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STIFFNESS DEGRADATION IDENTIFICATION OF STRUCTURES USING MODAL ANALYSIS

Hongying YUAN*, Kiyoshi HIRAO**,
Tsutomu SAWADA*** and Yoshifumi NARIYUKI****

A spatial domain, modal analysis method is presented to identify both the location and severity of stiffness degradation for damped structures by using lower measured modes. For small deterioration, it is difficult to detect the location of stiffness degradation using those lower measured modes. A two-stage least-square estimate procedure is then proposed to deal with such problem. In order to demonstrate the availability of the method, a ten-story shear building has been analyzed. The numerical result shows the proposed procedure is useful for detecting the minor stiffness degradation of structures.

1. INTRODUCTION

Identification or correction of structural parameters(mass, damping and stiffness) is often based on experimental data obtained from the real system. This kind of experimental data is a reference of the identification or correction. In most cases, responses or modal parameters(natural frequency, mode shape and damping ratio) are major candidates of those references. The commonly used references are summarized as follows.

In the time domain, a measured time-domain response is the simplest reference. The associated identification or correction method is often referred to as response fitting. This method has a long history of development with many applications[1),2),3)]. The modal parameters can also be extracted from the recorded response by performing various time domain modal analysis[4)]. However, in this domain, it is noticed that sometimes the accuracy

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* M.Eng., Graduate Student, Dept. of Civil Eng., The University of Tokushima, Japan.
** Dr.Eng., Professor, Dept. of Civil Eng., The University of Tokushima, Japan.
*** Dr.Eng., Asso. Professor, Dept. of Civil Eng., The University of Tokushima, Japan.
****Dr.Eng., Lecturer, Dept. of Civil Eng., The University of Tokushima, Japan.

of identification is greatly influenced by noise[1].

In the frequency domain, the frequency response function(FRT), as well as the fast Fourier transformation(FFT) of a time domain response(transfer function) is commonly used as references. Other references in the frequency domain include various modal parameters obtained by performing frequency-domain modal analysis. Usually, the parameters so obtained must have better signal-to-noise ratio[5]. Among these references, natural frequency is the most convenient one.

In the spatial domain[5], the references are directly measured mode shapes and other modal parameters. Because of intense computations involved, as well as the identification accuracy being highly dependent on measured mode shape[6], references from spatial domain are rarely used. However, with the recent improvement of vibrational testing technique[7],[8], the spatial-domain method has been made a certain development. As the instrument for earthquake record is installed only in a small number of important structures, but not most structures. Therefore, an evaluation of damage or an improvement of analytical model for structures through those spatial domain methods has a practical significance if the data obtained from ordinary vibrational testing is available.

For structures, measurement of modal parameters can lead to estimation of the elements of mass and stiffness matrices by using system identification techniques. However, in civil engineering, data in field test are rarely complete[9]. Only a limited number of degree of freedom and modes are observed in the response records. Adequate control of excitation is essential for precise mode shape measurement, but it may be difficult to achieve in the field. For example, the error in a measured mode shape could reach as high as 500%[10]. In spite of those difficulties, there are still many inspiring reports[8] of vibration testing for practical structures. This testing method is not only practical but also provides accurate modal parameters. Therefore, using those modal testing techniques, it is possible to obtain reliable experimental data for system identification.

There are many studies[6],[9] to identify the degradation of stiffness: Hearn and Testa[9] estimated the damage of welded steel frame and wire rope by using the ratios of changes in natural frequencies; Yuan and Hirao et al[6] estimated both the location and severity of damage for undamped structures by use of the lower measured modes. However, there are still several problems[8] in those current modal analysis methods. One of them is regarding to the sensitivity of detection, i.e., only the location of severe damage can be detected for practical structures[6],[8],[11]. An recent attempt for improving the sensitivity is that Yao et al[11] detected a location of minor damage for steel frame by using the strain mode shape(SMS) technique. In Yao's method, however, it needs a lot of measurement stations for complex structures.

In order to improve the sensitivity for detecting the location of damage, based on our previous study[6] for undamped structures, this study 1) presents a spatial domain, modal analysis method for damped structures; 2) proposes a two-stage least-square estimate procedure to detect the location of small degradation of stiffness; 3) evaluates the effect of measurement error of modal parameters(includes the damping ratio) on identification accuracy.

2. MODAL ANALYSIS INSPECTION FOR DAMPED STRUCTURES

(1) Basic formulations

A equation of motion for damped, free vibration system with N degrees of freedom is described as follows

$$[M]\{\ddot{Z}\}_p+[C]\{\dot{Z}\}_p+[K]\{Z\}_p=\{0\}; \quad p=1,2,\dots N \quad \dots\dots\dots\dots(1)$$

where $[M]$=mass matrix, $[K]$ =stiffness matrix and $\{Z\}_p$= the vector for pth mode of relative displacements.

The Eq. (1) is satisfied[12] by

$$\{Z\}_p=\{X\}_p e^{\mu_p t} \quad \dots\dots\dots\dots(2)$$

where μ_p is the pth complex natural frequency and $\{X\}_p$ is pth eigenvector corresponding to μ_p. Substitution of Eq. (2) into Eq. (1) yields

$$e^{\mu_p t}\ ([M]\{X\}_p\mu_p^2+[C]\{X\}_p\mu_p+[K]\{X\}_p)=\{0\} \quad \dots\dots\dots\dots(3)$$

Since $e^{\mu_p t}$ is non-zero, Eq. (3) becomes

$$[M]\{X\}_p\mu_p^2+[C]\{X\}_p\mu_p+[K]\{X\}_p=\{0\} \quad \dots\dots\dots\dots(4)$$

For damped structures, if the lower L modes have been measured, these modes can be expressed as follows

$$[\Omega^2]_{L\times L}=\mathrm{diag}\,(\mu_1^2,\mu_2^2,\dots\mu_L^2)\ ,\ [\Phi]_{N\times L}=(\{X_1\}\ ,\ \{X_2\}\ ,\dots\ \{X_L\}) \quad \dots\dots\dots\dots(5)$$

Replacing[13] the μ_p and $\{X\}_p$ by $[\Omega^2]$ and $[\Phi]$ respectively, Eq. (4) becomes

$$[M][\Phi][\Omega^2]+[C][\Phi][\Omega]+[K][\Phi]=[0] \quad \dots\dots\dots\dots(6)$$

Traditionally, Rayleigh damping has been assumed in most dynamic structural analysis because it is convenient for mathematical treatment. The damping matrix for Rayleigh damping may be written as

$$[C]=\beta_1[M]+\beta_2[K] \quad \dots\dots\dots\dots(7)$$

where β_1, β_2=scalars. For Rayleigh damping, substituting Eq. (7) into Eq. (4), yields

$$[M]\{X\}_p(\mu_p^2+\beta_1\mu_p)+[K]\{X\}_p(\beta_2\mu_p+1)=\{0\} \quad \dots\dots\dots\dots(8)$$

Eq. (8) can further be written as

$$[K]\{X\}_p=[M]\{X\}_p\ \{-(\mu_p^2+\beta_1\mu_p)/(\beta_2\mu_p+1)\} \quad \dots\dots\dots\dots(9)$$

For undamped system, a generalized eigenvalue problem is expressed as

$$[K]\{X\}_p=[M]\{X\}_p\omega_p^2; \quad p=1,2,\dots,N. \quad \dots\dots\dots\dots(10)$$

where ω_p is the pth undamped natural frequency, $\{X\}_p$ is the pth normal mode shape.

Note that the Eq. (9) is also a form of generalized eigenvalue problem. Therefore, the relation between ω_p and μ_p is as follows

$$\mu_p=\{-(\beta_1+\beta_2\omega_p^2)+\sqrt{(\beta_1+\beta_2\omega_p^2)^2-4\omega_p^2}\}/2 \quad \dots\dots\dots\dots(11)$$

Particularly for Rayleigh damping, because of[14]

$$2\xi_p\omega_p=\beta_1+\beta_2\omega_p^2 \quad \dots\dots\dots\dots(12)$$

where ξ_p=the pth damping ratio, Eq. (11) becomes

$$\mu_p=-\xi_p\omega_p+i\omega'_p; \quad i^2=-1 \quad \dots\dots\dots\dots(13)$$

where ω'_p=the pth damped natural frequency. Thus, the eigenvalue problem for Rayleigh damping is transformed into an undamped, generalized eigenvalue problem. Furthermore, the pth undamped natural frequency is obtained from

$$\omega_p = \omega'_p / \sqrt{1-\xi_p^2} \qquad \text{..........(14)}$$

Also for Rayleigh damping, the coefficients β_1 and β_2 can be obtained[14] as follows

$$\beta_1 = \omega_1 \omega_2 \, (2\xi_2 \omega_1 - 2\xi_1 \omega_2) \,/\, (\omega_1^2 - \omega_2^2)$$

$$\beta_2 = (2\xi_1 \omega_1 - 2\xi_2 \omega_2) \,/\, (\omega_1^2 - \omega_2^2) \qquad \text{..........(15)}$$

(2) The extension of modes

As the number of measured modes is often limited in civil engineering, it is difficult to detect the location of damage by using such lower measured modes for small deterioration. According to our previous study[6], the sensitivity of detecting the location of damage is closely related to the number of measured modes(known modes) of structures, and the detected location of damage remains unchanged within a certain range of measurement error. Furthermore, even an approximate modes may be used in the damage location detection. This fact leads us to estimate the unmeasured modes(unknown modes) and then to estimate the change of stiffness.

In steel or RC structures, cracks reduce stiffness without loss of mass, and a corrosion loss will affect stiffness to a much greater extent than it will affect mass. Therefore, $[\triangle M]$ can be taken as zero[9]. Paying attention to the condition $[\triangle M]=0$, the set of measured modes shapes can be extended as follows.

For every unknown $\{X\}_s (s=L+1, L+2, \dots, N)$ of damaged structures, the orthogonality condition becomes

$$\{X\}_r^T [M] \{X\}_s = 0; \quad r=1, 2, \dots, L \qquad \text{..........(16)}$$

$$\{X\}_s^T [M] \{X\}_s = 1 \qquad \text{..........(17)}$$

where $\{X\}_r$ = the rth measured mode shape and L = the number of measured modes. It is noticed that there are L equations in Eq.(16). Let $\{X\}_s = \{X_0\}_s + \{\triangle X\}_s$, in which $\{X_0\}_s$ is the sth mode shape of undamaged(a known vector) and $\{\triangle X\}_s$ is an unknown vector representing the change of the mode shape before and after the damage. Therefore, Eq.(16) can be written as

$$\{X\}_r^T [M] \{\triangle X\}_s = -\{X\}_r^T [M] \{X_0\}_s; \quad r=1, 2, \dots, L \qquad \text{..........(18)}$$

Also Eq.(17) becomes

$$\{X_0\}_s^T [M] \{X_0\}_s + \{\triangle X\}_s^T [M] \{X_0\}_s + \{X_0\}_s^T [M] \{\triangle X\}_s + \{\triangle X\}_s^T [M] \{\triangle X\}_s = 1 \qquad \text{..........(19)}$$

Theoretically, Eq.(19) is considered as a nonlinear planning problem. Since the $\{\triangle X\}_s$ is a vector, it is not difficult to prove the $\{\triangle X\}_s^T [M] \{X_0\}_s = \{X_0\}_s^T [M] \{\triangle X\}_s$. Also because of the existance of orthogonality(before damage), $\{X_0\}_s^T [M] \{X_0\}_s$ is just equal to 1. Neglecting second-order terms[9] in Eq.(19), the nonlinear planning problem is then transformed into a linear one. That is

$$\{X_0\}_s^T [M] \{\triangle X\}_s = 0 \qquad \text{..........(20)}$$

Eqs.(18) and (20) are a group of equations corresponding to every $\{\triangle X\}_s$. Furthermore there are N unknown components in every $\{\triangle X\}_s$, and there are L+1 known equations in Eqs(18) and (20). Accordingly, Eqs(18) and (20) can be expressed as the following form of matrix equation

$$[G] \{\triangle X\}_s = \{H\} \qquad \text{..........(21)}$$

where $[G]_{(L+1)\times N} = [\{X\}_1^T [M], \{X\}_2^T [M], \dots, \{X\}_L^T [M], \{X_0\}_s^T [M]]$ and $\{H\}_{(L+1)\times 1} = \{-\{X\}_1^T [M] \{X_0\}_s, -\{X\}_2^T [M] \{X_0\}_s, \dots, -\{X\}_L^T [M] \{X_0\}_s, 0\}^T$. Therefore, the least-square[15] estimate of the vector $\{\triangle X\}_s$ is

$$\{\triangle X\}_s = [G]^+\{H\} \qquad \cdots\cdots\cdots\cdot(22)$$

where $[G]^+_{N\times(L+1)}$ =the pseudoinverse matrix of [G]. Note that, the s varies from L+1 to N. The set of mode shapes can be extented for arbitrary order of mode shapes. Furthermore, let $\omega_s=\omega_{0s}+\triangle\omega_s$ and $\xi_s=\xi_{0s}+\triangle\xi_s$. In the case of Rayleigh damping, substituting the ω_s and ξ_s into Eq.(12), similar to Eq.(21), the $\triangle\omega_s$ can also be estimated from

$$2\omega_{0s}\triangle\xi_s+(2\xi_{0s}-2\beta_2\omega_{0s})\triangle\omega_s=\beta_1-2\xi_{0s}\omega_{0s}+\beta_2\omega_{0s}^2 \qquad \cdots\cdots\cdots\cdot(23)$$

where s=L+1, L+2,..., N. After ω^2_s and $\{X\}_s$ are estimated, the set of modes $[\Omega^2]$ and $[\Phi]$ can be extended by treating the ω^2_s and $\{X\}_s$ as the known modes.

(3) Two-stage least-square estimate procedure

The objective of the identification in this study is to obtain the stiffness degradation of damaged structures. The undamaged structural parameters(i.e. $[M_0]$, $[C_0]$ and $[K_0]$) are considered to be the known parameters(for example, structures remain in elastic stage before damage). Let $[K]=[K_0]+[\triangle K]$, in which $[\triangle K]$ is the change of stiffness matrix before and after the damage. Since the eigenvalue problem for Rayleigh damping has been transformed into the undamped eigenvalue problem. Substituting the [K]、[M] and set of L undamped natural frequencies $[\Omega^2]$ and mode shapes $[\Phi]$ into Eq.(10), $[\triangle K]$ is then expressed as

$$[\triangle K][\Phi]=[M][\Phi][\Omega^2]-[K_0][\Phi] \qquad \cdots\cdots\cdots\cdot(24)$$

In Eq.(24), there are N(N+1)/2 unknown coefficents of $[\triangle K]$ and N×L equations (L is number of measured modes). If L<(N+1)/2, the approximate solution of $[\triangle K]$ can be obtained. Also the least-square[15] estimate of the matrix $[\triangle K]$ is

$$[\triangle K]= ([M][\Phi][\Omega^2]-[K_0][\Phi])\ [\Phi]^+ \qquad \cdots\cdots\cdots\cdot(25)$$

where $[\Phi]^+_{L\times N}$ =the pseudoinverse matrix of $[\Phi]$.

In order to detect the location of damage, let's substitute the undamaged global stiffness matrix $[K_0]$, the mass matrix [M], the undamped natural frequency $[\Omega_L^2]$ and the measured mode shape $[\Phi_L]$ into Eq.(25). Furthermore, a percentage of change ratio $\triangle k_{pp}/k_{0pp}$(p=1, 2, ・・・, N) for diagonal stiffness coefficients are calculated. According to the magnitude of the change ratio, the node with a remarkable change ratio is therefore detected.

In this study, to estimate the change of stiffness directly(see Eq.(25)) is called one-stage estimate method. Furthermore, to estimate at first the unmeasured modes(see Eq.(22) and Eq.(23)) and then to estimate the change of stiffness is called two-stage estimate method.

3. SOLUTION OF UNKNOWN COEFFICIENTS

When the node in which the stiffness has been changed is detected, each non-zero stiffness coefficient in the column(or row) corresponding to this node in global stiffness matrix [K] is multiplied by an unknown coefficient α_K respectively. Thus, the damaged stiffness matrix [K] is expressed as $[K(\alpha)]$, in which $\alpha=\{\alpha_1, \alpha_2, \cdots, \alpha_{NF}\}^T$ and NF=the total number of α. As for the further details, see our pervious study[6]. Therefore, the damping matrix [C] is expressed as a function of α by substituting the [M] and $[K(\alpha)]$ into Eq.(7) for Rayleigh damping.

In the vibration equations of motion, there are N equations corresponding to each mode. If all those L measured modes are used, the L×N equations can be obtained. Substituting the measured $[\Omega^2]$, $[\Phi]$, together with [M], $[C(\alpha)]$ and $[K(\alpha)]$ into Eq.(10) for Rayleigh damping, and further arranging the L×N equations, the following new equations are therefore obtained

$$[A]\{\alpha\} = \{B\} \qquad \text{..........(26)}$$

where [A] and {B} = a known NF×NF matrix and NF×1 vector respectively. When $|[A]| \neq 0$, the exclusive $\{\alpha\}$ can be obtained. If $NF > L \cdot N$, it needs additional measured modes. After all the unknown coefficients $\{\alpha\}$ are solved, the $[K(\alpha)]$ is the identified result of [K]. Therefore, the serevity of damage can be identified.

4. EVALUATION OF IDENTIFICATION ACCURACY

Because the real damaged stiffness is an unknown value, it cann't be evaluated that the identified result is an exact one or not. However, on a certain significance of measured modes, the identified result could be evaluated by some formulae[5),9)]. If the system's stiffness matrix [K] and damping matrix [C] are identified, the modal parameters corresponding to this system can be obtained from Eq.(10). Similar to a least-square cost function[16)], a formula of error evaluation is defined as

$$J = \sum_{p=1}^{N} \{(\omega_p^{(M)} - \omega_p^{(I)})^2 + (\zeta_p^{(M)} - \zeta_p^{(I)})^T(\zeta_p^{(M)} - \zeta_p^{(I)})\} \qquad \text{..........(27)}$$

where $\omega_p^{(M)}$ = the measured pth natural frequency; $\zeta_p^{(I)}$ = the pth identified mode shapes. The value of J indicates whether the identified result is close to the measured one or not. In general, a recommended value of J should be[5)] less than 0.1.

5. NUMERICAL EXAMPLE

Fig. 1 shows a numerical example of a ten-story shear building[17)]. The structure is described as a FEM model with 11 nodes, 10 elements and 10 degree-of-freedom. Also the order of nodes agrees with the numbered levels in the model. The undamped natural frequencies and damping ratios of the structure are listed in Table 1. Furthermore, the initial (before damage) coefficents of stiffness (unit:$10^6 \times N/m$) are : $k_{11}=121.730$; $k_{12}=-59.260$; $k_{22}=113.400$; $k_{23}=-56.140$; $k_{33}=109.160$; $k_{34}=-53.020$; $k_{44}=99.810$; $k_{45}=-49.910$; $k_{55}=96.700$; $k_{56}=-46.290$; $k_{66}=90.460$; $k_{67}=-43.670$; $k_{77}=84.220$; $k_{78}=-40.550$; $k_{88}=77.980$; $k_{89}=-37.430$; $k_{99}=71.740$; $k_{9,10}=-34.310$; $k_{10,10}=34.310$; $k_{ps}=k_{sp}(p,s=1,2,...N)$. The other coefficents of stiffness are zero. The coefficents of mass(unit: $10^3 \times kg$) are: $M_{11}=179$; $M_{22}=170$; $M_{33}=161$; $M_{44}=152$; $M_{55}=143$; $M_{66}=134$; $M_{77}=125$; $M_{88}=116$; $M_{99}=107$; $M_{10,10}=98$; else $M_{ps}=0$ ($p \neq s$, $p,s=1,2,\cdots,N$). The damping ratios ξ_1 and ξ_2 are equal to 2% also the $\xi_3 \sim \xi_{10}$ are calculated by Eq.(15) and Eq.(12). In civil engineering, the stiffness degradation is considered as the result of seismic damage[18)] in both full-scale structures and small-scale models. In this example, it is assumed that, a damage occurs in element 3 (level 2-3) and element 8(level 7-8). If the damage is expressed as a

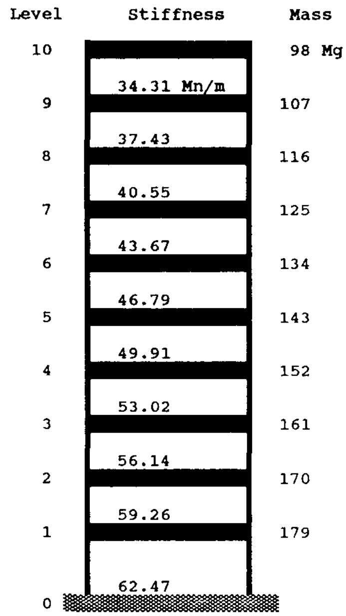

Fig. 1 Ten-storey shear building in numerical study.

Table 1 Natural frequencies and damping ratios

	BEFORE DAMAGE		AFTER DAMAGE	
N	f_p(Hz)	ξ_p	f_p(Hz)	ξ_p
1	0.500	0.020	0.480	0.054
2	1.326	0.020	1.264	0.037
3	2.151	0.027	2.097	0.035
4	2.934	0.035	2.838	0.051
5	3.653	0.042	3.500	0.060
6	4.292	0.049	4.076	0.068
7	4.836	0.054	4.791	0.078
8	5.272	0.059	5.109	0.083
9	5.590	0.063	5.288	0.085
10	5.787	0.065	5.633	0.091

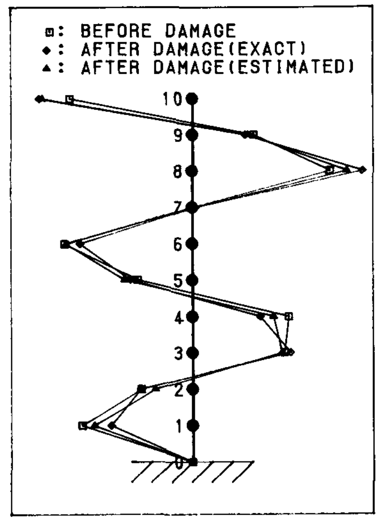

Fig. 2 The estimated 5th mode shape by using the 1～4th measured mode shapes.

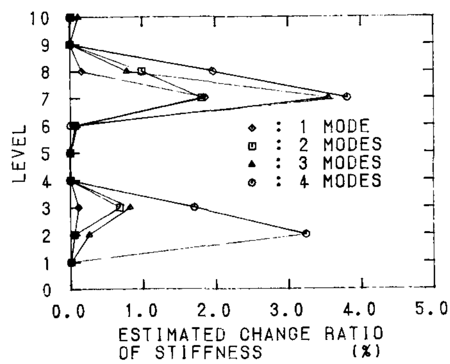

Fig. 3 Damage location detection by using the 1～4th measured modes.

Table 2 Identified coefficents of [K].

	BEFORE DAMAGE	AFTER DAMAGE		
NODE	EXACT	EXACT	IDENT.	E_{pp}
2	113.400	98.558	98.558	0.0
3	109.160	92.318	92.318	0.0
7	84.220	72.055	72.055	0.0
8	77.980	65.815	65.815	0.0

reduction in cross-section properties[9] in those elements, the coefficents of stiffness in nodes 2, 3, 7 and 8 corresponding to those elements will be degraded. Considering approximate 15% minor degradation of stiffness in those nodes, the coefficents of stiffnesses regarding to those nodes are therefore as follows: $k'_{22}=98.558$; $k'_{23}=-44.910$; $k'_{33}=92.318$; $k'_{77}=72.055$; $k'_{78}=-32.44$; $k'_{88}=65.815$. Also there is no stiffness change in other coefficents of stiffness. More, the coefficents regarding to Rayleigh damping(see Eq. (15))are: $\beta_1=1.31\times10^{-1}$ and $\beta_2=5.47\times10^{-3}$. For both before and after the damage, the calculated modal parameters are hereby taken as the measured ones. Moreover, all the calculations in this example are done by way of double precision.

In this example, the two-stage estimate method(two-stage least-square estimate procedure)is used and the 5~10th modes(unmeasured modes) are estimated respectively by using the 1~4th measured modes. The estimated 5th mode shape is hereby shown in Fig. 2 in which the estimated mode shape is closer to the real one comparing with the initial(before damage) mode shape. On the other hand, the estimated 5th natural frequency by using Eq. (23) is 3.642 Hz(exact: 3.500 Hz) which closes to the undamaged one(3.653 Hz). Also the calculated result shows that the undamaged natural frequencies can be taken as the extended ones in the two-stage estimate method. Furthermore, the calculated result shows that such conclusion is unavailable to mode shapes. All of these indicate once again the importance of mode shape in the identification procedure. The locations of damage are detected from Fig. 3 in which the nodes 2, 3, 7 and 8 are remarkable. Fig. 4 is the result of the detection of one-stage

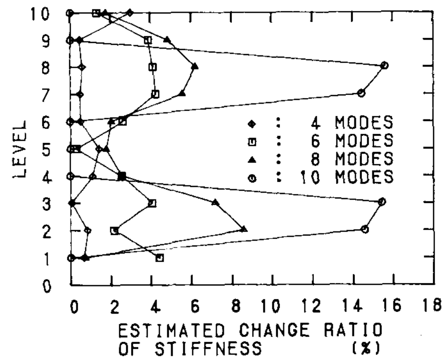

Fig. 4 One-stage estimate method on damage location detection for Rayleigh damping.

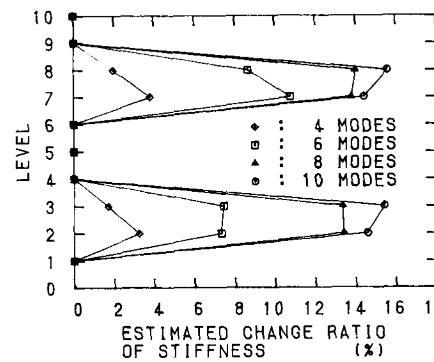

Fig. 5 Two-stage estimate method on damage location detection for Rayleigh damping.

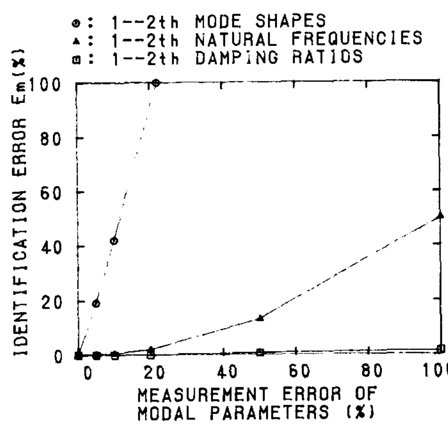

Fig. 6 The effect of measurement errors of natural frequency, mode shape and damping ratio on identification accuracy.

estimate method by using the 1~10th measured modes. In Fig.4, even though 8 modes are used, it seems there are stiffness changes in almost all the nodes. On the contrary, the result of the two-stage estimate method shows an excellent sensitivity in Fig.5. As the location of damage has been detected, the unknown coefficents α_K is therefore assumed on the nonzero stiffness coefficents associated with those detected nodes in global stiffness matrix(see Section 3). In the case of this example, the $[K(\alpha)]$ can be identified(see Eq.(26), N=10, NF=6, NF<2N) by using only the 1~2th measured modes. Paying attention to the identified diagonal stiffness coefficients, a percentage of identification error E_{pp} is defined as

$$E_{pp} = | \{(k_{pp})_{EXACT} - (k_{pp})_{IDENTIFIED}\} / (k_{pp})_{EXACT} | \times 100 \quad \text{..........(28)}$$

The identified result and error E_{pp} are shown in Table 2, in which the identified result is well satisfied. Substituting the identified [K] and [C], together with [M] into Eq.(10), the identified natural frequency and mode shape are then obtained. In this example, the error evaluation J (see Eq.(27)) is equal to 0.003.

The measurement error always exists in practical applications. In this paper, similar to the error of vibration testing data[19], the measurement error of a certain mode shape is defined: If a percentage of vibrational amplitude is increased/or decreased in one node, the same percentage is decreased/or increased in the neighbour nodes. Also, the error of natural frequencies(damping ratios) means that all the measured natural frequencies(damping ratios) are simultanously reduced(added) a percentage of respective frequency(damping ratio) itself. Furthermore, in order to examine the effect of measurement error on identification accuracy, a mean error of identification is defined as follows

$$E_m = (\Sigma E_{pp}) / ND \quad \text{..........(29)}$$

where the ND is the number of diagonal unknown stiffness coefficients among the NF(total number of unknown stiffness coefficents). From Fig.6, an effect of measurement error of damping ratio on identification accuracy is much smaller comparing with the natural frequency and mode shape. On the other hand, the measurement error of mode shape has a strong effect on the identification accuracy. Moreover, the calculated result shows that the detected location of damage remains unchanged even a 20% measurement error. This conclusion is similar to our previous study[6].

6. CONCLUSIONS

In this study, we first derived the basic formulations for structures with Rayleigh damping based on our previous method[6] for undamped structures. For small deterioration, a two-stage least-square estimate procedure was introduced to detect the location of damage. In the numerical example, we examined the effect of measurement error of modal parameters (natural frequency, damping ratio and mode shape) on identification accuracy for damped structures.

The main conclusions in this study are summed up as follows:

(1) A spatial domain, modal analysis method is presented to identify both the location and severity of damage for damped structures.

(2) The proposed two-stage least-square estimate procedure is effective to detect the loca-

tion of damage for damped structures, particularlly for a structure with minor damage.

(3) The measurement error of mode shape has a strong effect on identification accuracy. On the other hand, damping ratio has a comparatively weak one.

(4) As for the effect of measurement error on damage location detection, similar to the undamped case, within a certain range of measurement error, the detected location of damage remains unchanged.

Because of the complexities of real structures, sometimes it is difficult[21] to obtain the precise vibrational modes of structures. Also, as the dynamic behaviors of structures include not only normal mode shape(Rayleigh damping or proportional damping[14]) but also complex mode shape[8], an increasing attention is recently paid to nonproportional damping[20] by engineers. Therefore, it is expected to apply this method for real structures with nonproportional damping in future study.

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