JOURNAL OF STRUCTURAL ENGINEERING Vol.39A (March 1993) JSCE

Fatigue Life Analysis on Welded Joints under Various Spectrum Loadings

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Abstract

The Miner's rule or equivalent stress range concept is well accepted for fatigue design procedures of weldments under variable amplitude loading in various design recommendations. However, their applicability is still under discussion when fatigue life in long life region at low stress range level is to be evaluated. Stresses observed in highway and railroad bridges often fall in this range. A comparative study of fatigue crack propagation analysis on non-load carrying cruciform joints is carried out using fracture mechanics concept in order to clarify the effect of various variable amplitude spectrum loadings. Two types of analysis are made for both constant amplitude and variable amplitude loading conditions. First, fixed values of initial conditions are selected to investigate the effect of loading patterns. Then, the variation of the initial conditions is also considered by Monte Carlo simulation technique. The lower and upper bounds of the fatigue crack propagation life are observed under constant and various variable amplitude loading patterns. Also, comparison with the existing fatigue design curve of variable amplitude loading is carried out.

1. Introduction

Field measurements of stress ranges of highway and railroad bridges revealed that the stress ranges are normally small compared with the stresses due to static design load or constant amplitude fatigue limit [1]. If one estimates the fatigue life of the welded joints due to service loading by using Miner's rule or an equivalent stress range concept, it will normally result in very long fatigue life. However, in some bridges on main highways, stress ranges often exceed the constant amplitude fatigue limit due to frequent overloadings or due to heavy truck traffics. Some fatigue cracks have also been actually observed in these bridges [2].

In various fatigue design recommendations, the modified Miner's rule or the equivalent stress range concept is used in order to estimate the fatigue life of weldments subjected to variable amplitude (VA) loadings. Both of them give the same results when (a) the basic constant amplitude (CA) S-N diagrams are linear on log-log scale, (b) no constant amplitude fatigue limit is considered and, (c) the slope of S-N diagrams, k, is used to compute the equivalent stresses. The slope of S-N diagram

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for welded details normally has a fixed value of 3, hence the equivalent stress range is called the root-mean-cube (RMC) stress. The procedure is rather simple and easy to apply and, for this reason, it is widely used in various design recommendations [3),4)].

When the welded joints are subjected to large number of stress ranges at the lower stress range level, the specifications require one of the following three fatigue life assessments: (a) using modified Miner's rule, that is, using linearly extended S-N diagrams for VA loading conditions, (b) using modified slope (normally $k' = k + 2$) of S-N diagram for VA loading conditions, or (c) using linear S-N diagram, but introducing the cut-off limit, below which any stress ranges are neglected [3),4),5)]. The procedures are shown schematically in Fig. 1. Normally the latter two procedures give better

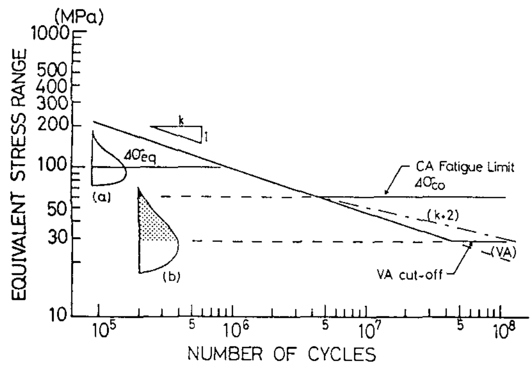


Fig. 1 Fatigue Life Assessments for Fatigue Design under VA loading

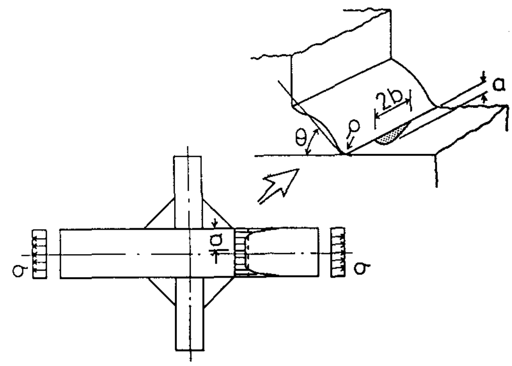


Fig. 2 Model of Fatigue Crack Emanating from Weld Toe of the Non-Load Carrying Fillet Welded Specimens

estimation of fatigue life, although they are somewhat more complicated than the first one. However, the application of these procedures is yet to be clarified, when the welded joints are subjected to extremely large number of stress cycles, and only a small portion of these cycles exceed the constant amplitude fatigue limit. Fatigue tests of weldments are underway [6)] in the U. S. but they require unduly long time for the test.

In this paper, the fracture mechanics analysis of fatigue crack propagation are carried out for both CA and VA loading conditions. Since the fracture mechanics is rather sensitive to the given initial conditions, such as initial crack size, crack shape, weld geometry and so on, a set of initial conditions is determined for CA loading from the comparison between analytical results and fatigue test data [6)]. Then, the same initial conditions are used for VA loading condition with various loading spectra. The same analysis is repeated considering the variation of initial conditions, so that effects of these parameters can be evaluated. The Monte Carlo simulation technique is used to give initial conditions under certain probability density functions to the analysis [7)].

2. Fracture Mechanics Analysis of Fatigue Crack Propagation

It has been widely known that the fatigue crack propagation of welded detail can be carried out by the linear elastic fracture mechanics approach. It consists of two basic equations.

For the fatigue crack growth rates of a given material, the following semi-empirical equation can be employed.

$$\frac{da}{dN} = C(\Delta K^m - \Delta K_{th}^m) \qquad \Delta K \geq \Delta K_{th} \tag{1}$$

and

$$\frac{da}{dN} = 0 \qquad \Delta K < \Delta K_{th} \tag{2}$$

where da/dN is the crack growth rate in $m/cycle$, ΔK is the stress intensity factor range in $MPa\sqrt{m}$ and ΔK_{th} is the threshold value of stress intensity factor range, below which no crack growth takes place. Throughout the analysis, fixed values of $m = 2.9$, $C = 9.69 * 10^{-12}\ m/(MPa\ \sqrt{m})^{2.9}$, and $\Delta K_{th} = 2.5\ MPa\sqrt{m}$ are used. They correspond approximately to the mean value of the fatigue crack growth rates for various structural steels obtained by the National Research Institute for Metals (NRIM) [17]. The values are obtained for CA loading, but it is assumed that they can also be applied to VA loading [8].

For the stress intensity factor range, the following expression is conveniently used to express ΔK for a crack emanating from the fillet weld toe, for example as shown in Fig. 2.

$$\Delta K = F_S \cdot F_E \cdot F_W \cdot F_G \cdot \Delta\sigma\sqrt{\pi\ a} \tag{3}$$

where F_S is free surface correction factor, F_E is elliptical crack shape correction factor, F_W is finite plate width (or thickness) correction factor, F_G is geometry correction factor, $\Delta\sigma$ is the applied nominal stress range in MPa, and a is the given crack size in m [9]. Known correction factors can be used to express F_S, F_E and F_W. For example, for a semi-elliptical crack emanating from fillet weld toe, they are:

$$F_S = 1.12 - 0.12\ a/b \tag{4}$$

$$F_E = 1/E(k)\ , \qquad E(k) = [1 + 1.464(a/b)^{1.65}]^{1/2} \tag{5}$$

and

$$F_W = \sqrt{sec\ (\pi\ a/2w)}\ (1 - 0.025(a/w)^2\ +\ 0.06(a/w)^4) \tag{6}$$

For geometry correction factor of the non-load carrying fillet welded specimens, two dimensional finite element analysis is carried out to obtain the stress distribution along the line of the expected crack propagation. Then F_G values are computed using the procedure mentioned in ref. 9.

One can obtain a first-order differential equation by substituting Eq. 3 into Eq. 1. If the applied stress is constant throughout the fatigue life, the resulting equation can be numerically integrated by Simpson's rule, and fatigue crack propagation life, N_p, can be obtained. If the stress range varies during the fatigue life, the first-order differential equation must be solved for the given crack size, for example, by Runge-Kutta method. A computer program, VARFAT, was developed based on CRACK program [16], which can be used to predict fatigue crack propagation behavior of various weldments subjected to VA loading. If the VA loading is fixed to a certain value, it becomes the crack propagation behavior due to CA loading.

3. Fatigue Life under CA and VA Loading

3.1 Fatigue Crack Growth under CA Loading

In the present study, fatigue crack propagation behavior of non- load carrying fillet welded specimens is analyzed as an example. A large number of fatigue test data under CA loading are available, and their fatigue crack propagation behavior is relatively well known [7),10),11]. Moreover, in recent years, fatigue tests under VA loading are also underway in the U. S. [6] The analytical results can be compared with these test data.

First, fatigue crack propagation analysis is carried out for a simple model under CA loading. The results are plotted in Fig. 3 with fatigue test results of non-load carrying fillet welded specimens tested by Klippstein *et al.*[6] and Albrecht *et al.*[12] In this analysis the weld profile is assumed by flank angle $\theta = 45°$ and weld toe radius $\rho = 0\ mm$. The fatigue crack propagation life, N_p, is computed for combination of initial crack size a_o and crack shape a/b, and compared with the test results. As shown in Fig. 3 for the fixed value of $a/b = 1/2$, the N_p with $a_o = 0.05\ mm$ is relatively close to mean fatigue life of automatically welded specimens, and that with $a_o = 0.1\ mm$ corresponds to the manually welded specimens. The initial crack size and the crack shape obtained here are relative values, while actual initial crack size is normally hard to define, if not impossible. However, they can be further utilized, when prediction of fatigue life is made for the same specimens subjected to VA loading.

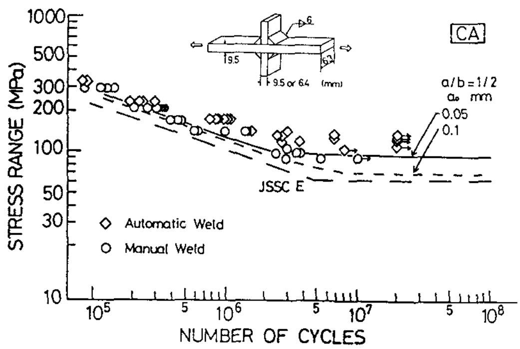


Fig. 3 Comparison between Analytical Result of N_p and Fatigue Test Result under CA Loading

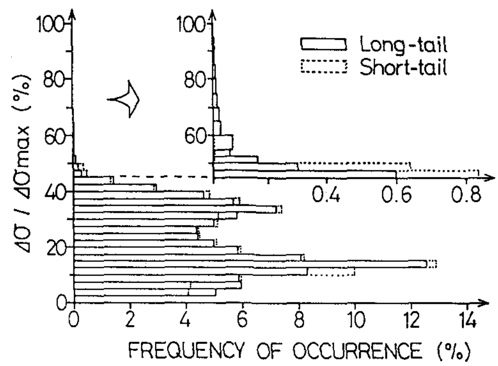


Fig. 4 VA Load Spectra Used in the Fatigue Test

3.2 Fatigue Crack Growth under VA Loading

As mentioned above, Klippstein *et al.* and Albrecht *et al.* are carrying out variable amplitude fatigue tests on the non-load carrying fillet welded specimens. Three types of stress range spectra are used in their fatigue tests; namely (a) long tail (LT), (b) short tail (ST), and c) medium tail. The root-mean-cube stress for these three spectra are almost identical. These stress range spectra represent the actual service stress observed in the highway bridges in the U. S. The N_p is computed for the LT and ST stress range spectra, as shown in Fig. 4, for the crack emanating from fillet weld toe. The initial conditions are $a_o = 0.05\ mm$, $a/b = 1/2$, $\rho = 0\ mm$, and $\theta = 45°$. The analytical results are plotted in Fig. 5. The ordinate is equivalent stress range, $\sigma_{r,eq}$. In this case it is the root-mean- cube (RMC) stress range $\sigma_{r,RMC}$, since $k = 3$ is used in calculating the equivalent stress range, as follows.

$$\Delta\sigma_{RMC} = \left(\frac{\Sigma\ n_i \Delta\sigma_i^3}{N} \right)^{1/3} \tag{7}$$

Also plotted in the Fig. 5 are the allowable stress ranges for CA and VA loadings of category E detail specified by the Fatigue Design Recommendation of Japanese Society of Steel Construction (JSSC) [3] and the VA loading test data of fillet welded specimens with automatic weld obtained by Klippstein *et al.*[6]

The analytical results show that in shorter fatigue life region, the two stress range spectra exhibit almost identical N_p, and they are also comparable to N_p for CA loading. In this region, the simple equivalent stress range concept can predict well the VA loading conditions. It is also observed that the difference in analytical N_p between the LT and the ST stress spectra is negligibly small in shorter fatigue life region, and that the LT shows less fatigue strength than that of the ST in the longer fatigue life region, although the difference is small. It is due to the fact that in this analysis the large stress ranges in the LT drive the fatigue crack faster, which yields the lower fatigue strength. $\Delta\sigma_{RMC}$ for the LT and the ST stress spectra are almost the same.

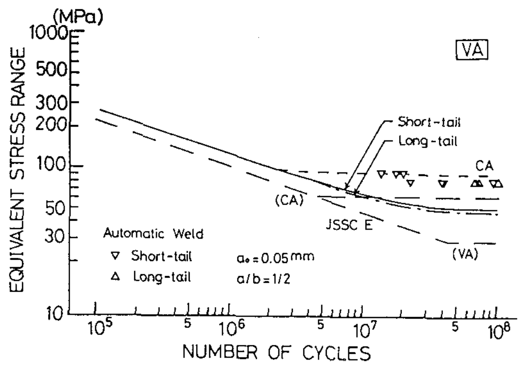


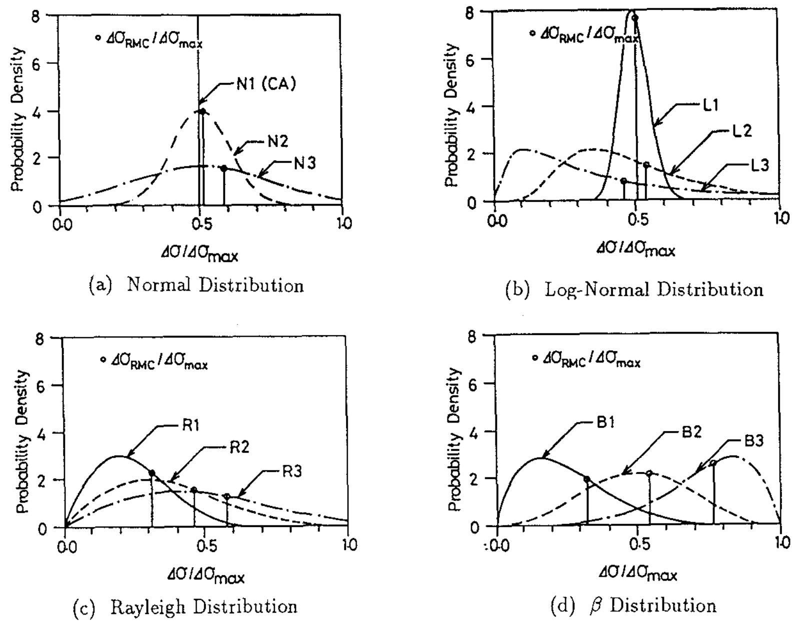
Fig. 5 Fatigue Crack Propagation Analysis for VA Loading

In the long life region, that is over about five million cycles, the computed S - N_p diagrams turn to parallel to the abscissa, and eventually yield to infinite N_p. The fatigue test data for VA loading seems to exhibit the same tendency. However, the computed N_p is much shorter, or computed fatigue strength at long life region is much lower than the test data. It may be due to the dominant crack initiation life at lower stress range region in early stage, or interaction effect in high-low sequence in VA loading in the test. Both of them are not considered in the analysis. The latter reason may be worthwhile to be further investigated, because the LT stress spectrum, which contains large stress ranges in the tail, exhibited longer fatigue life than that of the ST stress spectrum.

3.3 Fatigue Crack Growth under Various Spectrum Loadings

Steel structures are generally subjected to variable amplitude loading of various patterns. In order to see the effect of various spectrum loadings, four types of probability density functions are selected in the analysis, as shown in Fig. 6. They are: (a) Normal distribution, (b) Log-Normal distribution, (c) Rayleigh distribution and (d) β-distribution. The probability density functions are truncated by the random variable from 0 to 1 which represents the value of stress range divided by maximum stress range. In each distribution three different values of parameters are selected, so that they represent various loading patterns (see Appendix). For example, one can see the effect of stress variation around the mean value by changing the coefficient of variance in the Normal distribution. The Log-Normal distribution is used to represent the stress spectra in the railroad bridges in Japan, while Rayleigh distribution is used to express the stress spectra in the highway bridges in the U. S. The β-distribution is selected to emphasize the effect of skewness of the distribution. The N_p is computed for all stress spectra for the fixed values of $a_o = 0.05\ mm$, $a/b = 1/2$, $\rho = 0\ mm$ and $\theta = 45°$. The results are shown in Fig. 7. The allowable stress ranges of JSSC category E detail are also plotted for the comparison.

In general all stress spectra give the identical N_p in shorter fatigue life region, for example, less than two million cycles. In this region most of the stress range cycles in the stress spectra gives $\Delta\ K$ above $\Delta\ K_{th}$, and the equivalent stress range concept gives a good estimation of N_p under VA loading.



(a) Normal Distribution
(b) Log-Normal Distribution
(c) Rayleigh Distribution
(d) β Distribution

Fig. 6 Stress Range Spectra to Be Used in the Comparative Analysis

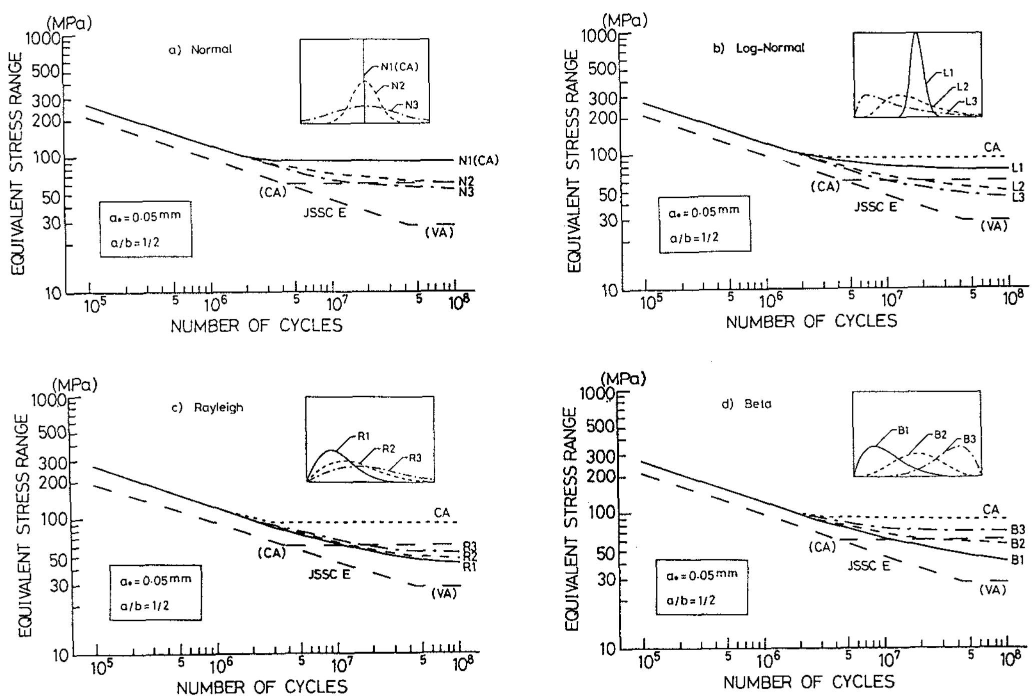


Fig. 7 Fatigue Crack Propagation Life Due to Different Stress Spectra

On lower RMC stress range level, or in longer fatigue life region, the computed N_p differs depending on the stress spectra. When the most stress ranges are in narrow band, such as N2 in the Normal distribution and L1 in Log-Normal distribution, the computed N_p is close to the computed run-out level due to CA loading. On the other hand, when the stress spectra contain a large number of lower stress ranges, and the long tail in high stress range region such as L3 in Log-Normal distribution, R1 in Rayleigh distribution and B1 in β distribution, the computed N_p is closer to what is given by the linear extrapolation of CA S-N diagram. Then the N_p tends to shift toward the infinite life. At the higher RMC stress level, most of the stress ranges above the CA fatigue life. But at the lower RMC stress level, N2, L3, R1 and B1 give the long tails in the high stress ranges which are above CA fatigue limit to cause a quick fatigue crack growth in the analysis.

4. Effect of Variation of Initial Conditions on Fatigue Life

4.1 Variation of Initial Conditions and Its Analysis

Fatigue crack propagation is largely affected by the weld toe geometry, such as ρ, θ, the assumed initial crack size, a_o, and crack shape, a/b, during its propagation. In order to evaluate the effect of these parameters on the N_p under CA and VA loadings, the N_p is computed for all possible combinations of the initial conditions. In the analysis, 1000 sets of possible combination of the initial conditions are determined from given probability distribution functions for ρ, θ ,a_o and a/b by Monte Carlo simulation technique. These probability functions are determined from measured data on the non-load carrying fillet welded specimens [7]. Then, the N_p is computed 1000 times by using the program VARFAT at a given CA or VA loading. The detail of this analysis is also described elsewhere [7].

4.2. Variation of Fatigue Life for CA Loading

The computed N_p is first plotted in Fig. 8, where the distribution of the computed N_p are shown at three stress range levels, 180, 90 and 60 MPa. At the other stress range levels, only the median N_p and the upper and the lower bounds corresponding to 95 % confidence limits are plotted. At higher stress range level, for example at $\Delta\sigma$ = 180 MPa, all the combinations of initial condition give finite life, while at the lower stress range level some of them yield to infinite life, which is plotted at 10^8 cycles. It is noted that the scatter of the N_p is larger at lower stress range levels, in which Δ K_{th} effect becomes dominant.

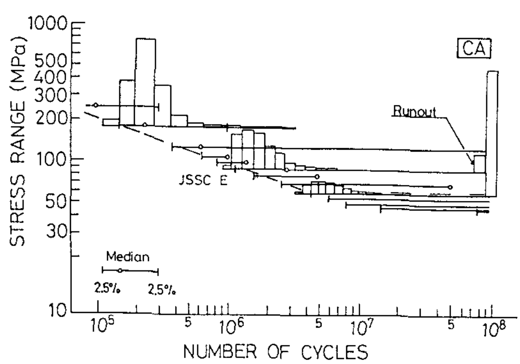


Fig. 8 Histogram and Median Values of N_p under CA Loading Considering Variation of Initial Conditions

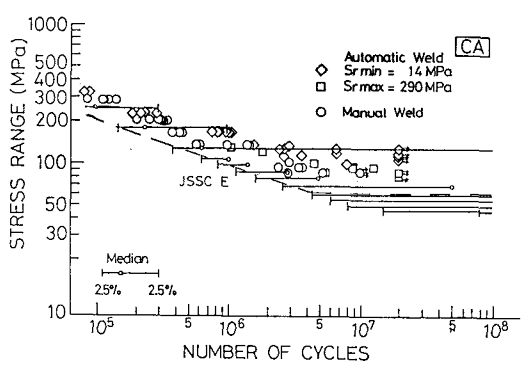


Fig. 9 Comparison between Analytical Result and Fatigue Test Data under CA Loading

The analytical results are also plotted in Fig. 9 with the CA test data for the comparison. Since the combinations of the initial conditions give a large scatter of the computed N_p, the test data generally fall in the scatter bands. It should be noted that the lower bound of N_p is close to the S-N design curve of category E detail of JSSC fatigue design recommendation. However, the computed fatigue limit is about 45 MPa, which is lower than the specified value of 62 MPa by JSSC. It implies that there is a slight possibility of fatigue cracking at lower stress range level, when poor fillet weld yields unfavorable combinations of the initial conditions, as assumed in the analysis.

4.3 Variation of Fatigue Life for VA Loading Used in the Test

The analysis on variation of fatigue life is also extended to the VA fatigue test, and the results are plotted in Fig. 10. Only the result on short tail (ST) stress distribution is shown, but the long tail (LT) stress distribution exhibits almost the same result. The median values, lower and upper bounds of computed N_p are shown for comparison. The test data fall within the scatter band. The histogram of the computed N_p distribute continuously from 2.5% lower bound to run out value. It is also noted that on the RMC stress level of about 29.6 MPa, the lower bound value seems to be coincided with the cut-off line at 29 MPa of VA design curve of JSSC. The comparison between computed lower bounds of fatigue life, which takes account of the variation in initial conditions, and JSSC design curve shows that they are very close to.

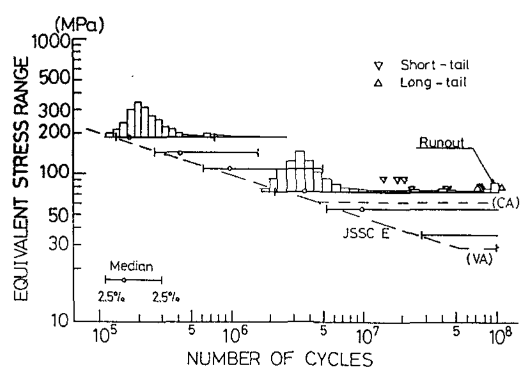


Fig. 10 Analytical Result under Test VA Loading by Considering the Variation of Initial Conditions

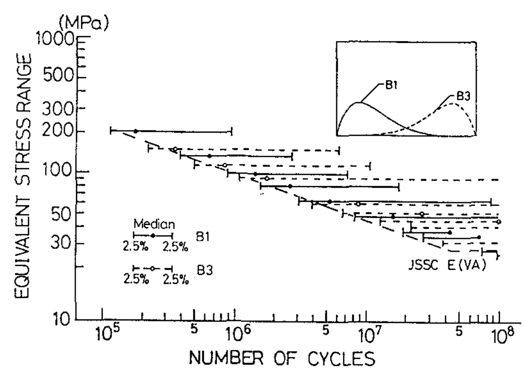


Fig. 11 Analysis of N_p under VA Stress Range Spectrum of β Distribution

4.4 Variation of Fatigue Life for β Distribution Spectrum of VA Loading

The same analysis is carried out for the stress spectrum expressed by β distribution. It is because the analytical results as shown in Fig. 7, indicate that the stress spectrum B1 gives the lowest fatigue strength, while the stress spectrum B3 gives the highest fatigue strength under VA loading. The analytical result is shown in Fig. 11. One can see that when we compare the median values of N_p for spectra B1 and B3, it gives almost the same fatigue crack propagation life in shorter fatigue life region. In longer N_p region over 5 million cycles, their N_p medians begin to separate. It also shows that the spectrum B3 gives the relatively wider scatter of N_p than that of spectrum B1. The analytical results are also compared with the allowable design curve for Category E detail specified by JSSC. The lower bounds of computed N_p for spectra B1 and B3 are below the design curve in shorter fatigue life region, for example, less than $5 * 10^6$ cycles. In the longer fatigue region, the lower bounds for spectrum B1 are closed to what is specified in JSSC. The computed fatigue limit of VA loading for spectrum B1 is about 26 MPa, which is slightly lower than the VA cut-off limit of 29 MPa specified by JSSC.

5. Summary of the Findings

Fatigue crack propagation analysis of non-load carrying fillet welded specimens is carried out by fracture mechanics. First, a set of initial conditions, such as, ρ, θ, a_o and a/b is selected, so that the computed N_p represents well the fatigue test data under CA loading. The same initial conditions are used to predict the fatigue test data under VA loading. The parametric analysis is also carried out to see the effect of VA loading spectra. Then, taking into account the variation of the initial conditions by Monte Carlo simulation, the same analysis is carried out for CA and VA loading, The followings summarize the findings.

1. The analytical result shows that the Miner's rule or equivalent stress range concept predict well the fatigue life under VA loading in the short life region, for example, less then two million cycles, by neglecting the type of stress range spectrum.
2. The computed N_p for VA loadings is much dependent on the VA loading spectrum patterns in the long life region, for example, over five million cycles. The long tail of spectrum in the high stress region causes a quick crack propagation in the analysis.
3. With consideration of the variation in the initial conditions for the analysis, the scatter bands of the computed N_p explain well the fatigue test data under CA loading.
4. For VA loadings, the scatter band of the computed N_p is even wider and the test data fall within the scatter band.
5. From the analysis on variation of fatigue life for CA loading, VA loading in the test and VA loading of β distribution, with the consideration of the variation in the initial conditions, the 2.5 % lower bounds are very closed to the existing design curve.
6. The analysis method used in this paper can directly compute the 97.5% confidence lower limit of fatigue propagation life which can be referred as fatigue design curve, if proper probability density distributions of initial conditions are given. This method can be extended to the analysis such as evaluation of residual life, prediction for test and definition of design curves, for other parameters and other load spectrum patterns.

Appendix

(a) Normal distribution:

$$f_N(x) = \frac{1}{\sqrt{2\pi}\sigma_x} e^{-\frac{(x-m_x)^2}{2\sigma_x^2}}$$

$$v_x = \frac{\sigma_x}{m_x}$$

where, m_x: mean value of random variable x
σ_x: standard deviation of x
v_x: coefficient of variation

(b) Log-Normal distribution:

$$f_L(x) = \frac{1}{\sqrt{2\pi}\sigma_{lnx}x} e^{-\frac{(lnx - m_{lnx})^2}{2\sigma_{lnx}^2}}$$

where,

$$m_{lnx} = ln(\frac{m_x}{\sqrt{1+v_x^2}})$$

$$\sigma_{lnx}^2 = ln(1+v_x^2)$$

(c) Rayleigh distribution:

$$f_R(x) = \frac{x}{\mu^2} e^{-\frac{x^2}{2\mu^2}}, \qquad x > 0$$

(d) β distribution:

$$f_\beta(x) = \frac{1}{B(r,t)} x^{r-1}(1-x)^{t-r-1} \qquad t > 0,\ t - r > 0,\ 0 \leq x \leq 1$$

where, $B(r,t)$ — β function

Table 1. Parameters Chosen for Each Spectrum

Normal		Log-Normal		Rayleigh		β	
N1	m_x=0.5, v_x=0.0	L1	m_x=0.5, v_x=0.10	R1	μ=0.2	B1	t=8, r=2
N2	0.5, 0.2	L2	0.5, 0.50	R2	0.3	B2	8, 4
N3	0.5, 0.5	L3	0.5, 1.31	R3	0.4	B3	8, 6

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(Received September 21, 1992)