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A Note on the Evaluation of Damage in Steel Structures under Cyclic Loading

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The concepts behind the different damage models currently used for evaluating damage in steel structures subjected to cyclic loading are analyzed. Suitable modifications are suggested to improve their performance and are verified against the existing damage models by using available test results. Using an elastic perfectly plastic model the effect of large deformations on different damage models are studied. Tentative values of structural parameters are suggested for cantilever box columns under a constant axial load and subjected to a variable transverse load.

1 Introduction

The quantification of damage in steel structures, due to severe earthquake ground motion has been the subject of extensive research. Many damage models based on a variety of parameters have been put forth[1)~5),7),8)]. Typical response parameters used are maximum displacement, cumulative plastic displacement and cumulative hysteretic energy dissipation. A damage index based on maximum ductility or maximum plastic deformation does not consider the effect of cycle number or the cumulative dissipated energy. Such an index performs well in case of cycles characterized by a single cycle of large displacement and others with small plastic displacements. On the other hand, a damage index based on total plastic dissipated energy does not consider its modality of achievement. It gives good results if the loading history is characterized by many cycles of large plastic deformations. Thus it appears that the two models are entirely different, each performing well under certain specific conditions. This led some authors to suggest damage functions based on both the maximum plastic displacement and the dissipated hysteretic energy. To these may be added another group of damage functions which consider the distribution of plastic cycles. Finally the order effects may be omitted in order to simplify the damage model. Based on the above discussion, we may conclude that a rational damage model should consider: (1)the maximum plastic deformation experienced, (2)the total dissipated hysteretic energy, (3)the distribution of cycles and, (4)the structural parameters.

Thus all the above damage models are approximations of the true damage, each stipulating certain conditions. The above models should not be considered to be different approaches, but should be looked upon as different approximations to the same problem. The evaluation of structural damage requires that the conditions under which such approximations can be made, and the consequences

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thereof, be thoroughly understood. This, when substantiated with experimental evidence, will lead to a formal damage theory.

In this paper the different damage models currently available are analyzed with reference to steel structures. The basic assumptions behind the damage models are examined to gain new insights regarding their performance under different loading histories. The structural constants β used in the Park and Ang model[1] and c used in the Krawinkler and Zhorei model[2),3] are evaluated from tests on cantilever steel box columns using different loading histories[6]. The Park and Ang model, the Krawinkler and Zhorei model and the Kato and Akiyama model[4),8] are compared for the case of response with a single large plastic displacement followed by constant amplitude cycling to understand the effect of the large plastic displacement.

2 Review of Previous Damage Models

A comprehensive review of the damage models currently available is given in Ref.[5] with reference to reinforced concrete structures. In this paper the models widely used for steel structures are examined.

2.1 Krawinkler and Zhorei model (low cycle fatigue model)

Krawinkler and Zhorei[2),3] proposed a damage model based on the concept of low cycle fatigue. They conducted experiments in which the specimens failed due to local buckling and crack propagation at weldments under the application of cyclic loads of constant amplitudes. Based on these experiments they suggested that the number of half-cycles to failure in steel structures, under cyclic loading of constant amplitude, can be expressed by a relation of the Coffin-Manson type as

$$N_{fi} = C^{-1}(\delta_i - \delta_y)^{-c} \tag{1}$$

where N_{fi} is the number of half-cycles to failure, δ_i is the constant deformation amplitude ($\delta_i \geq \delta_y$), δ_y is the yield deformation and C and c are structural constants.

For an arbitrary loading history, using Miner's rule of linear damage accumulation, we get

$$D = \sum_{i=1}^{N} \frac{1}{N_{fi}} = C \sum_{i=1}^{N} (\delta_i - \delta_y)^c \tag{2}$$

where D is the damage index ($D \geq 1$ indicates excessive damage or collapse) and N is the number of reversals.

Although c is a relatively stable parameter with suggested values between 1.6~1.8, C was found to be more difficult to determine and was of a random nature with considerable scatter. Some authors[7] evaluate C from the condition that $N_{fi} = 1$ for failure under monotonic loading. Thus,

$$D = \sum_{i=1}^{N} \left(\frac{\delta_i - \delta_y}{\delta_u - \delta_y} \right)^c \tag{3}$$

where δ_u is the ultimate displacement under monotonic loading.

This will give good results only if the deformations are comparable with the ultimate deformation under monotonic loading. Thus Krawinkler[3] kept C as it is even after normalizing by δ_y as

$$D = C\sum_{i=1}^{N}\left(\frac{\delta_i - \delta_y}{\delta_y}\right)^c \tag{4}$$

It is interesting to note that Krawinkler did not give any definite value for C. It seems more appropriate to model the structural characteristics by means of a hysteretic model and to use a value of unity for C.

Another difficulty associated with the model is that it can not distinguish the difference in hysteretic loops. Thus it cannot fully consider the different hysteretic models and is more suitable for elastic-perfectly plastic behaviour. Also under constant amplitude cycling, the shape and hence the area of hysteretic loops changes as cycling progresses due to deterioration in strength and stiffness. Further , in case of cycling with a nonzero mean deformation the model will not give accurate results. The above difficulties can be avoided using an appropriate hysteretic model and using a damage index of the form:

$$D = \sum_{i=1}^{N}\left(\frac{E_i}{E_{mon}}\right)^c \tag{5}$$

where E_i is the energy dissipated per half-cycle and E_{mon} is the energy dissipated under monotonic loading. For an elastic perfectly plastic model this is identical to equation (3).

The following may be concluded about the Krawinkler and Zhorei model.

1. It uses the total dissipated hysteretic energy as the damage parameter. In equation (3) this is implicite and in equation (5) it is explicite.

2. It does not give any special weightage to the maximum plastic deformation.

3. It does consider the distribution of plastic cycles when a value of $c > 1$ is used.

2.2 Kato and Akiyama model

Kato and Akiyama[4),8)] proposed a damage model which assumes failure to occur when the simple summation of plastic deformation equals two times that under monotonic loading:

$$D = \frac{1}{2}\sum_{i=1}^{N}\frac{\delta_i - \delta_y}{\delta_u - \delta_y} \tag{6}$$

The equation, although very simple, does not consider the distribution of cycles.

2.3 Park and Ang model

The Park and Ang damage model[1)] is a linear combination of the maximum deformation experienced, normalized by the ultimate deformation under monotonic loading and the total hysteretic energy dissipated, normalized by the factor $F_y\delta_u$:

$$D = \frac{\delta_{max}}{\delta_u} + \frac{\beta}{F_y\delta_u}\int dE \tag{7}$$

where δ_{max} is the maximum displacement experienced, δ_u is the ultimate displacement under monotonic loading, F_y is the yield strength, $\int dE$ is the dissipated hysteretic energy and β is a model parameter.

The model was originally presented without any theoretical basis but can be derived from the well known form of cumulative damage by making some assumptions and approximations. The cumulative damage is assumed to be expressed as

$$D = \sum_{i=1}^{N} \beta_i \left(\frac{E_i}{E_{mon}} \right) \tag{8}$$

This is nothing but the weighted sum of the dissipated energies with weight β_i, normalized by the energy dissipated under monotonic loading. In order to give more importance to the maximum plastic deformation, only that term is weighted by $(1+\beta)$ while the others are weighted by $\beta_i = \beta$ and so

$$D = \frac{E_{max}}{E_{mon}} + \frac{\beta}{E_{mon}} \sum_{i=1}^{N} E_i \tag{9}$$

where E_{max} is the energy dissipated in the half cycle corresponding to δ_{max}.

If an elastic perfectly plastic model is assumed, the following modified Park and Ang model is obtained

$$D = \frac{\delta_{max} - \delta_y}{\delta_u - \delta_y} + \frac{\beta}{F_y(\delta_u - \delta_y)} \int dE \tag{10}$$

and finally considering δ_y to be very small compared to δ_{max} and δ_u, one will get equation (7). The above derivation helps gain important insights regarding the model:

1. There is no basis for using the importance factor $(1+\beta)$ for the maximum deformation. In fact, since the damage index is normalized with respect to the case of monotonic loading, the maximum value of importance factor that can be used becomes unity, in order that D at collapse be less than or equal to one. This is the reason why equation (7) does not give a value of one for failure under monotonic loading.

2. Neglecting δ_y in comparison with δ_{max} and δ_u is justified only when δ_{max} is large. This is the reason why equation (7) does not give a value of zero under constant amplitude cycling at deformation amplitude δ_y.

3. By using $c = 1$, the model gives equal weight to all the other plastic cycles except the one producing δ_{max}. But experimental testing[3] shows that larger plastic deformations should be given more importance than smaller ones. Hence a value of $c > 1$ would be more appropriate.

3 Proposed Comprehensive Damage Model

Based on the above discussion, it is proposed to modify the Park and Ang model further as

$$D = (1-\beta) \left(\frac{\delta_{max} - \delta_y}{\delta_u - \delta_y} \right)^c + \beta \sum_{i=1}^{N} \left(\frac{E_i}{F_y(\delta_u - \delta_y)} \right)^c \tag{11}$$

This equation has two parameters c and β which have to be evaluated from experiments. It is based on the assumptions that,

1. The number of half-cycles to failure under constant amplitude cycling, N_{fi}, can be expressed as

$$N_{fi} = \left(\frac{E_{mon}}{E_i}\right)^c \tag{12}$$

2. The weighting or importance factor for maximum deformation is one and β for the other deformations. Also for simplicity, E_{max} is expressed as $F_y(\delta_{max} - \delta_y)$;

3. With the above two modifications Miner's rule of linear damage accumulation can be used:

$$D = \sum_{i=1}^{N} \frac{1}{N_{fi}} \tag{13}$$

The above damage model could be used in conjunction with a hysteretic model with loading, unloading and reloading rules and rules for considering deterioration of strength, stiffness and hysteretic energy dissipation capacity. In this way it will be suitable for any type of structure and can be used to evaluate damage using experimental results or by analytical modelling.

4 Comparative Study

In order to determine values for β and c, the results of tests conducted by Usami et al[6] on six identical cantilever box column specimens using different loading histories were used. Each specimen was subjected to a prescribed horizontal displacement history under a constant axial load of 0.2 times the squash load. Histories with groups of n displacement cycles were imposed on the specimen, each group having an increased cycle amplitude. n values of 1, 3, 5, 8 and monotonic and constant amplitude cycling were used(Fig. 1).

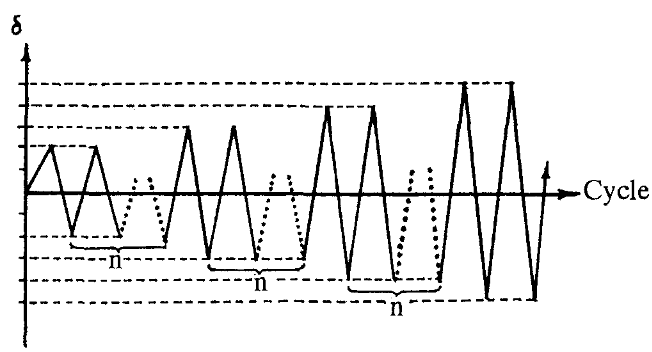


Fig.1 Variable Displacement Amplitude (Ref.[6])

The specimen was assumed to have failed when the envelope strength dropped to the theoretical yield strength H_y, the horizontal force corresponding to the first yield or the onset of local buckling[6].

The collapse thus defined may not be the real collapse of the specimen but is intended to represent a damage state at which the damage index is expected to become unity. The maximum displacement δ_{max} and energy dissipated in each test non-dimensionalized with respect to the theoretical value of elastic energy, and as calculated by equation (16) given later are shown in Table 1.

Table 1 Maximum displacement and energy dissipated

Test	δ_{max} mm	$\hat{E}_f$ eqn.(16)	$\hat{E}_f$ expt.
monotonic	121.6	49	51
n = 1	74.8	391	283
n = 3	66.2	457	430
n = 5	63.6	476	556
n = 8	58.3	514	597
constant amplitude	44.4	615	487
Note: δ_y = 6.56 mm; H_y = 82.68 KN			

Using equation (10) and assuming $D = 1$ at the above mentioned collapse point we get

$$\beta = \frac{(\delta_u - \delta_{max})H_y}{\int dE} \tag{14}$$

The value of β was evaluated in two ways–1)using the actual values of energy dissipated up to the assumed collapse point and 2)using the equation proposed by Usami et al[6] as

$$\int dE = \frac{1}{2}H_y\delta_y\hat{E}_f \tag{15}$$

where the energy dissipated nondimensionalized with respect to the theoretical value of elastic energy, $\hat{E}_f$, is given by

$$\hat{E}_f = -48.0\frac{\delta_{max}}{\delta_y} + 937 \tag{16}$$

The results are shown in Table 2 along with the mean value and the coefficient of variation (COV).

Table 2 β values for equation (10)

n	β(eqn.16)	β(actual)
1	0.0364	0.0503
3	0.0372	0.0394
5	0.0374	0.0320
8	0.0377	0.0325
constant amplitude	0.0383	0.0484
mean	0.0374	0.0405
COV	0.0186	0.2123

Also, using equation (3) and equation (5), the values of c were found by trial and error, so as to give $D = 1$ at the collapse point. These are shown in Table 3 for each loading history along with the mean value and COV. The c values obtained using equation (5) are higher than those obtained using equation (3) because the actual hysteretic behaviour is significantly different from the elastic-perfectly-plastic behaviour.

Table 3 c values for equations (3) and (5)

n	c(eqn.3)	c(eqn.5)
1	2.2	4.0
3	2.9	3.7
5	3.4	3.7
8	3.1	3.2
constant amplitude	3.0	3.1
mean	2.9	3.5
COV	0.15	0.11

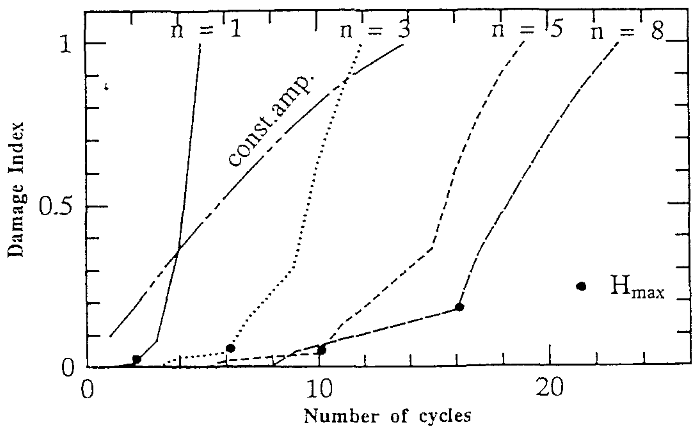
From Table 2 it can be observed that the β values are high for $n = 1$ and decrease as n increases. This is so because not much energy was dissipated for smaller values of n. For the same reason it can be seen from Table 3 that c values are higher for $n = 1$ and decrease as n increases, thereby trying to account for the deformation damage by increasing the importance of the higher displacements. Thus we can expect that the proposed equation, i.e., equation (11) will be able to account for the damage caused by dissipation of hysteretic energy and by excessive deformation in a better way. Based on the limited test results that are available, this was found to be true.

Using a value of 2 for c, so as to give more importance to the larger plastic deformations, and using the above mentioned test data, values of β were obtained so as to make the damage index given by the proposed damage index [equation (11)] unity at the presumed collapse point. These are shown in Table 4 along with the mean value and COV. It can be seen that the COV is less than that for β(actual) in Table 2.

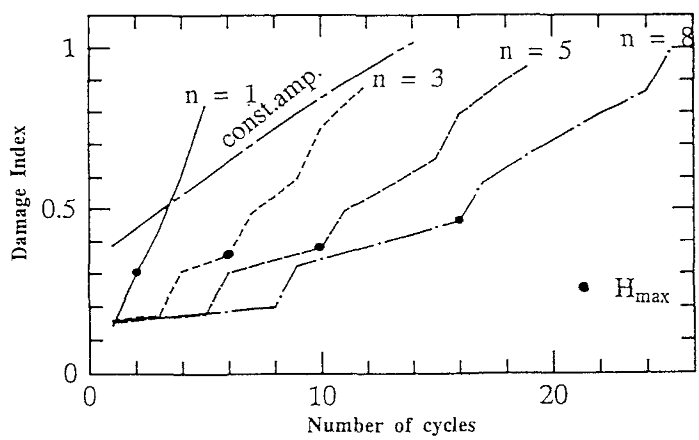
Table 4 β values for equation (11)

n	β
1	0.14
3	0.11
5	0.09
8	0.10
constant amplitude	0.12
mean	0.11
COV	0.17

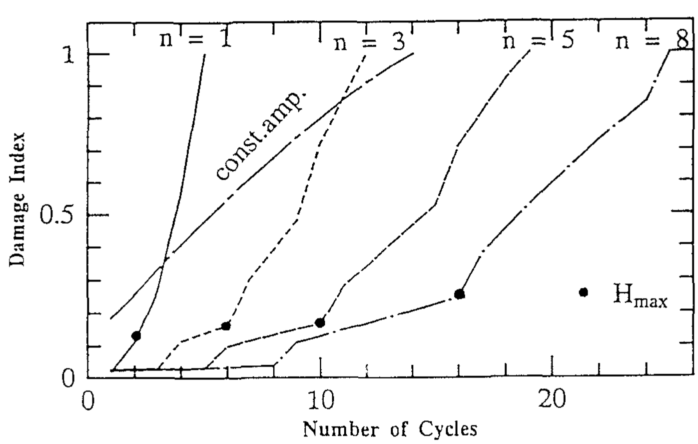
The variation of damage index with cycle number was studied using the actual values of dissipated energies obtained by tests conducted by Usami et al[6]. In Fig.2(a) the damage index was calculated using equation (5) and the c values given in Table 3. In Fig.2(b) the modified Park and Ang model was used [equation (10)], along with the β values given in Table 2. In Fig.2(c) the damage index was calculated using the proposed equation (11) with $c = 2$ and the β values given in Table 4. In Fig.2(a) the damage increases rather slowly in the beginning, i.e., up to the maximum horizontal load, H_{max}, and later very fast, showing the effect of higher values of c. As reported in Ref.[6], local plate buckling becomes visible at the maximum horizontal load, H_{max}. Thus, the increase in the damage index after H_{max} signifies the gradual deterioration of the test specimen's strength due to local buckling plus $P - \Delta$ effect. In Figs.2(b) and 2(c), the effect of considering deformation damage can be seen in the form of steps. Especially for $n = 8$, the effect of giving more importance to the maximum deformation and less importance to the dissipated hysteretic energy is more prominant.



(a) Low cycle fatigue model (Eq.5)



(b) Modified Park and Ang model (Eq.10)



(c) Proposed model (Eq.11)

Fig.2 Variation of damage index with number of cycles

The steep rises are due to the stepping up of deformation levels, while the cycling itself produces little damage. By comparing the three figures, one will realize that the proposed model provides the intermediate values of the damage index regardless of the loading programs (i.e., values of n). The validity of the model must be substantiated by tests like the hybrid (or pseudo-dynamic) test.

The modified Park and Ang model with $\beta = 0.04$, low cycle fatigue model with $c = 3$, Kato and Akiyama model and the proposed model were also compared for a case with a large plastic displacement $(\delta_i - \delta_y)$ (loop 0-1-2-3 in Fig.3), followed by cycling at a constant plastic deformation amplitude $m(\delta_i - \delta_y)$ (loop 3-4-5-6-7-2-3 in Fig 3). m values of 0.5, 1.0 and 2.0 were used, the last being symmetric cycling at constant deformation amplitude δ_i. An elastic perfectly plastic model with $\delta_u/\delta_y = 9$ was assumed. The number of cycles to failure, $N_f/2$ (excluding the initial cycle 0-1-2-3) was plotted against the plastic deformation amplitude as shown in Fig. 4.

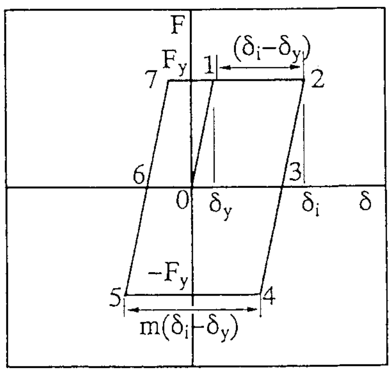


Fig.3 Assumed load–deformation hysteretic curves

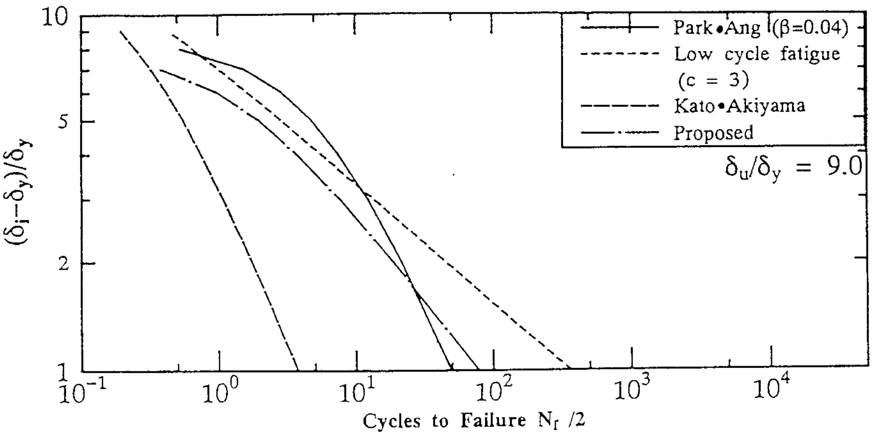
For a given value of m, the Park and Ang model and the low-cycle fatigue model differ more for smaller deformations than for larger deformations because the effect of deformation damage considered in the Park and Ang model is less in the regions of small plastic deformations. When a value of c greater than unity is used, the low-cycle fatigue model gives more importance to the larger displacements, thereby tending closer to the Park and Ang model.

For small values of m (i.e., $m \leq 1$), since the low cycle fatigue model does not consider the effect of maximum deformations, it differs more from the Park and Ang model and from the proposed model.

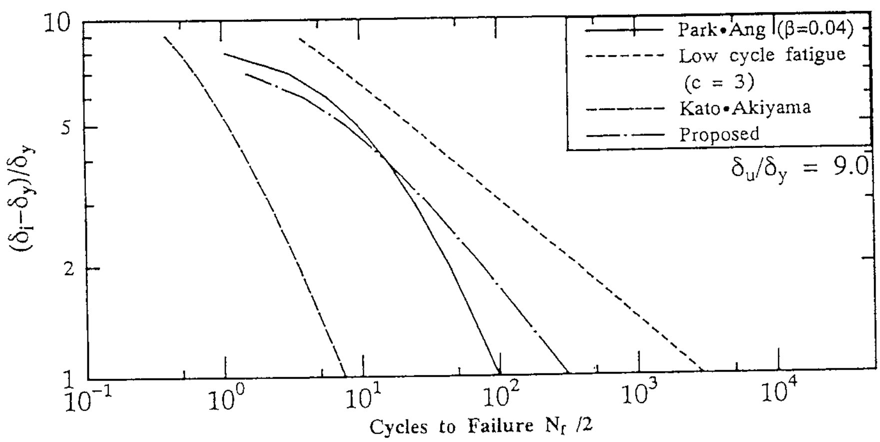
The proposed model uses a value of 2 for c and also considers the deformation damage. Thus it lies closer to the Park and Ang model for larger deformations, and tends towards the low-cycle fatigue model as the deformation decreases. This effect is more pronounced for smaller values of m.

5 Conclusions

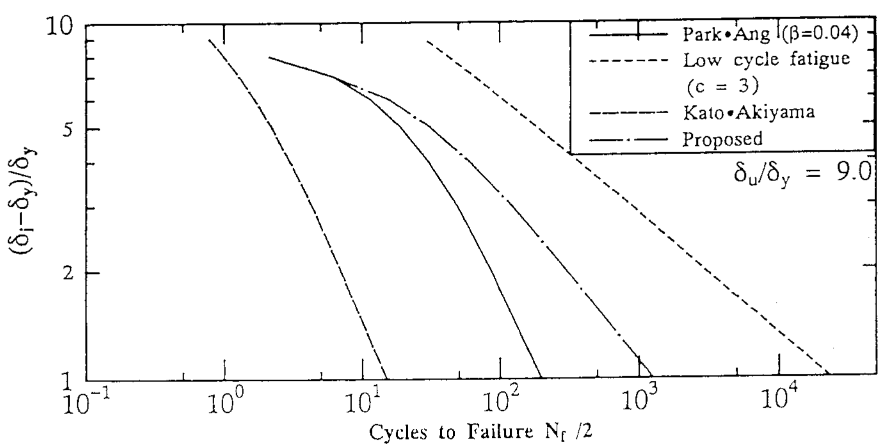
The following may be concluded about damage in steel structures:



(a) m = 2.0



(b) m = 1.0



(c) m = 0.5

Fig.4 Comparison of the numbers of cycles to failure

1. The assumptions behind the Park and Ang model[1] were analyzed. It was found that each assumption introduces certain types of errors and their combined effect is to make the determination of β very difficult. The model does not consider the distribution of the cycles which is very important in the evaluation of damage.

2. The Krawinkler and Zhorei model[2] (low cycle fatigue model) performs well for c values higher than unity and in cases where the cycles are of nearly the same amplitude. But it does not consider deformation damage which becomes important in case of a single large cycle and other cycles of much smaller amplitudes (i.e., $m \leq 1.0$ in Fig.4)

3. The new model [equation (11)] combining the advantages of the above two models was found to give better results within the limited test results available at Nagoya University. But the small number of test results with which it was verified makes it difficult to specify final values for the parameters. Thus, it will have to be tested further by carrying out experiments. The hybrid method of testing appears very attractive for the purpose.

In order to develope a formal damage theory by which structural damage can be quantified, the necessary concepts must be properly understood. Since damage depends on a number of parameters and since a complicated damage function would be difficult to use, certain simplifying assumptions and approximations will have to be made. But they should be based on proper logic and substantiated with experimental findings. Empirical formulations based on a small number of test data are of little use because of the wide range of structural systems used in practice. Thus efforts should be directed towards understanding the basic mechanism of damage and its relationship with the damage parameters.

References

1) Park,Y.J. and Ang,H.S., "Mechanistic Seismic Damage Model for Reinforced Concrete," J. Struc. Engg., Proc. ASCE, Vol.111, No.4, 1985.

2) Krawinkler,H. and Zhorei,M.,"Cumulative Damage in Steel Structures Subjected to Earthquake Ground Motions," *Computers and Structures,* Vol.16,No.1-4, 1983.

3) Krawinkler,H. and Nassar,A.A., "Seismic Design based on Ductility and Cumulative Damage Demands and Capacities", in "Nonlinear Seismic Analysis and Design of Reinforced Concrete Buildings", ed.by Fajfar,P. and Krawinkler,H., Elsevier, 1992.

4) Akiyama,H.,"Earthquake Resistant Limit-State Design of Buildings," University of Tokyo Press, 1985.

5) Chung,Y.S., Meyer,C. and Shinozuka,M., "Seismic Damage Assessment of Reinforced Concrete Members," Technical Report NCEER-87-0022, National Center for Earthquake Engineering Research, State University of New York at Baffalo, October, 1987.

6) Usami,T.,Banno,S.,Zetsu,H. and Aoki,T.,"An Experimental Study on Elasto-Plastic Behaviour of Compression Members under Cyclic Loading–Effect of Loading Program," *Journal of Structural Engineering,JSCE,* Vol.39A,1993 (in Japanese).

7) Cosenza,E. and Manfredi,G.,"Seismic Analysis of Degrading Models by means of Damage Functions Concept", in "Nonlinear Seismic Analysis and Design of Reinforced Concrete Buildings",

ed. by Fajfar,P. and Krawinkler,H., Elsevier, 1992.
8) Kato,B. and Akiyama,H.,"Aseismic Limit Design of Steel Rigid Frames," Proc. Architectural Institute of Japan, No.237, Nov., 1975.

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