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Uncertain Parameter Effects on Reliability of Offshore Platform

Katta VENKATARAMANA*, Kenji KAWANO** and Takahiro KOMASA***

* Dr. of Eng., Assoc. Professor, Dept. of Ocean Civil Eng., Kagoshima Univ., Korimoto, Kagoshima 890
** Dr. of Eng., Professor, Dept. of Ocean Civil Eng., Kagoshima University, Korimoto, Kagoshima 890
*** M. of Eng., Graduate Student, Dept. of Ocean Civil Eng., Kagoshima University

The loading processes in the offshore environment such as the wave force and the earthquake force are generally random in nature. The parameters which determine these forces are also random and provide significant roles on the dynamic response evaluations. In the present study, the effects of these uncertain parameters on the dynamic response of an offshore structure are investigated using the perturbation method and the spectral approach. The loading process is the wave force and the wave field is represented with the Bretschneider's power spectrum. Next, the first passage probabilities on the design's level crossing of the response are examined for the variations of the uncertain parameters.

Key Words: Offshore Structure, Dynamic Response, Reliability

1. Introduction

It is important to clarify the uncertainties involved in the dynamic response characteristics of offshore structures for more rational and reliable design. Sea wave force is one of the main dynamic loading acting on offshore structures and is generally expressed, for jacket and other similar slender structures, using the Morison equation. This equation contains several parameters which are functions of the various uncertainties of the ocean environment and therefore should be selected very carefully. The values of the parameters vary widely and it is necessary to investigate how the selection of these parameters influences the evaluation of dynamic response. Since the waves are usually random in nature, random vibration analysis technique is commonly employed[1] for the evaluation of dynamic response of offshore structures. Therefore spectral analysis approach can be applied and the effects of variations in the wave force parameters on the estimation of response can be determined.

In this paper, dynamic response analysis has been carried out for a jacket-type offshore platform including pile-soil foundation system for the input of sea wave loading. The dynamic soil-structure interaction is also included in the analysis. Small amplitude wave theory is used for computing the kinematics of flow field as the wave height is small compared to water depth. The wave loading is expressed using the modified Morison equation and the wave motion is represented by the Bretschneider's power spectrum. The effects, of the variations in the parameters of these expressions and also of the variations in the characteristics of the foundation-soil, on the response are estimated by perturbation method.

Next, the first passage probabilities on the design's level crossing of the response are determined for the variations of these parameters and their influence on the reliability of offshore structures is examined. In our earlier papers[2),3),4)], mainly the effects of parametric uncertainties on displacement responses was investigated whereas in the present paper the study is further extended to include the effects on stress responses. The reliabilities, with respect to the material strength of the structure, are also presented.

2. Governing equation of motion

For an offshore platform as shown in Figure 1, the superstructure can be represented with the finite element method. The dynamic characteristics of the soil-foundation system can be described using the impedance functions. If linearized Morison equation is used for modelling the external dynamic loading which is the wave force input, the dynamic equations of motion for the total system can be obtained by the dynamic substructure method. In this case the the drag force term of the wave force is nonlinear as it is proportional to the square of the relative velocity between the wave and the structure. Assuming that the distribution of the probability density function of the relative velocity is Gaussian process, using the equivalent linearization technique[5)], the equation of motion can be linearized. Therefore the equation of motion of the superstructure with the fixed-base condition for the input of sea wave loading is given as

$$[[M]+[K_m]]\{\ddot{u}\}+[[C]+[\bar{K}_D]]\{\dot{u}\}+[K]\{u\} = [K_M]\{\dot{v}\}+[\bar{K}_D]\{v\} \quad (1)$$

in which

$$[K_m] = [\diagdown\rho(C_M-1)V\diagdown], \quad [K_M]=[\diagdown\rho C_M V\diagdown]$$
$$[\bar{K}_D] = [\diagdown\sqrt{\frac{8}{\pi}}\rho C_D\frac{A}{2}\sigma_r\diagdown] \quad r=v-\dot{u}$$

The matrices $[M]$, $[C]$ and $[K]$ are respectively the mass matrix, the damping matrix and the stiffness matrix of the superstructure, $\{v\}$ and $\{\dot{v}\}$ are the velocity and the acceleration of the water particle, respectively. V and A are the volume and area respectively of each structural member normal to these flows. C_M and C_D are the inertia coefficient and the drag coefficient respectively. σ_r is the rms value of the relative velocity between the water particle and the structure. The dynamic response of the superstructure is generally determined using the first few vibrational modes. Therefore while applying the dynamic substructure method, the eigenvalue analysis of the superstructure with fixed-base condition is firstly carried out, only about 10 lowest vibration modes are selected and the equation of motion for the total system can be determined. Thus the number of degrees-of-freedom of the equation of motion for use in the dynamic response analysis of the total system is greatly reduced. The equation of motion determined in this way is finally expressed as[6)]

$$\begin{bmatrix} [I] & [\tilde{M}_{ap}] \\ [\tilde{M}_{pa}] & [\tilde{M}_p] \end{bmatrix} \begin{Bmatrix} \{\ddot{q}\} \\ \{\ddot{u}_p\} \end{Bmatrix} + \begin{bmatrix} [\diagdown 2\beta_{fj}\omega_{fj}\diagdown] & 0 \\ 0 & [\tilde{C}_p] \end{bmatrix} \begin{Bmatrix} \{\dot{q}\} \\ \{\dot{u}_p\} \end{Bmatrix} + \begin{bmatrix} [\diagdown\omega_{fj}^2\diagdown] & [0] \\ [0] & [\tilde{K}_p] \end{bmatrix} \begin{Bmatrix} \{q\} \\ \{u_p\} \end{Bmatrix} = \begin{bmatrix} [P_a] \\ [P_b] \end{bmatrix} \begin{Bmatrix} \{\dot{v}_a\} \\ \{v_a\} \end{Bmatrix} \quad (2)$$

in which

$$\begin{bmatrix} [P_a] \\ [P_b] \end{bmatrix} = \begin{bmatrix} [\Phi]^T[K_M] & [\Phi]^T[\bar{K}_D] \\ [G]^T[L]^T[K_M] & [G]^T[L]^T[\bar{K}_D] \end{bmatrix},$$
$$[\tilde{M}]=[M]+[K_m], \quad \{u_a^c\}=[\Phi]^T\{q\}$$

and the suffix a denotes the unrestrained nodal points of the superstructure and the suffix p denotes the pile-soil foundation, $[I]$ is the unit matrix, ω_{fj} is the natural frequency for the jth vibration mode of the structure with fixed-base condition, β_{fj} is the corresponding damping ratio which includes both the structural damping and the hydrodynamic damping, the matrix $[G]$ relates the displacements of the base nodal points and the pile-soil foundation, $[L]$ is the quasi-static transformation matrix, $\{u_a^c\}$ denotes the displacement due to the dynamic interaction, $[\Phi]$ is the modal matrix of the superstructure, and $\{q\}$ is the generalized displacement.

3. Equation of motion for uncertain parameters

By applying the equivalent linearization technique for the drag term of the wave force, the linearized equation of motion can be obtained. In this case, inertia coefficient and the drag coefficient are included in the wave force equation as parameters and these coefficients have generally variations. Assuming that these coefficients are random variables distributed statistically around their mean values, perturbation method can be applied and the coefficient matrices of the equation of motion of Eq.(2) become

$$\begin{aligned} [K_m] &= [K_m^{(0)}]+\varepsilon_1[K_m^{(1)}] \\ [K_M] &= [K_M^{(0)}]+\varepsilon_1[K_M^{(1)}] \\ [\bar{K}_D] &= [\bar{K}_D^{(0)}]+\varepsilon_2[\bar{K}_D^{(1)}] \end{aligned} \quad (3)$$

in which ε_1 and ε_2 are the random variables distributed statistically around their mean values for the inertia coefficient and the drag coefficient respectively. The wave motion can be represented by the energy spectrum of the Bretschneider type. This spectrum is a function of many factors such as wave number, mean wave height, mean wave period and so on. The effect of mean wave height on the dynamic response is expected to be larger and in the present analysis emphasis is placed on the variation of this parameter. The power spectrum of the wave energy for any depth is given as

$$S_{v_jv_j}(\omega)=S_{v_j}^{(0)}(\omega)+\varepsilon_3 S_{v_jv_j}^{(1)}(\omega) \quad (4)$$

in which ε_3 denotes the random variable distributed statistically around its mean value for the the mean wave height. The dynamic soil-structure interaction also provides important effects on the dynamic response evaluation of offshore structures. Generally the impedance function, which gives the relation of the displacement and the force on the dynamic soil-pile foundation system, is affected by the evaluation of the shear wave velocity of soil. Assuming that the shear wave velocity of a soil is a random variable distributed statistically around its mean value, the corresponding matrix on the shear wave velocity of the soil[6)] is given by

$$\begin{aligned} [K_p] &= [K_p^{(0)}]+\varepsilon_4[K_p^{(1)}] \\ [C_p] &= [C_p^{(0)}]+\varepsilon_4[C_p^{(1)}] \end{aligned} \quad (5)$$

in which ε_4 denotes the random variable distributed statistically around its mean value for the shear wave velocity in the soil. If the variations in the inertia coefficient, drag coefficient, mean wave height and shear wave velocity in the soil are expressed statistically in this way, the equation of motion for the total system of Eq.(2) can also be similarly expressed. Now the general coordinates in Eq.(2) can be modified with these random variables as

$$\{S\} = \left\{ \begin{array}{c} \{q\} \\ \{u_p\} \end{array} \right\} = \left\{ \begin{array}{c} \{q^{(0)}\} \\ \{u_p^{(0)}\} \end{array} \right\} + \sum_{i=1}^{4} \varepsilon_i \left\{ \begin{array}{c} \{q^{(i)}\} \\ \{u_p^{(i)}\} \end{array} \right\}$$
$$= \{S_o\} + \sum_{i=1}^{4} \varepsilon_i \{S_i\} \quad (6)$$

Substituting Eq.(6) in Eq.(2) and neglecting the terms containing higher than the first order in ε, the governing equation of motion for each random variable can be obtained as

$$[A_0]\{\ddot{S}_0\} + [B_0]\{\dot{S}_0\} + [D_0]\{S_0\} = [F_0]\{Z_0\} \quad (7)$$

and

$$\left.\begin{array}{rcl} [A_0]\{\ddot{S}_1\} + [B_0]\{\dot{S}_1\} & + & [D_0]\{S_1\} \\ & = & -[A_1]\{\ddot{S}_0\} + [F_1]\{Z_0\} \\ [A_0]\{\ddot{S}_2\} + [B_0]\{\dot{S}_2\} & + & [D_0]\{S_2\} \\ & = & -[B_2]\{\dot{S}_0\} + [F_2]\{Z_0\} \\ [A_0]\{\ddot{S}_3\} + [B_0]\{\dot{S}_3\} & + & [D_0]\{S_3\} \\ & = & [F_0]\{Z_1\} \\ [A_0]\{\ddot{S}_4\} + [B_0]\{\dot{S}_4\} & + & [D_0]\{S_4\} \\ & = & -[B_1]\{\dot{S}_0\} - [D_1]\{S_0\} \end{array}\right\} \quad (8)$$

Eq.7 corresponds to the mean values of each of the parameters. Each equation of Eq.(8) corresponds to the respective varying parameter namely the inertia coefficient, drag coefficient, mean wave height or the shear wave velocity in the soil. For all the cases the coefficient matrices on the left-hand side of the equations have the same form and hence it is easy to carry out the dynamic response analysis. (The details of the derivations and the expressions for the coefficient matrices in the above equations are given in Ref.2 and are omitted in this paper due to the space limitation.)

4. Random response analysis

The dynamic response analysis of offshore structure systems, considering the randomness of parameters, can be easily carried out applying the eigenvalue analysis again because the coefficient matrices on the left-hand side of Eqs.(7) and (8) are identical. Applying the classical modal analysis and neglecting the modal coupling effects since the first vibration mode is dominant, Eq.(7) can be transformed into a general coordinate system as

$$\{\ddot{y}_0\} + [\diagdown 2\beta_j\omega_j \diagdown]\{\dot{y}_0\} + [\diagdown \omega_j^2 \diagdown]\{y_0\} = [\Psi]^T[F_0]\{Z_0\} \quad (9)$$

in which

$$[\Psi]^T[A_0][\Psi] = [\diagdown 1 \diagdown], \quad [\Psi]^T[B_0][\Psi] = [\diagdown 2\beta_j\omega_j \diagdown]$$
$$[\Psi]^T[D_0][\Psi] = [\diagdown \omega_j^2 \diagdown], \quad \{S_0\} = [\Psi]\{y_0\}$$

Now applying the Fourier transform to both sides of Eq.(9),

$$\{\overline{y}_0\} = [H_0][\Psi]^T[F_0]\{\overline{Z}_0\} = [R_0]\{\overline{Z}_0\} \quad (10)$$

where

$$[H_0] = \left[-\omega^2[I] + i\omega[\diagdown 2\beta_j\omega_j \diagdown] + [\diagdown \omega_j^2 \diagdown]\right]^{-1}$$

Similarly the solution for Eq.(8) can be obtained by applying the Fourier transformation i.e.,

$$\{\overline{y}_i\} = [R_i]\{\overline{Z}_0\} \quad (i = 1,2,3,4) \quad (11)$$

where $\{\overline{y}_0\}$ and $\{\overline{Z}_0\}$ are respectively the Fourier transformations of $\{y_i\}$ and $\{Z_0\}$. Now for the input of the power spectral density function of the wave force, the response power spectral density function can be obtained in terms of the general co-ordinates $\{\overline{y}\}$. Next using the modal matrix for the general coordinate system, shown in Eq.(9), the power spectral density function of $\{S_i\}$ can be determined i.e.,

$$\left.\begin{array}{rcl} [S_{\overline{y}_j\overline{y}_k}(\omega)] & = & [R_j][S_{\overline{Z}_0\overline{Z}_0}(\omega)][R_k] \\ {[S_{\overline{S}_j\overline{S}_k}(\omega)]} & = & [\Psi][S_{\overline{y}_j\overline{y}_k}(\omega)][\Psi]^T \end{array}\right\} \quad (12)$$

in which $j,k = 0,1,2,3,4$. Since the general coordinate $\{S\}$ is a function of unknown parameters and these parameters have random characteristics with zero-mean, its power spectral density function can be expressed as

$$[S_{\overline{s}\overline{s}}(\omega)] = E[\varepsilon_j\varepsilon_k][S_{\overline{s}_j\overline{s}_k}(\omega)] \quad (13)$$

in which $E[\varepsilon_j\varepsilon_k]$ denotes the expectation of these variables. The covariance matrix of response $\{S\}$ is obtained using the inverse Fourier Transform.

$$[R_{ss}] = \int_{-\infty}^{\infty} [S_{\overline{s}\overline{s}}(\omega)]d\omega \quad (14)$$

The covariance matrices of the dynamic response of the offshore structure $\{u_a\}$ and $\{u_p\}$ are determined using the modal matrix $[\Phi]$ expressed in Eq.(2). The covariance matrix of response in Eq.(14) includes the effects of many uncertain parameters on the response evaluation, as given in Eq.(6). But their mean values are zero and hence if the covariance of uncertain parameters are given, the covariance matrix of responses including the uncertain parameter effects is determined using Eq.(14).

The covariance matrix of the resultant force of each element can be obtained using the element stiffness matrix $[K_e]$ and the corresponding power spectra of the displacement response $[S_{ee}(\omega)]$ as follows:

$$[R_{FF}] = \int_{-\infty}^{\infty} [K_e][S_{ee}](\omega)][K_e]^T d\omega \quad (15)$$

In this case one is also interested in the amount of peak response and in the first passage probability on design's level responses. Using the parameters of the response values obtained from the power spectral density function of the response $[S_x(\omega)]$, the first passage probability on level crossings can be expressed as[7),8)]

$$L(\lambda) = \exp\left[-\frac{1}{\pi}\sqrt{\frac{\alpha_2}{\alpha_0}} t_0 \exp\left(-\frac{\gamma^2}{2}\right) c_1\right] \quad (16)$$

where

$$c_1 = \frac{1 - \exp\left(-\sqrt{\frac{\pi}{2}} q_x \gamma\right)}{1 - \exp\left(-\frac{\gamma^2}{2}\right)}, \quad q_x = \left(1 - \frac{\alpha_1^2}{\alpha_0 \alpha_2}\right)^{1/2},$$

$$\alpha_i = \int_0^\infty \omega^i S_x(\omega) \mathrm{d}\omega \quad (i = 0, 1, 2)$$

and t_0 is the duration and γ is the ratio between design's level response and the rms response which includes the uncertain parameter effects. In the present study, the effects of the variation in each of the uncertain parameters, mentioned earlier, on the estimation of the dynamic response of offshore structures have been investigated using Eqs.(14) and (16).

5. Numerical results and discussions

The analytical model of an offshore structure is shown in Figure 1. The superstructure is 120m high and the water depth is 110m. The diameter of the main members is 2.0m and the thickness is 11mm. The foundation is supported by the pile and the soil, and the shear wave velocity in the soil is 100m/sec. The displacement of each nodal point has horizontal, vertical and rotational components in plane. The first and second natural frequencies of the total system are 3.51rad/s (1.79s) and 10.43rad/s (0.60s) respectively. The critical damping ratio of the first mode of the superstructure is 2%. The damping ratios obtained by the analysis of the total system for the first mode and the second mode are respectively 1.9% and 5.7% respectively. The wave force is represented with the modified Morison equation using the Airy wave theory. The response analysis is performed with equivalent linearization of the drag force term and reasonable convergence is attained in about three cycles of iteration.

Figure 2 shows the rms displacement at nodal point 1 (i.e., at the top node) and Figure 3 shows the rms stress (sum of axial and bending stresses) at the bottom end of the element between nodes 6 and 7 of the structure (i.e., at the bottom element) for the wave inputs of mean wave height $\overline{H}$ ranging from 3m to 9m and mean wave period $\overline{T}$ ranging from 5s to 15s. The response increases with the increase in mean wave height due to higher wave energy. Also when the mean wave period of the input wave becomes shorter, the corresponding peak of the wave force spectrum moves towards the natural period of the structure and consequently the response values become larger. Therefore it is important to determine accurately the natural period of the structure and the mean period of the power spectrum of the input wave.

Figures 4 and 5 show the effects on the rms displacements of the top node and on the rms stresses of bottom element respectively by the variations in the uncertain parameters for typical sea states ranging from moderate waves to severe storm conditions. The uncertain parameters considered are the inertia coefficient, drag coefficient, mean wave height and the shear wave velocity in the soil. All the uncertain parame-

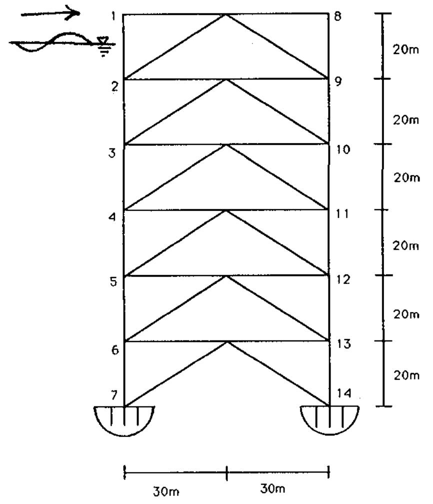


Figure 1 Analytical model of offshore structure-soil system

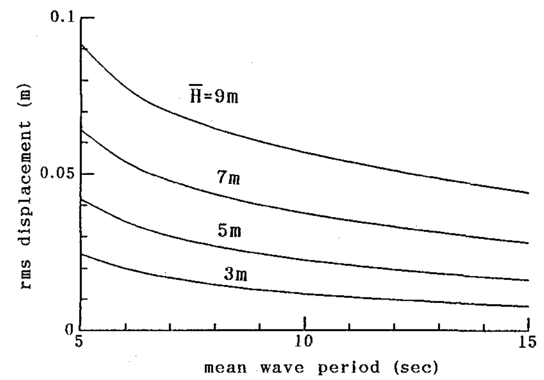


Figure 2 rms Displacements of offshore structure

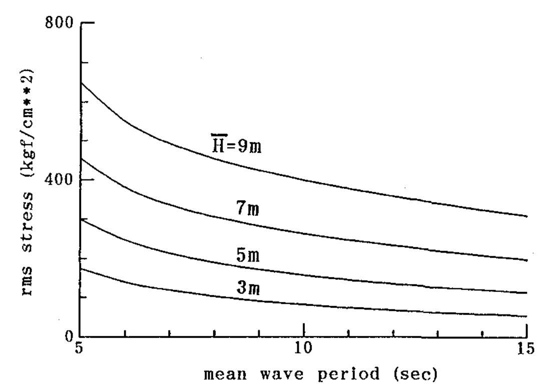


Figure 3 rms Stresses of offshore structure

ters are assumed to have the same variations. The abscissa denotes variations of each uncertain parameter and the ordinate denotes the rms response ratio between "without uncertain parameter" and "with each uncertain parameter". As the variations in each of the uncertain parameters increases, its influence on the response increases linearly. It is shown that the most important effects on the response are given by

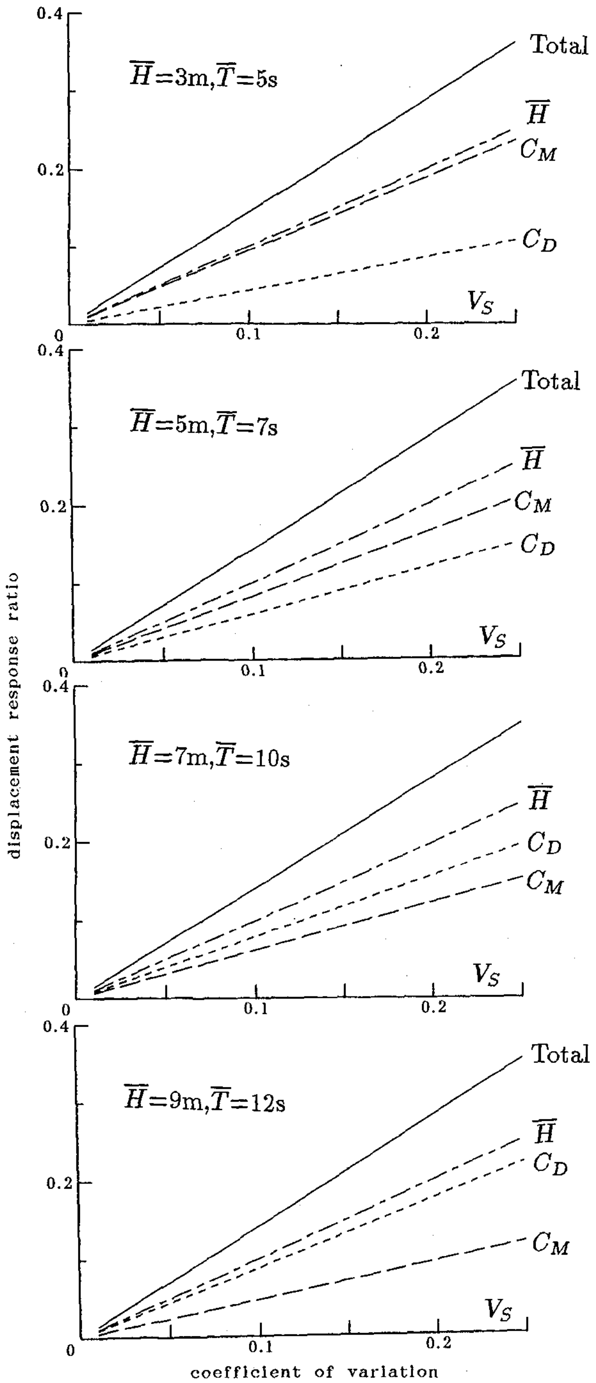


Figure 4 Uncertain parameter effects on response displacements

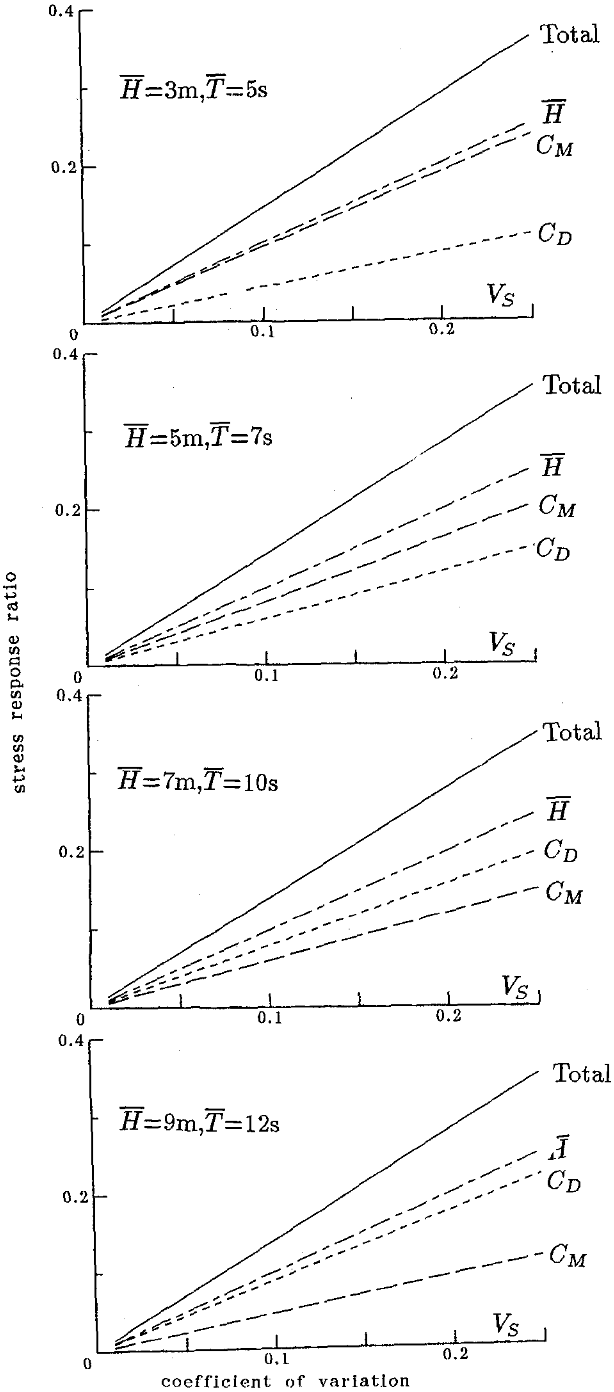


Figure 5 Uncertain parameter effects on response stresses

the variations of the mean wave height. The variations of the inertia and drag coefficients also provide similar effects as those of mean wave height. For the offshore structure model considered in this study, the effects of the variations in the shear wave velocity in the soil are small.

Next the variations in the inertia coefficient, drag coefficient and the mean wave height are investigated for different mean wave periods of the input wave. Figure 6 shows the effects of these uncertain parameters on the response stresses of bottom element (where maximum stresses are likely to occur) when the mean wave period varies from 5sec to 15sec and the mean wave height is 3m i.e., moderate waves (Figure 6(a)) and 7m i.e., severe waves (Figure 6(b)). The ordinate denotes the rms response ratio between "without uncertain parameter" and "with uncertain parameter". It is shown that the effect of the variation in the inertia coefficient on the response increases slightly as the mean wave period becomes longer, but the effect of the variation in the mean wave height is almost same for all the mean wave periods considered in the study.

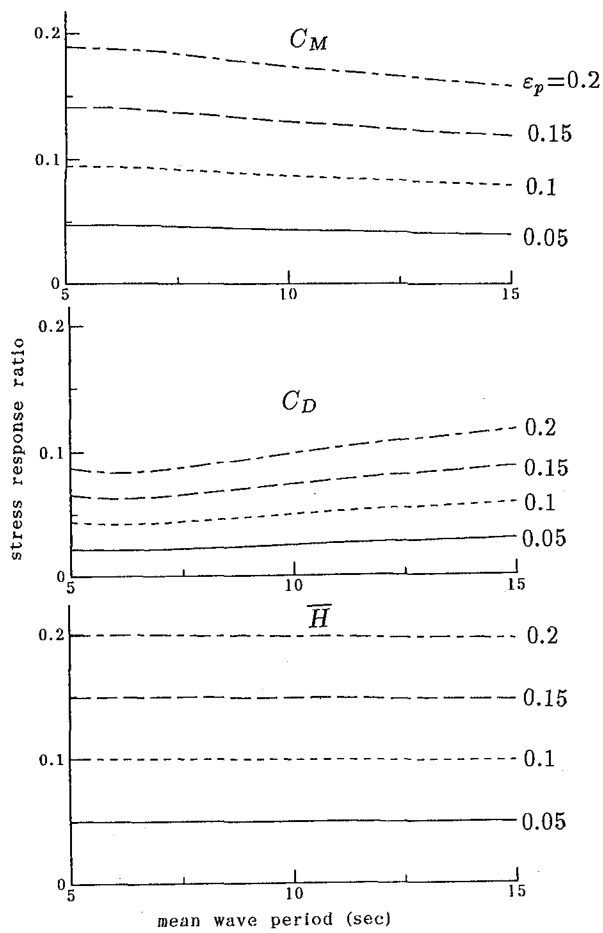


Figure 6(a) Uncertain parameter effects of C_M, C_D and $\overline{H}$ ($\overline{H}$=3m)

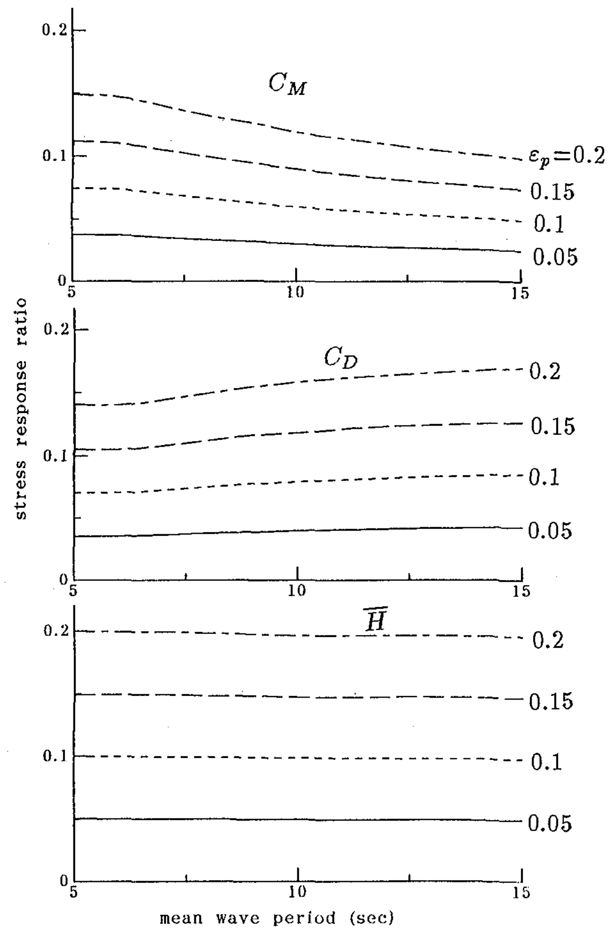
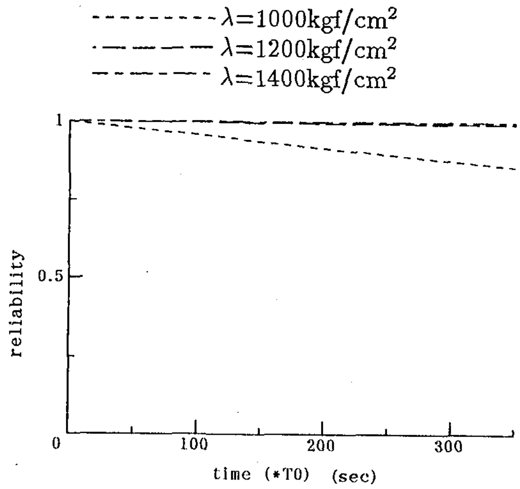


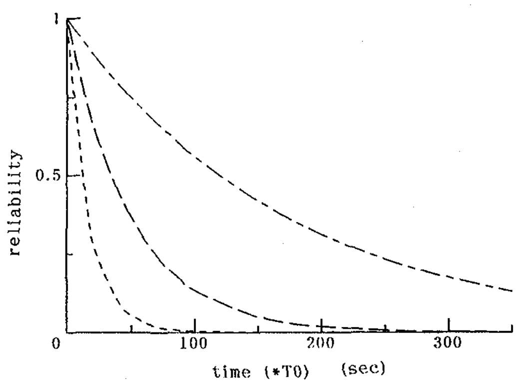
Figure 6(b) Uncertain parameter effects of C_M, C_D and $\overline{H}$ ($\overline{H}$=7m)

Figures 7 and 8 show the reliabilities on the level crossing of the rms stresses at the bottom element for different wave conditions. Mainly the results for severe wave conditions are shown. Three cases of barrier level λ i.e, λ=1000 kgf/cm^2, 1200 kgf/cm^2 and 1400 kgf/cm^2 are considered which represent the expected strength of the structure material. The abscissa denotes the duration of wave excitation. Figure 7 shows the reliability when there is no variation in the parameters. Figure 8 corresponds to the cases with varying parameters, the coefficient of variation of each uncertain parameter being 0.2.

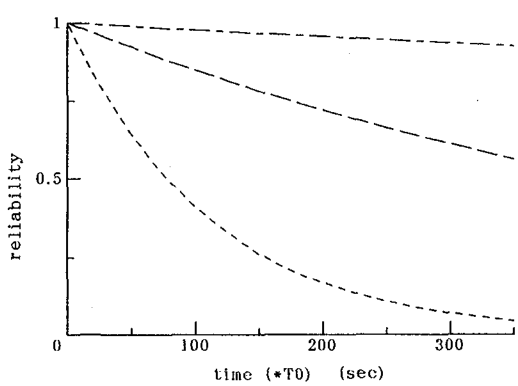
Generally the reliabilities decrease with increasing wave height due to higher wave forces. Also as the wave period becomes smaller and approaches the natural period of the structure, the reliability against the level crossing decreases due to the increase of response values. Further, as the duration time becomes longer, the effects of variations in the uncertain parameters and hence the probability of crossing the barrier level also increase.



(a) $\overline{H}$=7m,$\overline{T}$=10s, ε_p=0

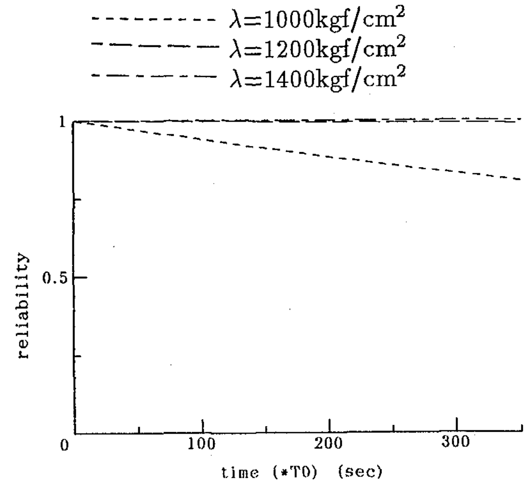


(b) $\overline{H}$=9m,$\overline{T}$=8s, ε_p=0

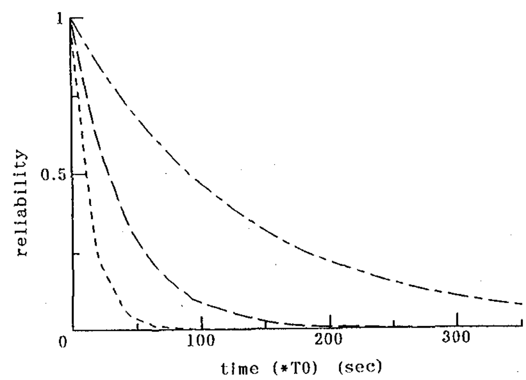


(c) $\overline{H}$=9m,$\overline{T}$=12s, ε_p=0

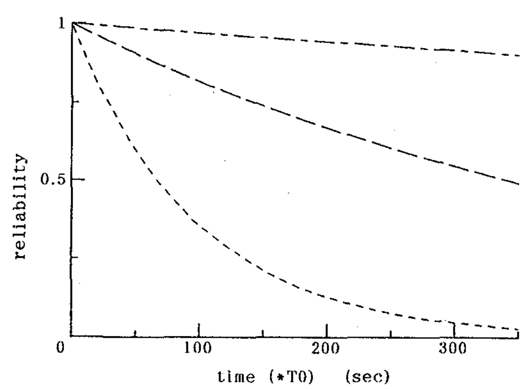
Figure 7 Uncertain parameter effects on level crossing probability



(a) $\overline{H}$=7m,$\overline{T}$=10s, ε_p=0.2



(b) $\overline{H}$=9m,$\overline{T}$=8s, ε_p=0.2



(c) $\overline{H}$=9m,$\overline{T}$=12s, ε_p=0.2

Figure 8 Uncertain parameter effects on level crossing probability

6. Conclusions

The effects of parametric uncertainties on the dynamic response of offshore structures and their reliabilities have been investigated. The main results of this research are summarized as follows:

- Among the uncertainties in the values of inertia coefficient, drag coefficient, mean wave height and shear wave velocity in the soil, the variations in the mean wave height have more significant effects on the response evaluation. It is important to clarify these effects on the responses for reliable response evaluation.
- The effects of the uncertain parameters on response evaluation are different for waves with different mean wave height and mean wave period. These effects are closely related to the natural frequency of the structure and the predominent frequency of the wave force.
- The results indicate that the variations in the uncertain parameters significantly affect the first passage probabilities across the design's response level and hence the reliability of offshore structures. As the duration of wave excitation becomes longer, the effects also increase correspondingly. Therefore, in order to perform reliable evaluation of the response of offshore structures, it is important to clarify the effects of parametric uncertainties on the response evaluation.

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