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DETECTION OF STIFFNESS DEGRADATION OF STRUCTURAL ELEMENTS FROM MEASUREMENT OF NATURAL FREQUENCIES AND MODE SHAPES

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This paper presents a method to detect the stiffness degradation of structural elements for undamped complex structures, by the use of modal analysis. In this method, the location of damage is detected by using the lower measured natural frequencies and pseudoinverse of mode shapes. In order to estimate the severity of damage, a certain unknown coefficients are consequently supposed in global stiffness matrix and solved by the arranged equations from vibration equations of structures. Moreover, a condensation technique is introduced for dealing with the case of incompletely measured system. Two numerical simulations are presented to demonstrate the availability of the method.

1. INTRODUCTION

The structure damage may be caused by many reasons, i.e. cracks, yieldings, corrosion losses, and concrete spalls, etc. All of these result in the degradation of stiffness. If the damage is severe, sometimes the degradation of stiffness may reach to 50%~70% [1]. So the damage estimation of structure is a long-standing concerned problem for engineers. From the tested result of damaged and undamaged natural frequencies, the global damage of structure can be evaluated [2]. By the identification of stiffness and damping of structure, also the damage of the structure can be estimated. There are many researches [3],[4],[5] in which the time-history methods are introduced to identify the structural parameters. However, these identification methods meet with the following difficulties: (1) The accuracy of the

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identified results will be greatly influenced by the existence of noise [6]; (2) To obtain the records of stress and strain history, a great number of instruments must be installed on the structure [2]; (3) The chance to obtain the strong earthquake records is rare; and (4) To analyse the whole motion history of structures will take a lot of computational time.

In order to avoid these difficulties, the method by the use of modal analysis was considerablely developed in recent years. Berman et al [7),8] developed the technique of identification for aerospace structures by use of vibrational tested results; Fritzen et al [9] studied the identification method for mechanical system; Hielmstand et al [10] estimated the structural parameters by introducing the complicated mutual residual energy method. As for the damage detection from vibration inspection in civil engineering, Shimada et al [11] investigated the vibrational characteristics and strengthening efficiency of damaged arch bridge. It is worth mentioning that Hearn and Testa [12] estimated successfully the damage of welded steel frame and wire rope by using the ratios of changes in natural frequencies. However, this method is limited on small deterioration of structure. Therefore, Hearn's method is not applicable for the case of severe damage, such as the structural damage caused by strong earthquakes.

As an attempt to improve the previous methods which are applied to detect the stiffness degradation of structural elements by using modal analysis, this paper presents a simple method which can be used to estimate both the location and severity of damage by use of the lower tested modes, i.e. natural frequencies and mode shapes. Two numerical examples are presented to demonstrate the availability of this method.

2. MODAL ANALYSIS INSPECTION

The equation of undamped free vibration system with N degrees of freedom, is described as follows

$$(K_0 - \omega_0^2 M_0) \boldsymbol{\phi}_0 = 0 \tag{1}$$

where K_0 and M_0 = the global stiffness and mass matrices, respectively; $\boldsymbol{\phi}_0$ = the normalized mode shape; and ω_0 = the natural frequency.

Changes in stiffness and mass matrices ($\triangle K, \triangle M$) produce changes in ω_0^2 and $\boldsymbol{\phi}_0$. For the perturbed system, Eq.(1) leads to

$$[(K_0+\triangle K)-\omega^2(M_0+\triangle M)]\ \boldsymbol{\phi} = 0 \tag{2}$$

where, $\omega^2=\omega_0^2+\triangle\omega^2$ and $\boldsymbol{\phi}=\boldsymbol{\phi}_0+\triangle\boldsymbol{\phi}$

Multiplied by $\boldsymbol{\phi}^T$, Eq.(2) becomes

$$\begin{aligned}\boldsymbol{\phi}^T(K_0+\triangle K)\boldsymbol{\phi} &= \boldsymbol{\phi}^T\omega^2(M_0+\triangle M)\boldsymbol{\phi} \\ &= \omega^2\boldsymbol{\phi}^T(M_0+\triangle M)\boldsymbol{\phi}\end{aligned} \tag{3}$$

As the $\boldsymbol{\phi}$ is the normalized mode shape, naturally the $\boldsymbol{\phi}$ satisfies

$$\boldsymbol{\phi}^T(M_0+\triangle M)\boldsymbol{\phi}=I \tag{4}$$

where, I is identity matrix.

From Eq.(4), Eq.(3) leads to

$$\boldsymbol{\phi}^T(K_0+\triangle K)\boldsymbol{\phi}=\omega^2 \tag{5}$$

For general engineering problems, only the lower modes can be obtained by vibration test.

If the lower L modes have been obtained, these modes are expressed as follows

$$\Omega_L^2 = \begin{bmatrix} \omega_1^2 & & & 0 \\ & \omega_2^2 & & \\ & & \ddots & \\ 0 & & & \omega_L^2 \end{bmatrix}, \quad \Phi_L = (\phi_1, \phi_2, \ldots \phi_L)$$

Replacing the ω^2 and ϕ by Ω_L^2 and Φ_L in Eq.(5) and Eq.(4) yields

$$\Phi_L^T \triangle K \Phi_L = \Omega_L^2 - \Phi_L^T K_0 \Phi_L \tag{6}$$

$$\Phi_L^T \triangle M \Phi_L = I_L - \Phi_L^T M_0 \Phi_L \tag{7}$$

where I_L is L×L identity matrix.

When L=N, the exact $\triangle K$ and $\triangle M$ in Eqs.(6),(7) can be obtained. However, since $L < N$, generally the exact $\triangle K$ and $\triangle M$ can not be obtained, only for their approximate solutions. Introducing pseudoinverse matrices Φ_L^+ and $(\Phi_L^T)^+$ [13], the least-squares solutions of $\triangle K$ and $\triangle M$ are consequently obtained as follows

$$\triangle K = (\Phi_L^T)^+ [\Omega_L^2 - \Phi_L^T K_0 \Phi_L] \Phi_L^+ \tag{8}$$

$$\triangle M = (\Phi_L^T)^+ [I_L - \Phi_L^T M_0 \Phi_L] \Phi_L^+ \tag{9}$$

The detailed steps to obtain the Φ_L^+ are expressed as follows

(1) Calculate all the eigenvalues $\lambda_1, \lambda_2, \ldots \lambda_L$ of product $\Phi_L^T \Phi_L$ $_{(L\times N \times N\times L = L\times L)}$

(2) Obtain the eigenvector matrix $Q=(Q_1, Q_2, \ldots Q_L)$ $_{(L\times L)}$ in which $Q_1, Q_2, \ldots Q_L$ correspond to $\lambda_1, \lambda_2, \ldots \lambda_L$.

(3) Obtain

$$\Pi_L = \begin{bmatrix} 1/\sqrt{\lambda_1} & & & \\ & 1/\sqrt{\lambda_2} & & \\ & & \cdots & \\ & & & 1/\sqrt{\lambda_L} \end{bmatrix}_{(L\times L)}$$

(4) Calculate the product $P = \Phi_L Q \Pi_L$ $_{(N\times L = N\times L \times L\times L \times L\times L)}$

(5) Calculate the pseudoinverse matrix $\Phi_L^+ = Q \Pi_L P^T$ $_{(L\times N = L\times L \times L\times L \times L\times N)}$

From the steps (1)~(5), it is easy to know that the main work for obtaining the Φ_L^+ is to calculate the eigenvalues and the eigenvectors of $\Phi_L^T \Phi_L$ and is to do the matrix multiplication. Moreover, since L is generally small, it only needs a short time of computation to obtain the Φ_L^+. Further details regarding the accuracy evaluation of the Φ_L^+ are available in the work of Masri et al[14),15)].

In steel structures, yieldings and cracks cause the degradation of stiffness without loss of mass, so $\triangle M=0$.

Deterioration of structures will alter stiffness and change the modes. The magnitude of change depends on both the location and severity of the deterioration and will affect each vibration modes differently, having a significant effect on certain modes and weak effect on others. This dissimilarity of the effect on each mode is the basis for detecting the damaged elements by modal analysis inspection.

If an undamaged structure is divided into a certain number of nodes and elements adequately as a FEM model of that structure, the location of damage can be detected by surveying the change of coefficients in global stiffness matrix for the structure before and after damage. The practical procedure is discribed as follows:

Substituting the undamaged global stiffness matrix K_0, measured natural frequency $\Omega_L{}^2$, measured mode shape Φ_L and pseudoinverse matrix $\Phi_L{}^+$ into Eq.(8), a percentage of change ratio $\triangle k_{ii}/k_{0ii}$ (i=1, 2, ..., N) for diagonal stiffness coefficients are calculated. According to the magnitude of the change ratio, the node in which the ratio is remarkable is therefore found not and recorded.

3. SOLUTION OF UNKNOWN COEFFICIENTS

When the node in which the stiffness has probably been changed is found out, each non-zero stiffness coefficient in the column(or row) corresponding to this node in global stiffness matrix is multiplied by an unknown coefficient α_K respectively. For instance, if the stiffness coefficients are multiplied by α_K in the *i*th and *j*th columns, the global stiffness matrix therefore becomes

$$K(\alpha) = \begin{bmatrix} k_{1,1} & \dots & \alpha_1 k_{1,i} & \dots & \alpha_{NI+1} k_{1,j} & \dots & k_{1,N} \\ k_{2,1} & \dots & \alpha_2 k_{2,i} & \dots & \alpha_{NI+2} k_{2,j} & \dots & k_{2,N} \\ \cdot & & \cdot & & \cdot & & \cdot \\ \cdot & & \cdot & & \cdot & & \cdot \\ \cdot & & \cdot & & \cdot & & \cdot \\ k_{N,1} & \dots & \alpha_{NI} k_{N,i} & \dots & \alpha_{NI+NJ} k_{N,j} & \dots & k_{N,N} \end{bmatrix} \tag{10}$$

where, NI, NJ are the numbers of non-zero stiffness coefficients in *i*th and *j*th columns respectively, and the total number of α is NF=NI+NJ.

It is worth mentioning that in 2-D/or 3-D structures, the stiffness coefficients in two /or three directions of axes should be supposed by α_K corresponding to each node (see Section 4).

In the vibration equations of structures, there are N equations corresponding to each mode. If all the L measured modes are used, the L・N equations can be obtained. Substituting $K(\alpha)$ into the vibration equations and arranging the L・N equations, the following new equations can be obtained

$$A\alpha^* = B \tag{11}$$

where, $\alpha^*=(\alpha_1, \alpha_2, \dots \alpha_{NF})$, A and B are known NF×NF matrix and NF×1 vector respectively.

When $|A| \neq 0$, the exclusive α^* can be obtained. If NF>L・N, it needs addtional measured modes. After all the unknown coefficients α^* are solved, the $K(\alpha)$ is the identified result of the global stiffness matrix.

4. GUYAN REDUCTION

Because of the complexity of the actual structure, not all the degrees of freedom can be measured. Such measurement system is called incomplete measurement system. For example the measurement of rotation is comparatively difficult. Therefore in the procedure of structural identification, it is necessary to exclude the rotational degrees of freedom from the stiffness matrix. In vibrational equations of structures, if the rotational degrees of freedom and translational ones in X and Y axes have been segregated. Subsequently the rotational degrees of freedom have been excluded from the vibrational equations, the *condensed* stiffness matrix for the translational degrees of freedom is therefore expressed as

$$K^*_{tt} = K_{tt} - K_{tr} = \begin{bmatrix} K_{XX} & K_{XY} \\ K_{YX} & K_{YY} \end{bmatrix} - \begin{bmatrix} K_{XR}K_{RR}^{-1}K_{RX} & K_{XR}K_{RR}^{-1}K_{RY} \\ K_{YR}K_{RR}^{-1}K_{RX} & K_{YR}K_{RR}^{-1}K_{RY} \end{bmatrix} \tag{12}$$

where, the subscripts X, Y and R indicate the translation in X, Y axes and rotation respectively.

Obviously, K^*_{tt} is the translated result of the K_{tt}. Therefore, Guyan Reduction remains the corresponding nodal relationship between K^*_{tt} and K_{tt}. When a damage occurs on an element, *for instance* the modulus of elasticity E or cross-section properties reduces [12], all the submatrices in Eq.(12) lead to change. In other words, even the condensed stiffness matrix K_{tt}^* is identified, the uncondensed stiffness matrix K_{tt} is not identified yet. Therefore, the change of stiffness for nodes is not estimated. Furthermore, the degradation of the structural element which corresponds with these nodes is not evaluated. However, a calculated result of a simply-supported beam in this paper shows that there is no change in axial stiffness in the procedure of Guyan Reduction. It can be explained from the Eq.(12), if the axial direction is the X direction and K_{XR} is zero, the $K_{XR}K_{RR}^{-1}K_{RX}$ is also zero. Once the axial stiffness shows a significant change before and after damage, it indicates the degradation on modulus of elasticity or cross-section properties of structural elements which correspond to the nodes, so the degradation of the elements can be evaluated. Therefore, it may be considered to be necessary to remain the axial degrees of freedom for the damage evaluation procedures of structures.

5. APPLICATIONS

According to the method described above, an analytical program has been developed and applied to the damage estimation of the following problems:

<u>Example 1</u>: Fig.1 shows a simply-supported steel beam with 13 nodes, 12 elements, and 23 translational degrees of freedom. The stress-strain relationship of the material is shown in Fig.2. It is supposed that the loads acted on the beam increase monotonically. Once some elements in the beam are yielded, there will occur the stiffness degradation on these elements. When a percentage of damage ratio of axial stiffness ($=100 \times [(k_{0ii} - k_{ii}) / k_{0ii}]_{AXIAL}$) in node 7 (the center of span) reaches 55 %, Guyan Reduction is done for the global stiffness matrix and the vibrational modes are consequently obtained as the measured modes of damaged

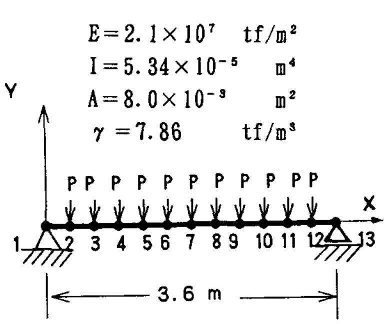

Fig.1 Simply-Supported Beam

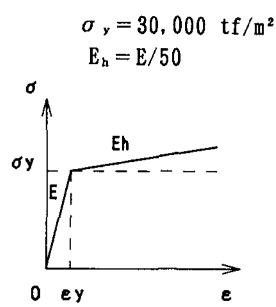

Fig.2 Stress-Strain Relationship of material

state. The change ratios of vertical and axial stiffness for each node are calculated by Eq. (8). The calculated result shows that change ratio of the axial stiffness is much smaller than the vertical one.

Table 1 Identified Results of Stiffness (tf/m)

NODE	UNDAMAGED		DAMAGED		
	UNCONDENSED	CONDENSED	IDENTIFIED	EXACT	E_{Rii}
7-X	0.11200E+07	0.11200E+07	0.50540E+06	0.50540E+06	0.0 %
7-Y	0.99735E+06	0.59649E+06	0.19788E+06	0.15515E+06	27.5 %
8-X	0.11200E+07	0.11200E+07	0.73920E+06	0.73920E+06	0.0 %
8-Y	0.99735E+06	0.59648E+06	0.27988E+06	0.21558E+06	29.8 %
9-X	0.11200E+07	0.11200E+07	0.10465E+07	0.10465E+07	0.0 %
9-Y	0.99735E+07	0.59642E+07	0.50970E+06	0.42909E+06	18.8 %

○ : 7 MODES
▲ : 6 MODES
□ : 5 MODES
+ : 4 MODES
◇ : 3 MODES
× : 2 MODES
• : 1 MODE

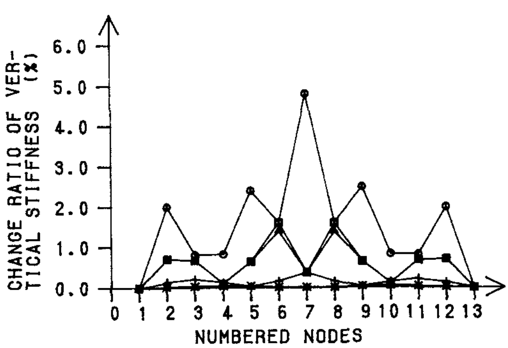

Fig. 3 Estimated Result of Used 1 ~ 7 Modes

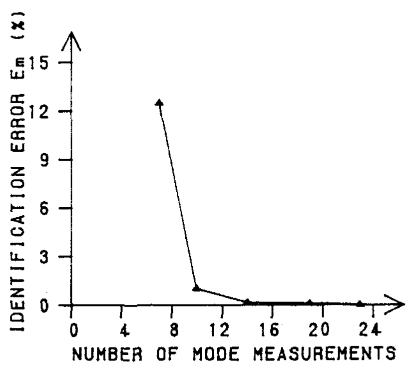

Fig. 4 Variation of Identification Error with Number of Tested Modes

Therefore the change ratios of vertical stiffness for all the nodes of the structure are shown in Fig. 3 in which the 1 mode, 2 • • • 7 modes indicate the 1th mode, 1th~2th • • • 1th ~7th modes, respectively. Also this indication is available in the following paragraphs. Among these 13 nodes, the change ratios of 7 nodes, i.e. 7, 5, 9, 2, 12, 6 and 8 are remarkable. So the change of stiffness is presumed on these nodes. Afterwards the unknown coefficient α_K is supposed on the stiffness coefficients for these nodes in the condensed global stiffness matrix. If the α_K is supposed on the coefficients regarding all the 7 nodes (14 degrees of freedom), the total number of unknown coefficients NF is greater than the number of arranged equations L • N under the condition of used 7 modes. Therefore, the α_K is only supposed on the coefficients for partial nodes. However, if the α_K is supposed on the positions for nodes 7, 5, 9, 2 and 12, the identified result of vertical diagonal stiffness of node 2 in the global stiffness matrix is -0.77299×10^6 tf/m. Obviously, this is an unreasonable result. Subsequently, the α_K is supposed on the positions for nodes 7, 5, 9, 6, 8, and the identified result of diagonal stiffness coefficients of nodes 7, 8, 9 (symmetrical about 7, 6, 5) is listed in Table 1.

Focusing attention on the identified diagonal stiffness coefficients, the percentage of identification error E_{Rii} is defined as

$$E_{Rii} = | [(k_{ii})_{EXACT} - (k_{ii})_{IDENTIFIED}] / (k_{ii})_{EXACT} | \times 100 \quad (i=1, 2, ... N) \tag{13}$$

The calculated E_{Rii} is also shown in Table 1 in which all the results are calculated by way of the double precision. Furthermore, the mean error of identification is defined as

$$E_m = (\Sigma E_{Rii}) / ND \tag{14}$$

where, the ND is the number of diagonal unknown coefficients among the NF. The Σ means the summation of these ND unknown coefficients. The the calculated result of the E_m is shown in Fig. 4. With the increment of the number of measured modes, the E_m will tend to zero. When the stiffness coefficients of all the nodes have been identified, the E_m should be zero. So the main reason of error is whether or not the α_K is supposed on the real positions in the changed stiffness matrix.

In order to discuss the sensitivity to detection of severity of damage, Fig. 5 shows the relationship between the damage ratio of axial stiffness and change ratio of vertical stiffness for the results of used 4, 5, 6, 7, 8, 9, 10 and 23 modes (where 23 modes implies that all the modes regarding the 23 degrees of freedom are measured and the result of 23 modes means the exact solution). When the damage ratio of axial stiffness in node 7 reaches 79 %, Fig. 5 shows that it is possible to detect the damaged node by using some lower measured modes, for instance, 4 or 5 modes. However, when the severity of damage is small, it is comparative-difficult to detect the location of damage by the lower measured modes.

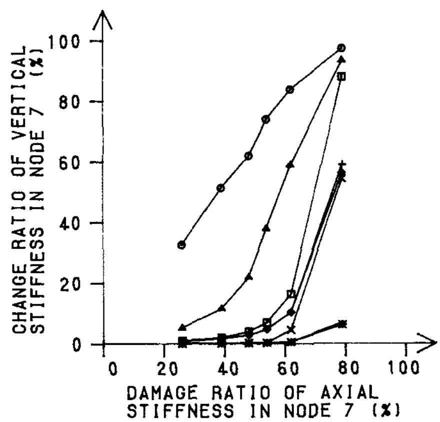

Fig. 5 Relation between Damage Ratio of Axial Stiffness And Change Ratio of Vertical Stiffness

It is easy to know from Eqs. (8), (10) and (11) that a error of measured mode has a direct effect on the damage location detection and identified result. Therefore, the effect of error in measured modes on the identified result will be discussed in the next Discussion 1 and 2.

Discussion 1: In the actural procedure of mode measurements, the reasons resulting in errors and the types of errors may be complicated and varied. In this paper, therefore, a simple one of the errors, a proportional error, as similar ot reference [10], is used to examine the effect of measurement error on the identifition results. In this paper, the measurement error of a certain mode shape is defined: If a percentage of vibrational amplitude is increased /or decreased in one node, the same percentage is decreased/or increased the neighbour nodes. Similarly, the proportional error of natural frequencies means that all the measured natural frequencies are simultanously reduced a certain percentage of respective frequency itself. When the proportional error of natural frequencies ranges from 0, 5, 10, 20, 50 to 100 percentage respectively, the vertical stiffness change ratios of used 7 modes are shown in Fig. 6. Furthermore as Fig. 6 shows, the location of maximum change ratio of vertical stiffness in node 7 remains unchanged under the condition of these proportional errors. Also when the proportional error of mode shapes ranges from 0, 5, 10, 20 to 50 percentage respectively, the vertical stiffness change ratios of used 7 modes are shown in Fig. 7 and the location of maximum change ratio of vertical stiffness in node 7 also remains unchanged under these proportional errors of mode shapes.

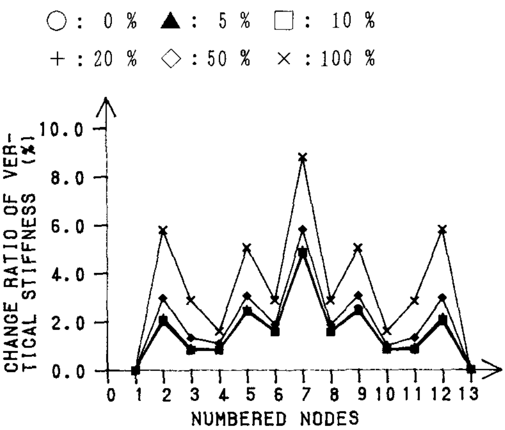

Fig. 6 Effect of Proportional Errors of Natural Frequencies on Damage Location Detection

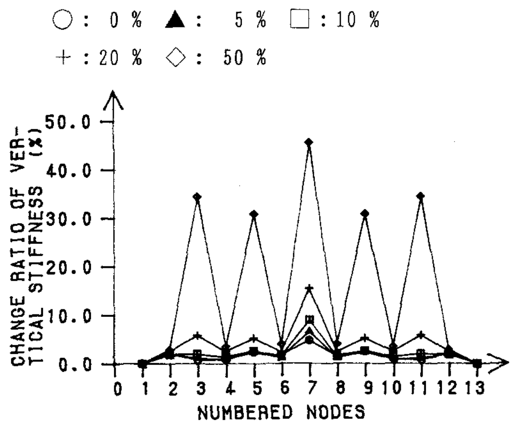

Fig. 7 Effect of Proportional Errors of Mode Shapes on Damage Location Detection

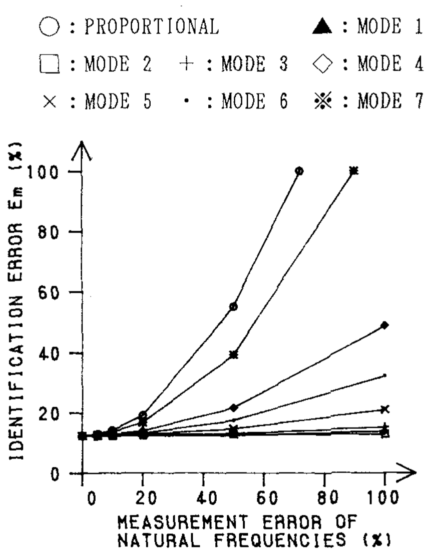

Fig. 8 Effect of Measurement Errors of Natural Frequencies on E_m

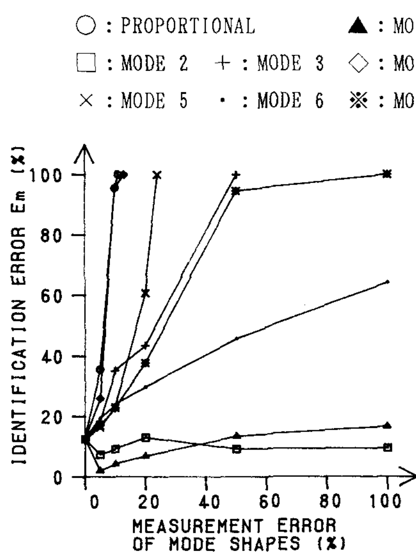

Fig. 9 Effect of Measurement Errors of Mode Shapes on E_m

Discussion 2: As for the effect of the measurement error of modes on the identified result, the effects of proportional error, imposed on the all the 1th~7th natural frequencies on the E_m are shown in Fig. 8. Also the effects of proportional error imposed on only *i*th mode shape among the 7 mode shapes on the E_m are shown in Fig. 9. Figs. 8 and 9 show that some modes have strong effect on the E_m (modes 4, 5 and 3 etc.), and some modes have comparative weak effect (modes 1 and 2). Also under the condition of same percentage of the proportional errors, mode shape has the greater effect on the E_m than natural frequency does. For instance, the 10 % proportional error of natural frequencies (see Fig. 8) and mode shapes (see Fig. 9) leads to about 16 % and 95 % of the E_m respectively.

Example 2: Fig. 10 presents a bowstring steel truss with 12 nodes, 25 elements and 21 degrees of freedom. The nominal areas of the members are as follows: Bottom chords 0.06 m^2, top chords 0.0312 m^2, verticals 0.024 m^2, and diagonals 0.024 m^2. The stress-strain relationship of all materials are elastic-perfectly-plastic model. It is supposed that P_Y is the constant load of 20 tf and P_X increases monotonically. When the damage ratio of axial stiffness in node 2 reaches 89 % (the elements 1-2 and 2-3, i.e. ① and ② have been yielded), the vibrational modes are calculated as the modes of damaged state. Fig. 11 shows the estimated results of used 1~4 modes. Also the estimated results of vertical stiffness show that there are significant change on nodes 2, 8, 9 and 3. Therefore the α_K is supposed on these nodes. The identified results of axial stiffness of node 8 and vertical stiffness of node 2 in the diagnoal coefficients of global stiffness matrix are about +7 and +176 times of undamaged ones if the α_K is supposed on nodes 2, 8 and 2, 9 respectively. Obviously, these results are also unreasonable.

Therefore, the α_K is supposed on nodes 2, 3 and the identified results and the E_m are shown in Table 2 in which the accuracy of identification is satisfied well.

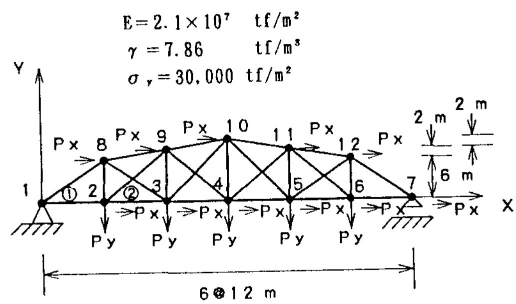

Fig. 10 Bowstring Truss

Table 2 Identified Results of Stiffness (tf/m)

NODE	UNDAMAGED	DAMAGED		
		IDENTIFIED	EXACT	E_{Rii}
2-X	232168.1	24161.8	24481.9	1.3 %
2-Y	95737.8	94928.6	95310.7	0.4 %
3-X	258938.7	154568.5	154073.6	0.3 %
3-Y	84575.2	84094.3	84563.3	0.6 %

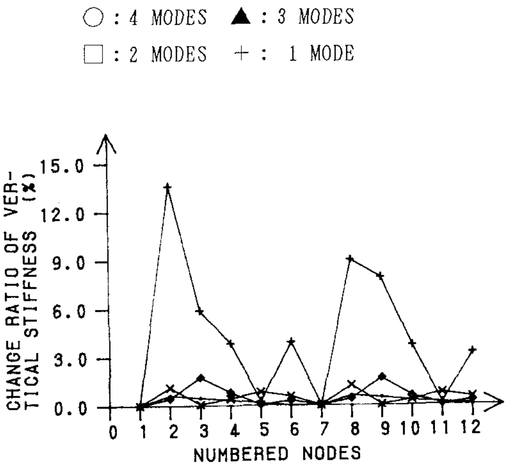

Fig. 11 Estimated Result of Used 1～4 Modes

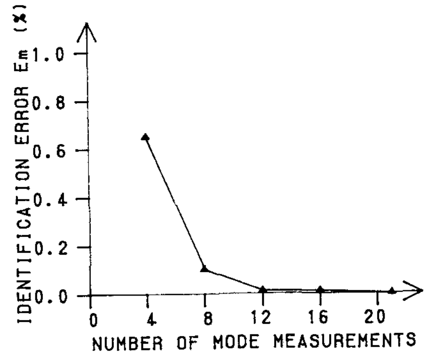

Fig. 12 Variation of Identfication Error with Number of Measured Modes

As same as the Example 1, the effect of error in measured modes on the identified result will be discussed by the same way in the next Discussion 3 and 4.

Discussion 3: When the proportional error of natural frequencies ranges from 0, 5, 10, 20, to 50 percentage respectively, the vertical stiffness change ratios of used 4 modes are shown in Fig. 13, furthermore the location of maximum change ratio of vertical stiffness(in node 2) remains unchanged under the condition of these proportional errors. Also the vertical stiffness change ratios of used 4 modes are shown in Fig. 14, When the proportional error of mode shapes ranges from 0, 5, 10, 20 to 50 percentage respectively. Moreover the location of maximum change ratio of vertical stiffness (also in node 2) remains unchanged under the condition of these proportional errors of mode shapes.

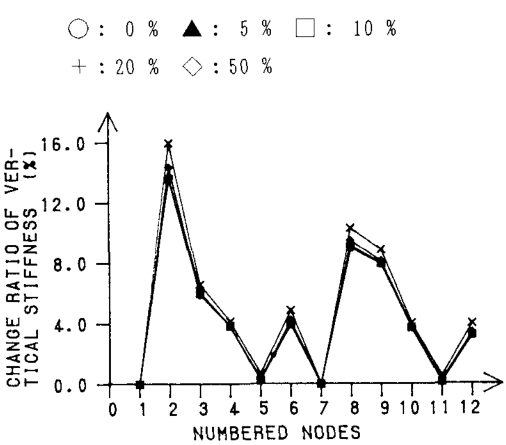

Fig. 13 Effect of Proportional Errors of Natural Frequencies on Damage Location Detection

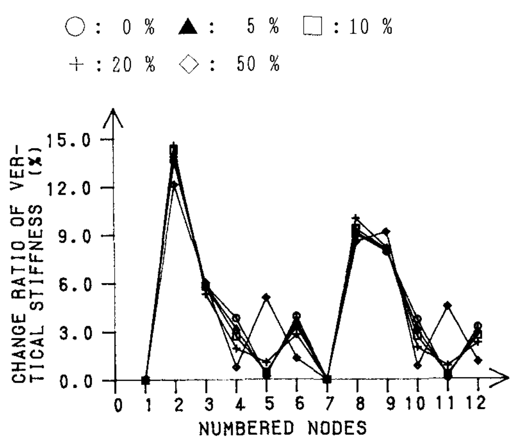

Fig. 14 Effect of Proportional Errors of Mode Shapes on Damage Location Detection

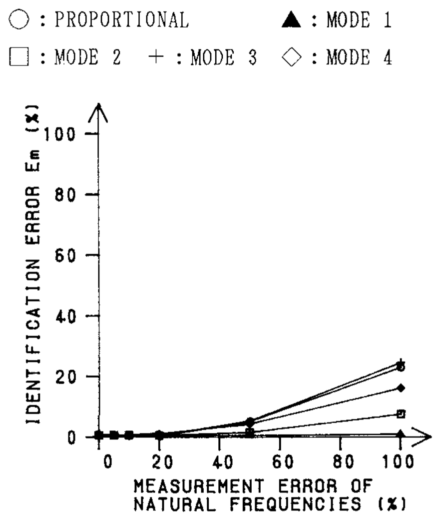

Fig.15 Effect of Measurement Errors of Natural Frequencies on E_m

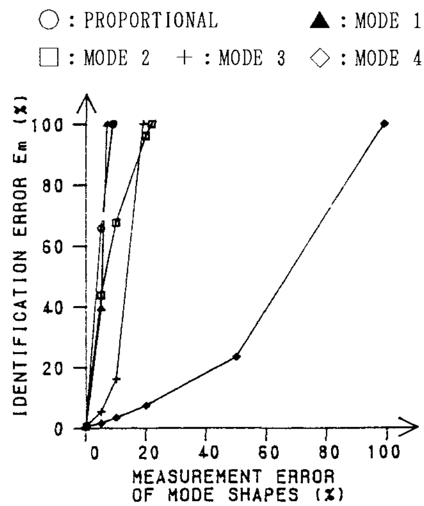

Fig.16 Effect of Measurement Errors of Mode Shapes on E_m

Discussion 4: Being similar to Discussion 2, the effects of proportional error, as well as the measured errors of the 1th~4th natural frequencies on the E_m are shown in in Fig.15. Also the effects of proportional error, as well as the measured errors of the 1th~4th mode shapes on the E_m are shown in Fig.16. Besides, Figs. 13 and 14 show that some modes have strong effect on the E_m (modes 1,2 and 3), the other has comparative weak one (mode 4). Moreover, it is evident that mode shape has the greater effect on the E_m than frequency does.

6. CONCLUSIONS

The main conclusions in this paper can be summed up as the following six points:

1) A method to detect the stiffness degradation of structural elements by the use of modal analysis inspection has been demonstrated. By this method, both the location and severity of damage can be estimated.
2) Guyan Reduction is used to deal with the case of incompletely measured system. The calculations on the simply-supported beam shows that the change of vertical stiffness may be used to detect the possible damage location. The change of axial stiffness may be used to evalute the actual degradation of structural elements for such kind of structures.
3) For the structure with severe damage, it is possible to detect the damage locations by using the lower measured modes. The required numbers of measured modes are about 1/4 ~ 1/5 of the whole degrees of freedom N.
4) As for the effect of the number of measured modes on the identification result, as Figs. 4 and 12 show, the identification error decreases as the number of mode measurements taken increases.
5) The effect of the measurement errors on damage location detection is also discussed

in this paper. The location of maximum change ratio of vertical stiffness remains unchanged under the condition of certain ranges of proportional errors of modes.

6) As to the effect of measurement errors on the identification results, this paper shows that some modes have strong effect on the E_m and some modes have comparative weak ones. Under the condition of same percentage of the proportional measurement error, mode shape has a greater effect on the identification result than natural frequency does.

Damping ratio may be an important parameter in the modal analysis for damage detection of structure. As the research [12] pointed out, damping change is clear evidence of damage in the structure. Furthermore, damping may be the only indication of distress when frequencies, as well as mode shapes, are insensitive to damage estimation. Therefore, it is expected that by the use of this method the research can be done on the damage estimation aspect for damped structures.

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