Skin friction of tapered piles related to stress-dilatancy effect and cavity expansion theory

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1 INTRODUCTION

The tapered pile is characterized by the difference in axial diameters of the pile at its top and bottom. A slight increase in tapering angle of the pile results in higher skin friction comparing with conventional straight piles. In this paper, effects of stress-dilatancy have been adopted in cavity expansion theory. In general one of the soil parameters (angle of internal friction or dilatancy angle) is assumed to be constant for the ease in computation. In the proposed model, this drawback has been improved and calculated the skin friction successfully. The results are verified using parametric study on different types of model tests, prototype tests, and real type pile tests.

2 STRESS DILATANCY EFFECT AND CAVITY EXPANSION THEORY

In stress-dilatancy relation, secant angle of internal friction, rate of dilatancy towards critical states and both effective stress and soil density parameters are interdependent by their strength parameters. Therefore, either rate of dilatancy or secant angle of internal friction cannot be avoided in the analysis. Moreover, density and confining pressure are affected through tapered pile penetration. Confining pressure increases with increasing relative density and angle of internal friction with decreasing dilatancy together with pile penetration (Manandhar 2010).

The stress-dilatancy relation established by Bolton (1986, 1987) has been introduced in Yu and Houlsby's (1991) cavity expansion theory to check the bearing behavior of different types of piles.

For a plane strain, the stress-dilatancy relation can be expressed in the following term:

$$\phi'_{max} - \phi'_{cv} = 0.8\psi_{max} = 5\, I_R^o \quad (1a)$$

$$I_R = I_D(10 - \ln p') - 1 \quad (1b)$$

Where, ϕ'_{max}, ϕ'_{cv}, ψ_{max}, and I_R^o are maximum angle of friction, angle of friction at critical states, maximum dilation angle and relative dilatancy index at plane strain respectively. The relative dilatancy index I_R is a function of relative density I_D and mean effective stress p' as shown in Eq. (1b). The mean effective stress can be simply defined as the mean radial and hoop stresses explained in the cavity expansion theory as follows:

$$p' = \frac{\sigma_r + \sigma_\theta}{2} \quad (1c)$$

$$p' = \frac{1}{2}\left[\frac{Y}{\alpha'-1} + Ar^{-\frac{(\alpha'-1)}{\alpha'}} + \frac{Y}{\alpha'-1} + \frac{A}{\alpha'} r^{-\frac{(\alpha'-1)}{\alpha'}}\right] \quad (2a)$$

Replacing the constant of integration A, the above equation is simplified into the following form as:

$$p' = -p_0 b^{\frac{(\alpha'-1)}{\alpha'}} r^{-\frac{(\alpha'-1)}{\alpha'}} \quad (2b)$$

The general Eq. (2b) can be simplified in the elastic-plastic region (Manandhar 2010). At the boundary of plastic region, the effective mean stress can be modified into the following structure as:

$$p' = -p_0 R \quad (3)$$

Where p', p_0, b, r, α and R are described in detail by Manandhar (2010).

3 SKIN FRICTION OF TAPERED PILES

In this section, a method explained by Kodikara and Moore (1993) has been utilized to see the effectiveness of the behavior. The skin friction-displacement relation can be segmented into three zones. At first, as elastic deformation when pile-ground interaction bonded together. Secondly, when slip has occurred at the pile-ground interface and ground still behaves elastic deformation. And finally, when slip has occurred and plastic zone has developed to obtain elastic perfectly plastic pile-ground interface. Assuming the small section of the pile as shown in Fig. 1(a), the stresses acting normal and parallel to the pile-ground interface respectively are shown by σ_n and τ_n as:

$$\tau_n = \sigma_n \tan(\phi_i + \alpha) + c_i \quad (4)$$

The vertical and radial components of the stresses at the interface (τ_0 and σ_0) govern this state in the form:

$$\tau_0 = \sigma_0 \tan(\phi_i + \alpha) + \frac{c_i \sec^2\alpha}{(1 - \tan\alpha \tan\phi_i)} \quad (5)$$

When slip occurs at the pile-ground interface and the vertical pile movement u_p at any point X on the pile-ground interface is greater than the vertical ground movement u_g at the corresponding point Y on the interface. As shown in Fig. 1(b), pile displaced from point X to X' while at the same time, ground moves from point Y to Y' in such a way that the component of lateral movement (radial expansion) v, can be determined in the form:

$$v = (u_p - u_g)\tan\alpha \quad (6)$$

For small taper angles, the increase in radial stress $\Delta\sigma$ for radial expansion may be calculated from cylindrical cavity expansion theory. When the pile-ground is continuously yielding, the vertical shear stress τ_x acting on the pile wall can be expressed as:

$$\tau_x = (\sigma_0 + \Delta\sigma)\tan(\phi_i + \alpha) + c_i' \quad (7)$$

Where, $c_i' = \frac{c_i \sec^2\alpha}{(1 - \tan\alpha \tan\phi_i)}$ (7a)

In Eq. (6), the ground deformation u_g may be related to shear stress τ_x. Using this approximation, the τ_x-u_p relationship for this phase can be written as:

$$\tau_x = \frac{K_e \tan\alpha \tan(\phi_i + \alpha) u_p + \sigma_0 \tan(\phi_i + \alpha) + c_i'}{1 + \frac{K_e \zeta r_m}{G} \tan\alpha \tan(\phi_i + \alpha)} \quad (8)$$

When (u_p > $(u_p)_Y$) or $\sigma > \sigma_Y$, it indicates a plastic zone adjacent to pile wall. In this case it will explain the influence of plastic zone which will extend further with more pile deformation. The corresponding vertical shear stress, τ_x can be expressed in terms of tangent gradient (K_p) as:

$$\tau_x = \left(\sigma_Y + \int_{vY}^{v} K_p\, dv\right)\tan\left(\phi_i + \alpha\right) + c_i' \qquad (9)$$

4 PARAMETRIC STUDY

In order to determine the skin friction in closed form, the variables were iteratively determined using a load transfer method. The results of small scale model test, proto type test together with full scale test have been validated through the results from the calculation in parametric study.

Two different types of sands Toyoura sand (TO) at high relative density (80 %) and K-7 sand at medium relative density (60 %)have been considered for the analyses. The chromium plated steel model piles, one straight (S) and two taper-shaped (T1 and T2), with equal lengths of 500 mm and same tip diameters of 25 mm were used for pile penetration.

Parameters of Fanshawe brick sand and pile materials have been adapted from Sakr et al. (2004, 2005, 2007) as a prototype. The cylindrical fiber-reinforced polymer (FRP) FC pile and another three tapered FRP composite tapered piles have been considered for analyses.

Similarly, a real type pile material has been taken for analyses. For this, the pile used by Rybnikov (1990) was accomplished. The top and bottom radii of piles were 200 mm and 100 mm (1.2^o tapering angle), one tapered pile with corresponding radii of 250 mm and 100 mm (2^o tapering angle) and for last two tapered piles had radii of 300 mm and 100 mm (2.4^o tapering angle) respectively.

Angle of tapering and normalized ratios of average vertical shear stress at 0.1 settlement ratio for all soils have been studied as the main effective key variables (Manandhar 2010).

Fig. 2(a) shows the effect of tapering angle in four types of soil by taking the ratios of average vertical shear stress divided by average vertical shear stress of straight pile. In the figure, with increase the tapering angle of the pile, average vertical shear stress increases. The parametric study shows that the most tapered angle shows 236 % increase in Fanshawe brick sand, 331 % in K-7 sand, 287 % in TO sand and 295 % increase by Rybnikov pile on sandy ground respectively. Similarly, the total skin friction of tapered pile divided by straight pile for all types of soils at settlement ratios of 0.1 was shown in Fig. 2(b). The total skin friction of maximum tapered pile of Fanshawe brick sand was about 6.5 times to more than 10 times in Rybnikov type full scale pile loading test at maximum tapering angle while compared with conventional straight piles.

5 CONCLUSION

Among different key variables, effects of tapering angle has been put in this paper as the parametric study on average vertical shear stress and total skin friction at 0.1 settlement ratios of pile penetration. It has been shown that the proposed model support strongly the general behavior of tapered piles to evaluate the skin friction. New proposed and extended form of cavity expansion theory on non-associated flow rule introducing Bolton's stress-dilatancy relationship can successively evaluate the skin friction of tapered piles without altering the basic formula of the cavity expansion theory. It has the ability to introduce the interdependent functions of confining pressure and relative density in angle of internal friction and dilation angle.

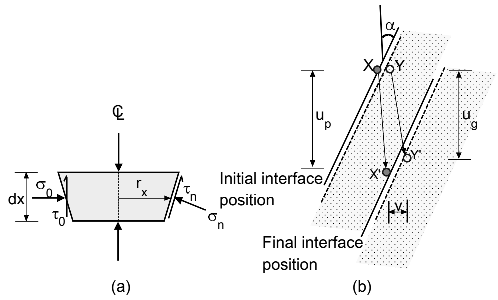

Fig. 1 (a) Segment of pile-ground interface; and (b) kinematics of initial and displaced position

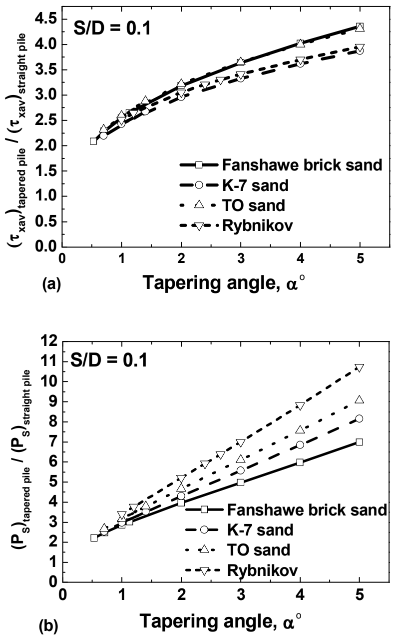

Fig. 2 Effect of angle of tapering on normalized average vertical shear stress (a) and (b) the total skin friction at 0.1 settlement ratios

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