Fully Explicit time integration of ISPH for large-scaled tsunami simulations

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1. Introduction

After the Great East Japan Earthquake in 2011, the large-scale simulation for tsunami becomes significantly important for disaster prevention and mitigation for the next millennium tsunamis. In our research group, the simulation for a potential danger area Kochi City has been done by an incompressible smoothed particle hydrodynamics (ISPH) method. However, ISPH method results in a very high computational cost both in time and memory. In this research, a fully explicit version of ISPH method where originally a semi-implicit time integration is used is proposed to overcome the difficulties in the large-scale simulation. In this paper, at first the precision of explicit ISPH (EISPH) is validated by comparing with the result of ISPH.

2. Methodology

2.1 Governing equation

The governing equations for incompressible fluid simulation include continuity equation (1) and Navier-Stokes equation (2):

$$\nabla \cdot \boldsymbol{u} = 0 \tag{1}$$

$$\frac{d\boldsymbol{u}}{dt} = -\frac{1}{\rho}\nabla P + \nu\nabla^2\boldsymbol{u} + \boldsymbol{g} \tag{2}$$

where ρ, P, $\boldsymbol{u}$, ν and $\boldsymbol{g}$ stand for the density, pressure, velocity, kinematic viscosity and gravitational acceleration respectively.

2.2 SPH formulation

In SPH method, the fluid domain is discretized into a finite number of particles which represent the fluid volumes. The physical quantity of a particle located at $\boldsymbol{x}_i$ can be approximated from the summation of its neighbouring particles (3):

$$\langle f(x_i)\rangle = \sum_{j=1}^{N}\frac{m_j}{\rho_j} f(x_j) W(\boldsymbol{r}_{ij}, h) \tag{3}$$

where f($\boldsymbol{x}_i$) and f($\boldsymbol{x}_j$) are the physical values of particle i and j, m_j and ρ_j mean the mass and density of particle j, and $W(\boldsymbol{r}_{ij},h)$ means a kernel function with smooth length h and the distance between two particles $\boldsymbol{r}_{ij}$.

2.3 Projection method

In ISPH method, to update the velocity and position of each particle, the governing equation is solved in projection method which divides Navier-Stokes equation into two parts predictor and corrector.

1) Predictor

In predictor, the velocity is updated explicitly into an intermediate value $\boldsymbol{u}^*$ (4) and (5):

$$\Delta\boldsymbol{u}^* = \left(\boldsymbol{g} + \nu\nabla^2\boldsymbol{u}\right)\Delta t \tag{4}$$

$$\boldsymbol{u}^* = \boldsymbol{u}^n + \Delta\boldsymbol{u}^*. \tag{5}$$

2) Corrector

In corrector, the intermediate velocity $\boldsymbol{u}^*$ is updated implicitly into the current value $\boldsymbol{u}^{n+1}$ (6) and (7):

$$\Delta\boldsymbol{u}^{**} = -\frac{1}{\rho}\nabla P^{n+1}\Delta t \tag{6}$$

$$\boldsymbol{u}^{n+1} = \boldsymbol{u}^* + \Delta\boldsymbol{u}^{**}. \tag{7}$$

Then the current velocity is used to update the position of each particle (8):

$$\boldsymbol{r}^{n+1} = \boldsymbol{r}^n + \boldsymbol{u}^{n+1}\Delta t. \tag{8}$$

2.4 Pressure Poisson equation (PPE)

In ISPH method, to calculate the pressure value, pressure Poisson equation has to be solved. In this research, the source term of PPE proposed by Asai et al [1] is applied (9):

$$\left\langle \nabla^2 P_i^{n+1}\right\rangle = \frac{\rho^0}{\Delta t}\left\langle \nabla\cdot\boldsymbol{u}_i^*\right\rangle + \alpha\frac{\rho^0 - \left\langle \rho_i^n\right\rangle}{\Delta t^2}. \tag{9}$$

The difference between ISPH and EISPH is the solution to PPE.

1) The solution to PPE in ISPH

The left hand side of PPE can be discretized in SPH form, and the following equation (10) can be derived:

$$\sum_{j+1}^{N} m_j \frac{2}{\rho^0}\frac{\boldsymbol{r}_{ij}\cdot\nabla W(\boldsymbol{r}_{ij},h)}{\boldsymbol{r}_{ij}^2+\eta^2}\left(P_i^{n+1} - P_j^{n+1}\right) = \frac{\rho^0}{\Delta t}\left\langle \nabla\cdot\boldsymbol{u}_i^*\right\rangle + \alpha\frac{\rho^0 - \left\langle \rho_i^n\right\rangle}{\Delta t^2}. \tag{10}$$

where η is a small parameter equal to $0.01\boldsymbol{r}_{ij}$ to avoid singular denominator.

Then the equation (10) can be simplified into equation (11):

$$P_i^{n+1} = \frac{B_i + \sum_{j=1}^{N} A_{ij} P_j^{n+1}}{\sum_{j=1}^{N} A_{ij}} \tag{11}$$

where

$$A_{ij} = m_j \frac{2}{\rho^0} \frac{\boldsymbol{r}_{ij} \cdot \nabla W(r_{ij}, h)}{\boldsymbol{r}_{ij}^2 + \eta^2} \tag{12}$$

$$B_i = \frac{\rho^0}{\Delta t} \langle \nabla \cdot \boldsymbol{u}^* \rangle + \alpha \frac{\rho^0 - \langle \rho^n \rangle}{\Delta t^2}. \tag{13}$$

2) The solution to PPE in EISPH

According to the basic theory of EISPH proposed by Barcarolo et al [2], in a small time increment, the current pressure value P_j^{n+1} is assumed to be equal to the previous one P_j^n. Equation (11) can be changed into equation (14):

$$P_i^{n+1} = \frac{B_i + \sum_{j=1}^{N} A_{ij} P_j^{n}}{\sum_{j=1}^{N} A_{ij}}. \tag{14}$$

3. Validation

1) Dam-break model

Dam-break model as shown in Fig.1 is simulated here to validate the accuracy of EISPH compared with the original ISPH with the same resolution models. In this model, the particle diameter is 10cm and the number of particles is 214932. EISPH can evaluated a smoothed pressure field, the pressure level on the sensor is closed to the result by ISPH as shown in Fig.2.

2) Large-scale simulation of Kochi City

To confirm whether EISPH can resolve the difficulties in large-scale simulation. The simulations of Kochi City have been done with different particle diameter 4meters and 2meters. The number of particle of different particle diameter is about 34,000,000 and 144,000,000 respectively, while the time cost is 24hours and 12hours. The simulation result with 4meter particle diameter at 450 second is shown in Fig.3 for the ISPH and Fig.4 for the EISPH. From the comparisons between these figures, there are almost no difference in the velocity distribution and in the pressure evaluation. Finally, 2meter particle diameter model can solve only with the EISPH because of high memory cost in the ISPH.

4. Conclusion

1) From the results of the simulations of dam- break and Kochi City, EISPH shows high precision compared with ISPH.

2) In EISPH method, the difficulties of large-scale simulation in computational cost can be resolved.

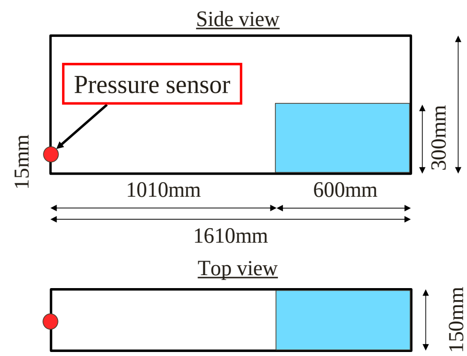

Fig.1 Dam-break model

Fig.2 Result of pressure value

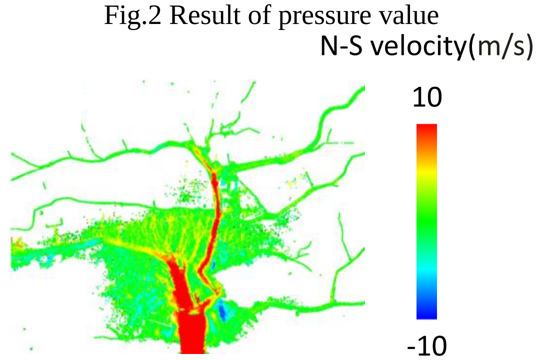

Fig.3 Simulation result of Kochi City in ISPH

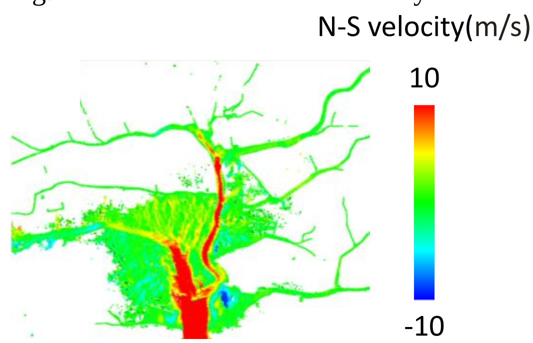

Fig.4 Simulation result of Kochi City in EISPH

Reference

[1]Mitsuteru Asai et. al.: *International Journal for Applied Mathematics*, Vol.2012, Article ID 139583, 24 pages, 2012.

[2]D.A.Barcarolo, et. al. *Blucher Mechanical Engineering Proceedings*, vol.1, num.1, 2014.