*. If P_{u1}^* is less than P_{u2}^*, define $L_1=L_2^*$, $L_2=L_1^*$, $P_{u1}=P_{u2}^*$ and $P_{u2}=P_{u1}^*$. Otherwise, $L_1=L_1^*$, $L_2=L_2^*$, $P_{u1}=P_{u1}^*$, and $P_{u2}=P_{u2}^*$.
4. If L_1 is less than L_2 Choose L_3^* greater than L_2. Otherwise, choose L_3^* less than L_2.
5. Calculate the ultimate load of the cylinder with L_3^* (P_{u3}^*)
6. If P_{u3}^* is less than P_{u2}, define $L_1=L_2$, $L_2=L_3^*$, $P_{u1}=P_{u2}$, and $P_{u2}=P_{u3}^*$. Go to step 4.
7. If P_{u3}^* is greater than P_{u2}, define $L_3=L_3^*$ and $P_{u3}=P_{u3}^*$.
8. Construct quadratic equation by using three points, i.e., $Q_1(L_1, P_{u1})$, $Q_2(L_2, P_{u2})$, and $Q_3(L_3, P_{u3})$.
9. Calculate L_4 that gives minimum value of P_u. Calculate the ultimate load of the cylinder with L_4 (P_{u4}) by using finite displacement analysis. We have $Q_4(L_4, P_{u4})$.
10. Choose the smallest ultimate load among Q_1 Q_2 Q_3 Q_4 to update $Q_2(L_2, P_{u2})$. Update $Q_1(L_1, P_{u1})$ and $Q_3(L_3, P_{u3})$ by using the points that located in the left and the right of the current Q_2, respectively.
11. Return to step 8 or finish if a suitable ultimate load was obtained.

The plotted results in this paper are obtained from these steps. The idea of using the minimum ultimate strength is based on the assumption that the local buckling or collapse occurs at one critical part of structures that has minimum level of ultimate strength.

All Figs. 2-11 show that the patterns of curve are similar. Although the material has three clear states, i.e., elastic, perfectly plastic, and plastic with strain hardening state, but the curves have only two parts, after increasing with high slope then change to near horizontal slope before collapse. From these figures, we can observe that the normalized ultimate strength (P_u/P_Y) is vary to radius-to-thickness ratio (R/t), initial deflection-to-length ratio and material. The differences are seen more clear in Fig. 12 which the normalized ultimate strength is plotted with respect to R_t parameter. The design formulae of Plantema[12]), JSCE[13]), and ECCS[14]) are also plotted in the figures as comparative data.

This figure shows that the normalized ultimate strength of the cylinders decrease with the increasing R_t parameter, and w_o/L parameter. The curve of the perfect cylinders with SM400 shows almost coincide with that of the perfect cylinders with SM570. The cylinders with initial deflection in accordance to material show that the curves of cylinders with SM400 drop lower than those with SM570. This phenomenon is due to the reality that the reducing of ultimate strength of the cylinders with initial imperfection is more sensitive with an increase in R/t. In fact the cylinders with SM400 have R/t twice of those with SM570 for same R_t parameter.

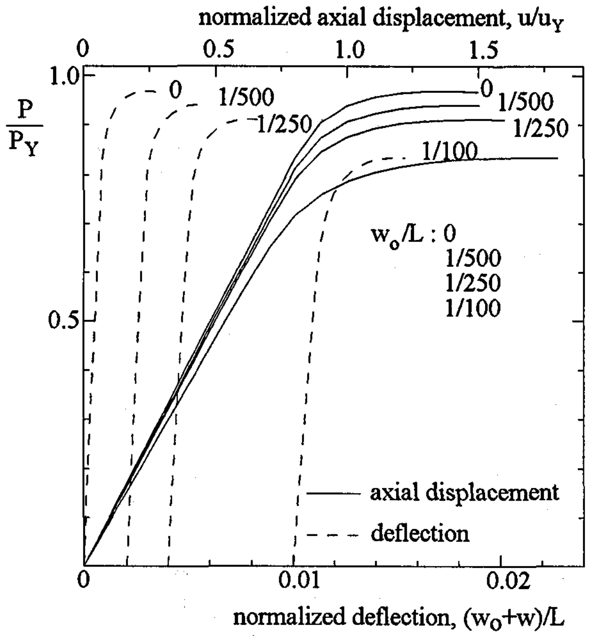

Fig. 2 Load-displacement curves of cylinders with SM400 and R=23 cm

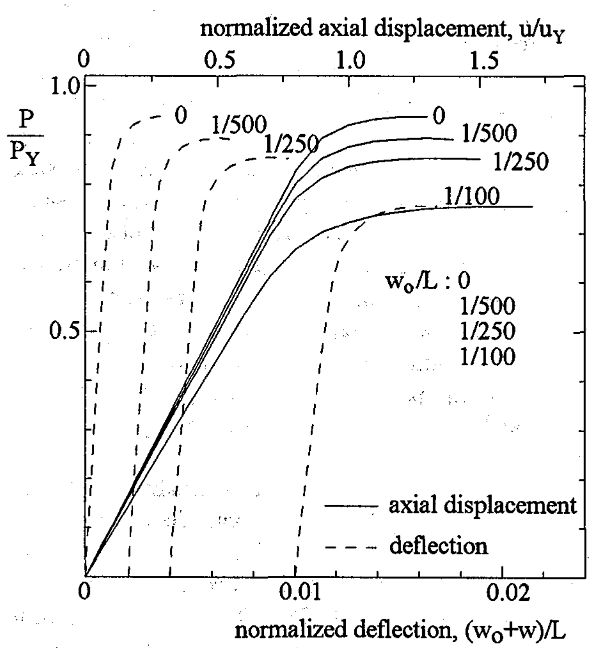

Fig. 3 Load-displacement curves of cylinders with SM400 and R=40.0 cm

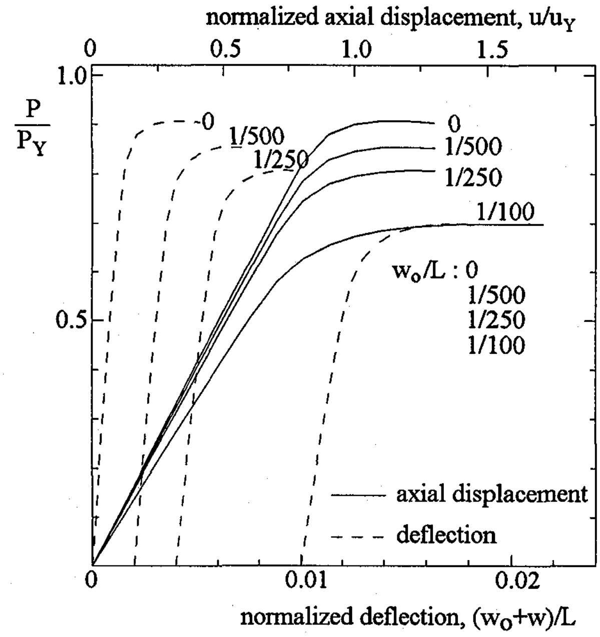

Fig. 4 Load-displacement curves of cylinders with SM400 and R=56.0 cm

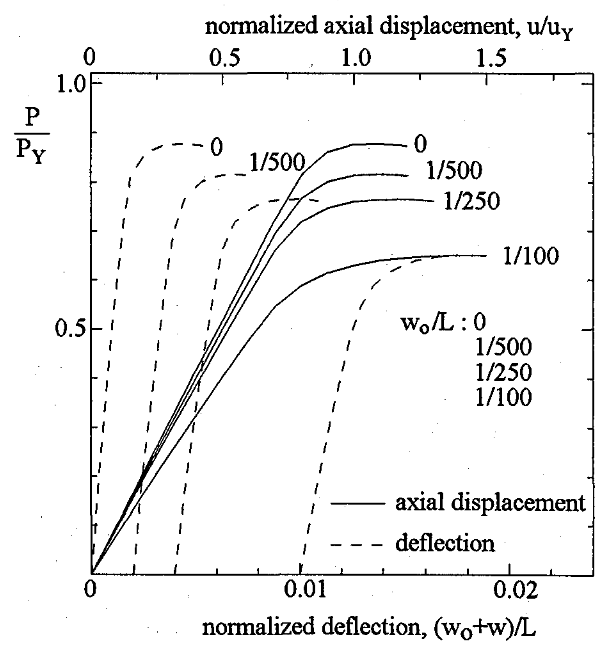

Fig. 5 Load-displacement curves of cylinders with SM400 and R=72.0 cm

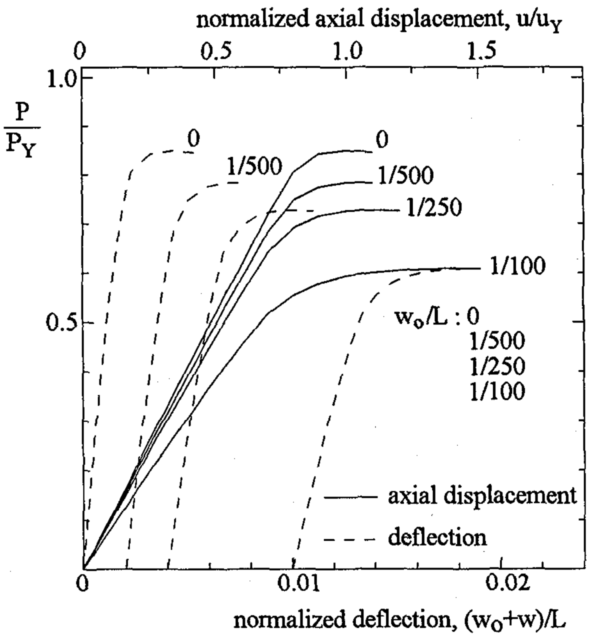

Fig. 6 Load-displacement curves of cylinders with SM400 and R=88.0 cm

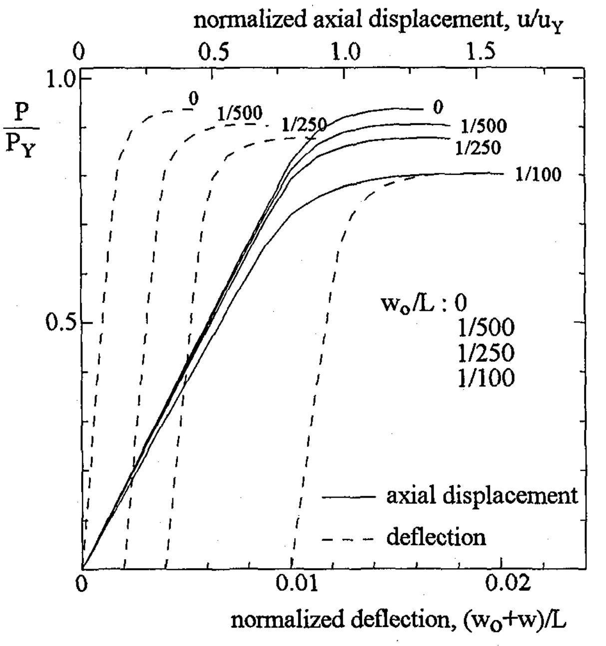

Fig. 8 Load-displacement curves of cylinders with SM570 and R=39.0 cm

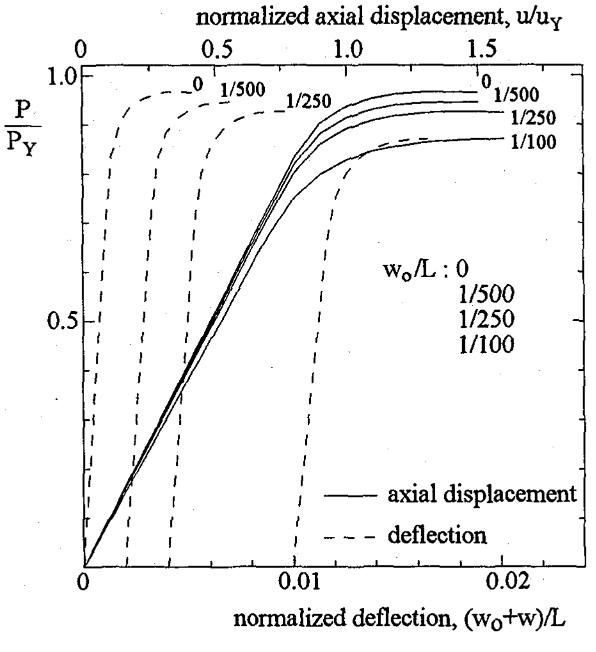

Fig. 7 Load-displacement curves of cylinders with SM570 and R=23.0 cm

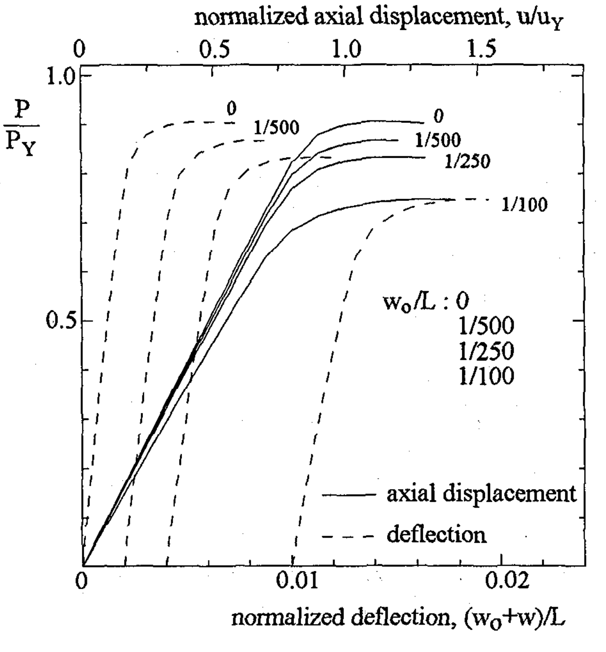

Fig. 9 Load-displacement curves of cylinders with SM570 and R=55.0 cm

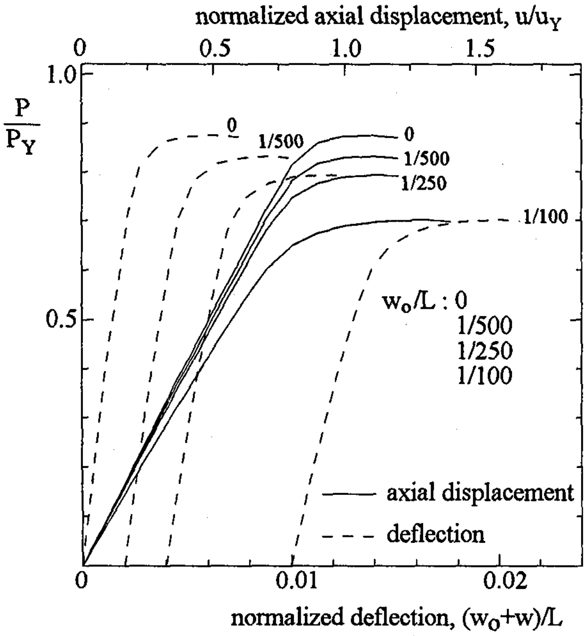

Fig. 10 Load-displacement curves of cylinders with SM570 and R=71.0 cm

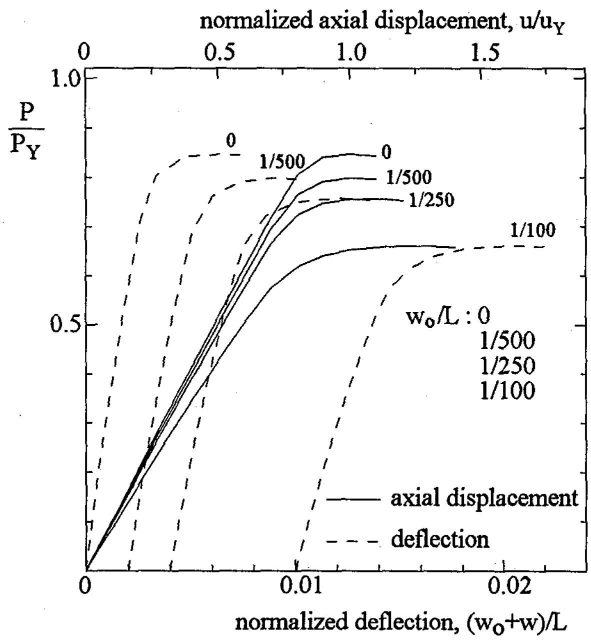

Fig. 11 Load-displacement curves of cylinders with SM570 and R=87.0 cm

In comparison with JSCE design criterion which its formulae are written in Eqs. (2) and (3), the present curves of cylinders with w_o/L=1/250 show close to the JSCE curve. Those of the cylinders with w_o/L=0 and w_o/L=1/500 show considerably higher and those of the cylinders with w_o/L=1/100 show considerably lower than the JSCE curve.

$$\frac{\sigma_{cul}}{\sigma_Y}=\frac{P_u}{P_Y}=\begin{cases}1.0 & R_t\leq 0.091\\ 0.665+\dfrac{0.0304}{R_t} & 0.091<R_t\leq 0.355\end{cases}\tag{2}$$

$$R_t=1.65\frac{\sigma_Y}{E}\frac{R}{t}\tag{3}$$

The formulae of the Plantema design criterion is written as follows.

$$\frac{\sigma_{xc}}{\sigma_Y}=\begin{cases}1.0 & \alpha_p\geq 8\\ 0.75+0.031\alpha_p & 2.5\leq\alpha_p<8\\ 0.33\alpha_p & \alpha_p\leq 2.5\end{cases}\tag{4}$$

$$\alpha_p=\frac{E}{\sigma_Y}\frac{t}{2R}\tag{5}$$

According to R_t parameter Eqs. (4) and (5) can be written as follows.

$$\frac{\sigma_{xc}}{\sigma_Y}=\begin{cases}1.0 & R_t\leq 0.103125\\ 0.75+\dfrac{0.025575}{R_t} & 0.103125<R_t\leq 0.33\\ \dfrac{0.27225}{R_t} & R_t\geq 0.33\end{cases}\tag{6}$$

The present curves of cylinders with w_o/L=1/500 show close to the Plantema curve. Those of cylinders with w_o/L=0 show higher and those of cylinders with w_o/L=1/250 and w_o/L=1/100 show considerably lower than the Plantema curve.

All cylinders used in this study are short. Based on the calculation of minimum ultimate load, length-to-radius ratio is obtained in the region between 0.12 to 0.39. It can be investigated that the cylinders satisfy $\alpha_e\sigma_{cr}\geq 0.5\sigma_Y$. Hence, the ECCS design criterion formula that satisfies for this specification can be written as follows.

$$\sigma_u=\sigma_Y\left[1-0.4123\left(\frac{\sigma_Y}{\alpha_e\sigma_{cr}}\right)^{0.6}\right]\tag{7}$$

When $w_o/L\leq 0.01$, knockdown factor α_e for cylinders with pure axial load given as follows.

$$\alpha_e=\frac{0.83}{\sqrt{1+0.01R/t}}\qquad R/t<212\tag{8}$$

$$\alpha_e=\frac{0.70}{\sqrt{1+0.01R/t}}\qquad R/t>212\tag{9}$$

The ECCS approach is plotted in the Fig. 12 as blank circle for cylinders with SM400 and black circle for those with SM570, and shows close agreement with the present results of cylinders with w_o/L=0.01.

Based on the present results of the cylinders without initial imperfection and using linear regression that give smallest error compared with nonlinear regression, we may write the following formula for calculating the ultimate strength of the cylinders.

$$\frac{P_u}{P_Y} = (1 - 0.46\, R_t)\, \alpha_s \tag{10}$$

where α_s is knockdown factor that depends on initial-to-length ratio and radius-to-thickness ratio as written as follows.

$$\alpha_s = 1 - 0.312 \left(\frac{w_o}{L}\frac{R}{t}\right) + 0.089 \left(\frac{w_o}{L}\frac{R}{t}\right)^2 \tag{11}$$

Eq. (11) is derived using polynomial quadratic regression of ultimate strength of the imperfect cylinders-to-ultimate strength of the perfect cylinders ratio in accordance to $(w_o/L)(R/t)$. The curves of Eq. (10) and others' design criterion formulae are shown in Fig. 13.

The suggested formula as shown in Eq. (10) gives more reasonable results than those of the Plantema and JSCE, due to the formula has the knockdown factor that affect the ultimate strength. The ECCS criterion as shown in Eq. (7) also has the knockdown factor, but the factor is specified for w_o/L=0.01 or Eqs. (8) and (9) are valid for w_o/L=0.01 only. While, in the suggested formula w_o/L may vary from 0.0 to 0.01. Another feature of the suggested formula shows that the ultimate strength can be calculated relatively easier than that of the ECCS criterion.

5. Conclusions

Using finite displacement method on the basis of isoparametric shell element, the ultimate strength of cylindrical steel shells under axial compression was studied. The cylinders with mild steel SM400 and high strength steel SM570 through R_t parameter and w_o/L parameter were evaluated. Main conclusions of the present study are as follows:

1. The normalized ultimate strength of the cylinders decreases with an increase in R_t parameter, and w_o/L parameter. For small initial deflection, the curves are close to linear line.
2. The curve of the perfect cylinders made of SM400 almost coincides with that of the perfect cylinder made of SM570, but for the cylinders with initial deflection in accordance to material show that the curves of cylinders with SM400 drop lower than those with SM570. This phenomenon shows that the ultimate strength of the imperfect cylinders is depend on R/t factor. The reduction of ultimate strength of the cylinders with initial imperfection is more sensitive with increasing R/t.
3. The ultimate strength versus R_t curves of present results were compared to the Plantema curve, the JSCE curve, and the ECCS approach. It was obtained that the present curves for w_o/L=1/500 are close to the Plantema curve, and those for w_o/L=1/250 are close to the JSCE curve. Whereas, those for w_o/L=1/100 are close to the ECCS approach. It should be noted that there is no explanation about initial deflection that used on the Plantema and JSCE design criteria. While, the ECCS criterion is based on w_o/L=1/100.

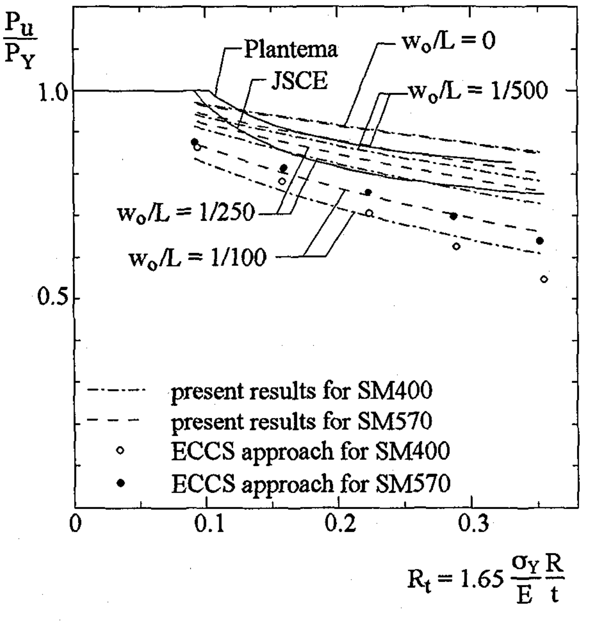

Fig. 12 Ultimate strength vs R_t curves of cylinders

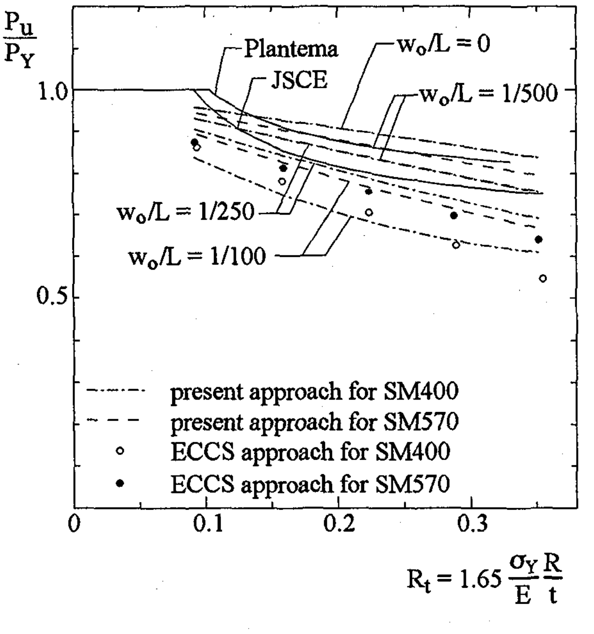

Fig. 13 Ultimate strength vs R_t curves of cylinders

4. The suggested formula can be used to asses the ultimate strength of the cylinders with various material (with yield plateau), R_t (0.092-0.355), and w_o/L (0-1/100). However, a small error may occur due to the choice of polynomial quadratic regression for knockdown factor α_s. An alternative different nonlinear regression may be proposed to overcome the problem.

Acknowledgment

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