Numerical Simulation of Equivalent Permeability and Fractal Characteristics of Fractured Rock Masses Using Discrete Element Method

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1. Research background and objective

Understanding the hydraulic characteristics of fractured rock masses is significantly important for practices in various fields of rock engineering, such as slopes, dam foundations, underground excavations and nuclear waste disposals. To evaluate the permeability of fractured crystalline rocks, the discrete fracture network (DFN) approach is widely used. The objective of this paper was to firstly establish a DFN approach based on the Monte Carlo method. By means of this approach, the equivalent permeability of rock fracture networks was calculated with varying model sizes and rotation angles, and the representative elementary volume (REV) size of these networks were assessed. Fractal dimension was calculated and utilized to describe the relationship between fractal characteristics and equivalent permeability of fracture networks.

2. Generation of DFN models based on Monte Carlo method

Table 1 shows the basic parameters involved in the fracture network generation processes. Four sets of fractures were assumed in the networks, in which, the trace lengths, dip/dip directions and apertures of rock fracture sets followed the log-normal distributions. In this study non-persistent fractures with dead ends were deleted during DFN generation.

Table 1 Geometric parameters used for DFN generation

Joint set	Tendency (°) Avg. /Dev.	Dip angle (°) Avg. /Dev.	Trace length (m) Avg. /Dev.	Fracture density/m²
1	332.5/7.5	67.5/7.5	5.63/0.45	0.35
2	17.5/6.5	62.5/3.5	4.35/0.35	0.17
3	57.5/13.0	75.0/8.5	3.82/0.28	0.11
4	285.5/13.5	70.0/9.0	4.54/0.22	0.08

3. Calculation of equivalent permeability

3.1 Boundary conditions

The equivalent permeability tensor, k_{ij}, was calculated according to the classical Darcy's law. Figs. 1 and2 show the generic boundary conditions for the calculation of permeability tensor. Each fracture network has four boundaries, numbered with BC1-BC4. The equivalent permeability k_{xx} and k_{yy} in the two boundary conditions can be calculated as follows:

$$Q_x = A\frac{k_x}{\mu}\frac{\partial P}{\partial x} \quad (1) \qquad Q_y = A\frac{k_y}{\mu}\frac{\partial P}{\partial y} \quad (2)$$

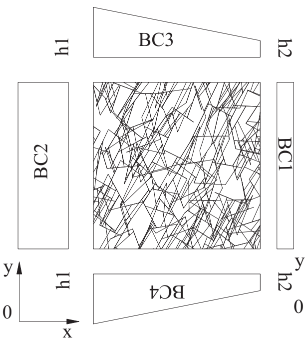

Fig. 1 The type 1 boundary condition

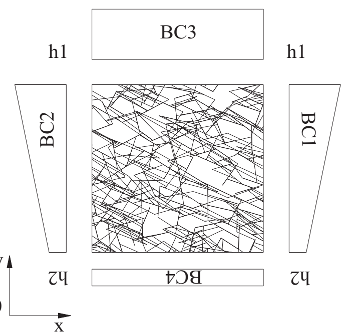

Fig. 2 The type 2 boundary condition

where Q_x and Q_y are the flow rates in the x- and y- directions, respectively, A is the cross-sectional area, k_x and k_y are permeability in the x- and y-directions respectively, μ is the dynamic viscosity, and P is the hydraulic pressure.

3.2 Calculation of REV

During the calculations of fluid flow, stress effects were not considered so that the equivalent permeability of models at 0° and 180° rotation angle was almost identical. Additionally, the values of k_x and k_y at rotation angle of 0° equal to those of k_y and k_x at rotation angle of 90°, respectively, due to the two orthogonal boundary conditions. Herein, the directional equivalent permeability k_{xx} and k_{yy} can be calculated through the DFN models with rotation angles of 0°, 15°, 30°, 45°, 60° and 75°. Fig. 4 shows the results and variation tendency of equivalent permeability with the change of model side length that can be used to determine the reasonable REV size. When side lengths are small (e.g., less than 40m), the variances of equivalent permeability are significantly large. Both the permeability of k_{xx} and k_{yy} become stable after the side lengths of DFN models become

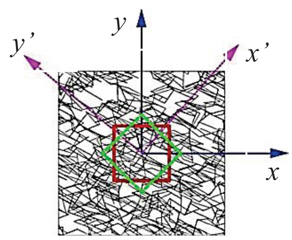

Fig.3 Extraction and rotation of DFN model based on an original large model

larger than 40m. Therefore, a DFN model size of 40m can be regarded as a REV for practical uses.

4. Calculation of permeability tensor

By using the type 1 boundary condition described in section 3, the penetration ellipses with fracture network side lengths of 5m,10m, 20m, 30m, 40m, 50m, 60m were calculated, respectively, based on the least square method. According to the major axis and minor axis of penetration ellipse, penetration principal value K_1, K_2 and main percolation direction θ for each fracture network were obtained as shown in Table 2, where the semi-major axis=$1/\sqrt{K_1}$, and the semi-minor axis=$1/\sqrt{K_2}$. As a consequence, the main percolation direction about this fracture network is approximately 67°.

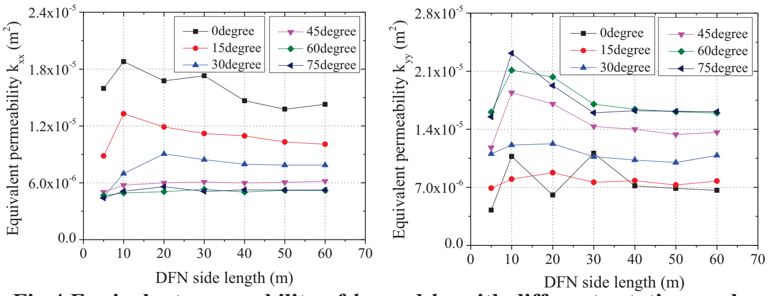

Fig.4 Equivalent permeability of k_{xx} and k_{yy} with different rotation angles and DFN side lengths

Table 2 Penetration principal value and direction

DFN side length (m)	K_1(m²)	K_2(m²)	θ（rad）	θ(°)
5	3.72E-06	1.33E-05	1.113	63.759
10	4.76E-06	1.86E-05	1.156	66.240
20	4.64E-06	1.80E-05	1.172	67.145
30	5.19E-06	1.59E-05	1.168	66.921
40	4.85E-06	1.52E-05	1.166	66.813
50	4.89E-06	1.45E-05	1.182	67.747
60	4.88E-06	1.48E-05	1.165	66.738

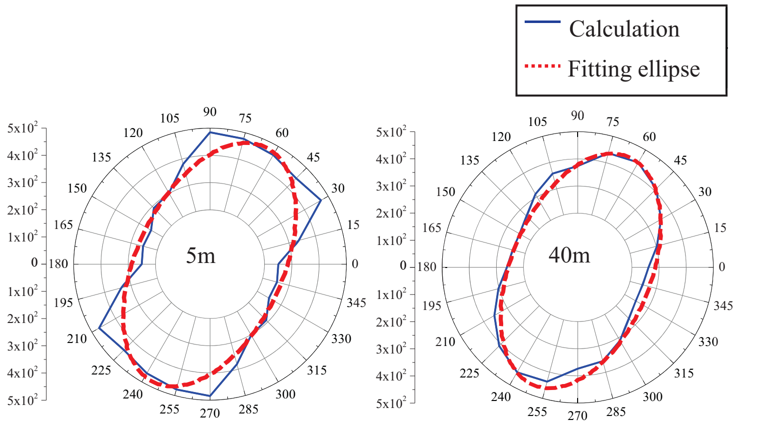

Fig.5 Relationship between side length of network models and permeability tensor

5. Analysis of equivalent permeability of DFN using fractal properties

In this paper, the box-counting method was utilized to determine the fractal dimension that can serve as one effective indicator to describe the geometrical distribution characteristics of rock fractures. According to the study of Falconer (1990), fractal dimension D_{Box} in the box-counting method can be computed using the following equation:

$$D_{Box} = \lim_{\delta \to 0} \frac{\log N_\delta}{-\log \delta} \qquad (3)$$

where, D_{Box} is the fractal dimension; N_δ is the number of boxes needed to cover the object; and δ is the box size.

Fractal dimension and equivalent permeability of rock fracture networks with number densities of $0.6\rho_0 \sim 1.4\rho_0$ (ρ_0 is the original number density) were calculated as shown in Figs. 6 and7. With increasing number densities of fractures, the equivalent permeability and fractal dimensions increase, too. Their regression equations were presented in Figs. 6-7.

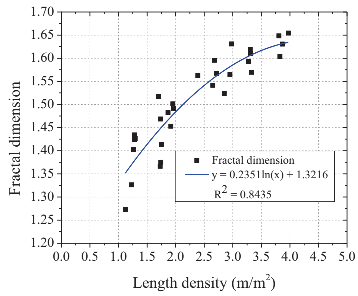

Fig.6 Relationship between length density and fractal dimension

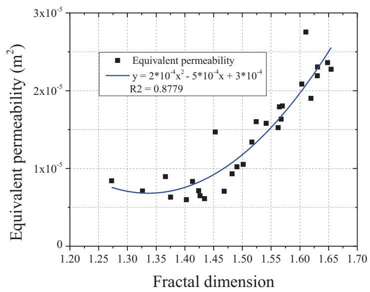

Fig.7 Relationship between fractal dimension and equivalent permeability

6. Conclusions

In this study, REV for the current FDN models was identified as 40m. Based on the fitting of percolation ellipses, the principle value and direction of percolation were investigated. The correlations between fractal dimension and equivalent permeability showed a second order linear relation.

References

Falconer, K.J., (1990). Fractal Geometry. John Wiley and Sons, London.