Three-dimensional analysis of slopes reinforced by piles using Monte Carlo method

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1 INTRODUCTION

An analysis based on upper bound limit analysis and traversal algorithm was implemented to assess the critical yield acceleration of 2D slopes without reinforcement (Chang et al. 1984). This method was later adopted to analyze the directed sliding mechanism for geosynthetics reinforced soil structures. The previous researches were analyzed involving two-dimensional (2D) plane-strain failure mechanisms. However, it was commonly acknowledged that 2D solutions were conservative to analyze slope stability compared with 3D solutions. In this paper, 3D analysis of the slopes is considered.

Michalowski and Drescher (2009) have proposed a class of 3D admissible rotational failure mechanisms for slopes. In this paper, these 3D failure mechanisms are adopted. Furthermore, the Monte Carlo method (Hammersley and Handscomb 1964) is used to find the yield acceleration of the reinforced slope.

2. CRITICAL ACCELERATION FOR 3D SLOPES REINFORCED WITH PILES

In this study, the soil of the slopes is considered to be homogeneous and isotropic. The failure surface in the 3D slopes is assumed to be the curvilinear cone (the shape is similar to a horn), with upper and lower contours defined by log-spirals (Michalowski and Drescher 2009), which is shown in Fig. 1. The shape of the failure surface is smooth, and has one symmetry plane. The trace of the mechanism on the symmetry plane is described by two log-spirals, AC and A′C′

$$r = r_0 e^{(\theta-\theta_0)\tan\varphi} \tag{1}$$

$$r' = r'_0 e^{-(\theta-\theta_0)\tan\varphi} \tag{2}$$

where r_0 is the radius of the log spiral with respect to angle θ_0, shown as Fig. 1. φ is the internal friction angle of the soil. The location of the 'horn' for a specified slope is uniquely determined by the angles θ_0, θ_h, the ratio r_0'/r_0, and the resistance force F_p. The failure soil mass rotates as a rigid body about the point O with angles velocity ω. In order to allow the 3D failure surface to transitioned to plane-strain mechanisms, the 3D failure surface model is split from the symmetry plane, and then separated laterally into two halves. Additionally, a plane with a width of b is inserted (Fig. 2). This 'plane insert' modification has been proposed by Michalowski and Drescher (2009).

The rate of work of the failing soil weight in block CDEFGQ (Fig. 2) during an incipient rotation about point O is calculated as

$$W_\gamma = W_\gamma^{2D} \cdot b + W_\gamma^{3D} \tag{3}$$

where the superscript 3D denotes the work rates for the 3D portion of the failure mechanism and 2D relates to the plane insert (Fig. 2). The details of the equation used in calculations can be find in the references (e.g. Michalowski and Drescher 2009; Chang et al. 1984).

Once the slope is subjected to horizontal shaking, the rate of the inertial force W_s needs to be considered in the energy balance equation. According to the pseudo-static approach, the horizontal force acting at the center of gravity is calculated to represent the effect of the earthquake loading on the failing soil mass. Rate W_s is calculated as the product of a seismic coefficient k and the weight of potential failing soil mass.

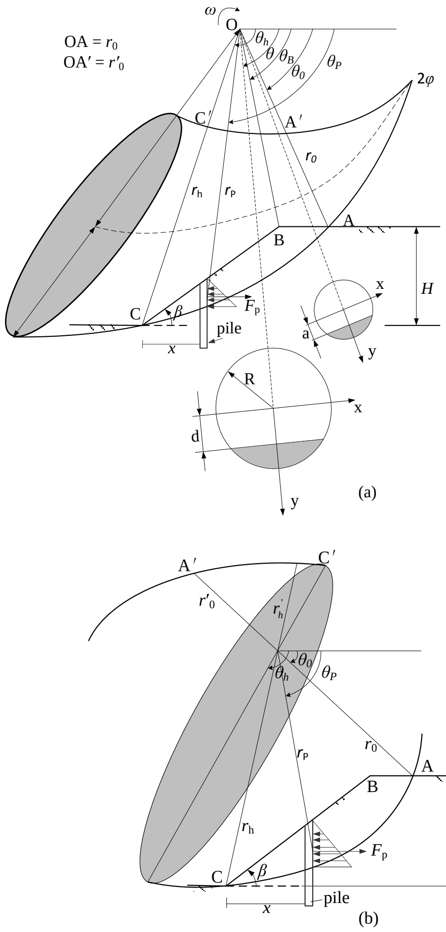

Fig. 1 Three-dimensional rotational toe-failure mechanism in stabilized slopes: (a) a 'horn-shape' surface; (b) alternative mechanism (based on Michalowski & Drescher, 2009)

$$W_s = W_s^{2D} \cdot b + W_s^{3D} = k\gamma r_0^3 \omega b(f_1^s - f_2^s - f_3^s) + W_s^{3D} \tag{4}$$

where the superscript has the same meaning with Eq. (3); k is the seismic coefficient; γ is the unit weight of the soil; the coefficient $f_1^s \sim f_3^s$ and W_s^{3D} can be found in the references.

Considering the resistance provided by the piles, the total energy dissipation rate D is the sum of D_c and D_p, shown as follows

$$D = D_c + D_P = D^{2D} \cdot b + D^{3D} + D_P \tag{5}$$

where D_p is the dissipation rate induced by the reinforcement; D_c is the rate of work dissipation caused by soil cohesion. Additionally, the work dissipation rate caused by soil cohesion involves two terms: the 3D term (D^{3D}) and the plain-strain term (D^{2D} b). The soil reinforcement plastically deforms at the slope limit state, and the rate of dissipated work associated with the reinforcement plastic deformation depends on its distribution throughout the unstable soil height. In order to simplify the calculation, the work dissipation rate induced by each pile is assumed to be the same, which equals to that caused by the pile embedded on the symmetry plane of the composite 3D slope (referring to Fig. 2). Based on the distribution of the lateral force acting on piles (referring to Fig. 1.) the rate of work dissipation caused by the piles can be calculated as follow

$$D_P = F_P n \sin\theta_p r_P \omega \tag{6}$$

in which F_p is the total lateral force exerted on a stabilizing pile due to the plastically deforming layer around the pile; θ_p specifies the position of the stabilizing piles (Fig. 1); r_p is the radius of F_p about the rotation center; n is the number of the piles; n is the number of piles.

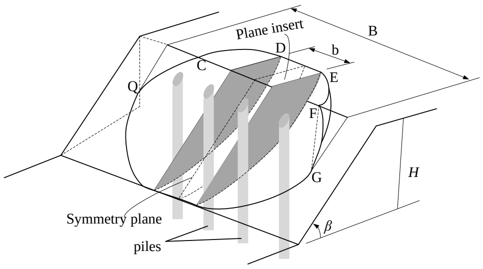

Fig. 2 Schematic diagram of 3D rotational failure mechanism with limited width B for slopes stabilized with piles

To evaluate the force F_p, a theory developed by Ito and Matsui (1975) to specifically estimate the pressure acting on the passive piles is adopted in this present work. The lateral force per unit thickness of the layer acting on the pile proposed in their work is integrated to calculate the force F_p. The equation can be found in the source reference.

In order to obtain the critical acceleration coefficient k_c, according to the upper-bound theory, we let the rate of internal work dissipation equal to the external rate of work, then the upper bound of k is calculated as

$$k' = \frac{D^{2D}\cdot b + D^{3D} + F_P n \sin\theta_P r_P \omega - (W_\gamma^{2D}\cdot b + W_\gamma^{3D})}{\gamma r_0^3 \omega b(f_1^s - f_2^s - f_3^s) + W_s^{3D}} \tag{7}$$

3. THE MONTE CARLO METHOD

A random search approach based on Monte Carlo method (Hammersley and Handscomb 1964) is used to find the least upper bound to the critical acceleration coefficient k_c. In this work, 200,000 trials are performed to find k_c for each condition.

For a specified slope (the properties and geometry are given), shown as Fig. 3, comparing the critical seismic acceleration coefficient k_c analyzed by Li et al. (2010) for 2D slopes, under the condition of the parameter B/H equaling to 15 in composite 3D mechanisms, a verification of the accuracy of k_c using the random trials method is listed in Table 1. The comparisons shown in Table 1 imply that Monte Carlo method is available in finding the least upper bound for 3D toe-failure mechanism. In addition, the critical acceleration coefficient of the stabilized slopes is listed in Table 2, comparing with the conditions of slopes without reinforcement. Table 2 indicates that for the same soil properties, the constraint of the width significantly affects the results of the critical acceleration coefficient. When B/H = 2, and φ = 15 °, the value of k_c is 0.210, while the ratio of B/H increases to 5, k_c decreases to 0.147 with φ remains 15 °. When B/H = 10, the 2D failure mechanism is considered to be acting on the slopes. It is clear that the critical acceleration coefficient of the slope is significantly reduced by the stabilizing piles.

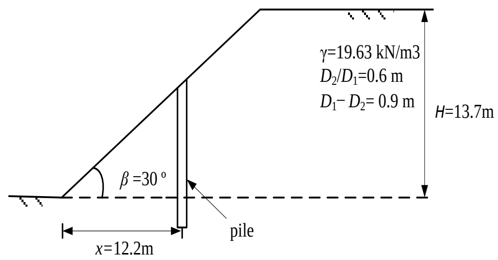

Fig. 3 Geometry, reinforcement, and soil properties of the symmetry plane of the example slopes

Table 1 Comparison of the numerical results

	B/H	$b/$H	k_c	
			Present study	Li et al. (2010)
φ = 10 °, c = 23.94	15	13.10	0.060	0.061
φ = 15 °, c = 23.94	15	12.63	0.165	0.182
φ = 15 °, c = 18	15	12.96	0.087	0.089

Table 2 Numerical results of the critical coefficient

	Without piles				With piles			
B/H	2		5		2		5	
c (kPa)	24		24		24		24	
φ (°)	15	10	15	10	15	10	15	10
b/H	0.717	0.560	3.417	3.327	-	0.758	3.553	3.681
k_c	0.210	0.106	0.147	0.042	-	0.181	0.219	0.145

4. CONCLUSIONS

This paper attempts to develop a method to analyze the critical yield acceleration coefficient of 3D slopes reinforced with piles under earthquake loading. The yield seismic acceleration coefficient is derived from the upper bound theorem. The random trials method (Monte Carlo method) is introduced to determine the least upper bound of the acceleration coefficient k_c. It is found that the yield accelerations of 2D mechanism (B/H = 10) are less than that in 3D mechanism with the same soil properties.

REFERENCES

1. Chang, C. J., Chen, W. F., and Yao, J. T. P. (1984). “Seismic displacements in slopes by limit analysis.” Journal of Geotechnical and Geoenvironmental Engineering, ASCE, 110(7),860–874.
2. Hammersley, J.M., and Handscomb, D.C., (1964). “Monte Carlo method.” Methuen Co. Ltd., London
3. Ito, T., and Matsui, T., (1975). “Methods to estimate lateral force acting on stabilizing piles.” Soils and Foundations, 15, 43–59.
4. Li, X., He, S., and Wu, Y., (2010). “Seismic displacement of slopes reinforced with piles.” Journal of Geotechnical and Geoenvironmental Engineering, ASCE, 136, 880-884.
5. Michalowski, R. L., and Drescher, A. (2009). “Three-dimensional stability of slopes and excavations.” Geotechnique, 59(10), 839–850.