Performing Transient Analysis by imposing Dirichlet Boundary Condition using Model Order Reduction (MOR) based on Krylov subspace method

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1. Introduction

Recently, Model Order Reductions (MOR) method based on Krylov subspace has been introduced using moment matching to create projectors that guarantee the reduced system interpolates the original system at given frequency.

Two methods to apply MOR for second order system are by converting the model into first order and direct projection to second order with structure preserving. Z. Bai[1] is using moment matching of Arnoldi method in second order Krylov subspace known as Second Order Arnoldi (SOAR) with structure preserving. The overview paper Z.Bai[1] and references theorem to derive reduce system can be referred.

Although some researchers prove the mathematic and show several numerical examples, this method still has a couple of problems when applying to engineering field. One of the major problems is to handle with Dirichlet boundary condition (DBC) as it's important for seismic response analysis. In this paper, procedures to apply DBC in the MOR have been introduced and analytical analysis is conducted to validate the efficiency of method.

2. Governing Dynamic Equation for Diriclet Boundary Condition

The matrix form governing equations after discretize with the consideration of DBC can be derives in dynamic equation form as follows,

$$\sum N:\begin{cases}\mathbf{M\ddot{u}(t)}+\mathbf{D\dot{u}(t)}+\mathbf{Ku(t)}=\alpha(t)\mathbf{F}\\ \mathbf{y(t)}=\mathbf{l}^T\mathbf{u(t)}\end{cases} \tag{1}$$

where $\mathbf{M}\in\Re^{N\times N}$, $\mathbf{D}\in\Re^{N\times N}$, $\mathbf{K}\in\Re^{N\times N}$ indicate mass, damping and stiffness matrix respectively. While $\mathbf{\ddot{u}(t)}\in\Re^N,\mathbf{\dot{u}(t)}\in\Re^N,\mathbf{u(t)}\in\Re^N$ and $\mathbf{F}\in\Re^N$ are acceleration, velocity, displacement and force vector respectively.

Using partition method to arrange and store the sparse matrix to constraint reduced system, it's become as;

$$\begin{bmatrix}\mathbf{M}_F & \mathbf{M}_{CF}\\ \mathbf{M}_{CF} & \mathbf{M}_C\end{bmatrix}\begin{Bmatrix}\mathbf{\ddot{u}}_F\\ \mathbf{\ddot{u}}_C\end{Bmatrix}+\begin{bmatrix}\mathbf{D}_F & \mathbf{D}_{CF}\\ \mathbf{D}_{CF} & \mathbf{D}_C\end{bmatrix}\begin{Bmatrix}\mathbf{\dot{u}}_F\\ \mathbf{\dot{u}}_C\end{Bmatrix}+\begin{bmatrix}\mathbf{K}_F & \mathbf{K}_{CF}\\ \mathbf{K}_{CF} & \mathbf{K}_C\end{bmatrix}\begin{Bmatrix}\mathbf{u}_F\\ \mathbf{u}_C\end{Bmatrix}=\begin{Bmatrix}\mathbf{f}_F\\ 0\end{Bmatrix} \tag{2}$$

where C-nodes are constraint nodes and F-nodes are unconstraint (free) nodes. The system matrix then is reduced to solve unconstraint nodes with an effect of constraint nodes for prescribed displacement (Directlet BC) on the RHS.

Applying Multiple Input Multi Output (MIMO) functions specify by $\alpha_i\in\Re^l$ which are distributed to every force vector in RHS which have prescribed value. The formulation of RHS of dynamic equation then become a system matrix of constraint node,

$$\begin{aligned}&\mathbf{M}_F\mathbf{\ddot{u}}_F+\mathbf{D}_F\mathbf{\dot{u}}_F+\mathbf{K}_F\mathbf{u}_F\\ &=\left(\alpha_F(t)\mathbf{f}_F-\alpha_M(t)\mathbf{M}_{FC}\mathbf{\ddot{u}}_C-\alpha_D(t)\mathbf{D}_{FC}\mathbf{\dot{u}}_C-\alpha_K(t)\mathbf{K}_{FC}\mathbf{u}_C\right)\\ &=\left[\mathbf{f}_F\mathbf{f}_M\mathbf{f}_D\mathbf{f}_K\right]\begin{bmatrix}\alpha_F(t)\\ \alpha_M(t)\\ \alpha_D(t)\\ \alpha_K(t)\end{bmatrix}\end{aligned} \tag{3}$$

3. Dimension of Reduction using BSOAR

The state vector u(t) of original system can be projected by another state vector z(t) constrained in second order Krylov subspace of $\varsigma_n(\mathbf{DK}^{-1},\mathbf{MK}^{-1};\mathbf{q_o})=span\{\mathbf{Q}_n\}$, with relation equation as $\mathbf{u(t)}\approx\mathbf{Q}_n\mathbf{z(t)}$. Thus, the reduced system then become as;

$$\sum n:\begin{cases}\mathbf{M}_n\mathbf{\ddot{z}(t)}+\mathbf{D}_n\mathbf{\dot{z}(t)}+\mathbf{K}_n\mathbf{z(t)}=\alpha_l(t)\mathbf{F}_n\\ \mathbf{\tilde{y}(t)}=\mathbf{l}_n{}^T\mathbf{z(t)}\end{cases} \tag{4}$$

where $\mathbf{M}_n=\mathbf{Q}_n^T\mathbf{MQ}_n,\mathbf{D}_n=\mathbf{Q}_n^T\mathbf{DQ}_n,\mathbf{K}_n=\mathbf{Q}_n^T\mathbf{KQ}_n$ are n x n matrix, and $\{\mathbf{F}\}_n$ and $\mathbf{l}_n{}^T$ are n x l vectors, with $\mathbf{F}_n=\mathbf{Q}_n^T\mathbf{F}$ and $\mathbf{l}_n=\mathbf{Q}_n^T\mathbf{l}$. To extent into MIMO, modification of algorithms is known as BSOAR procedure developed by Yiqin L [2] described in Algorithm 1.

4. Numerical experiments

A small model with dimension of 0.2x0.2x0.2 m square hollow for 8 stacks slender is created. Material property is isotropic, with Young modulus density, ρ and poison ratio, ν are 196 GPa, 7.95 x 10^3 kg/ m^3 and 0.3 respectively. The coefficients for Newmark-B are β=0.25 and γ=0.5. The model is prescribed displacement at 1 mm at bottom's constraint node under sinusoidal load history as below;

$$\alpha_1 = \sin(2\Pi f t)$$
$$\alpha_2 = 2\Pi f \cos(2\Pi f t)$$
$$\alpha_1 = -4\Pi^2 f^2 \sin(2\Pi f t) \qquad (5)$$

Algorithm 1: BSOAR procedure

```
Q0=[q1,q2,…,ql]
p0 = 0
for j =1,2,…,m1xl do
    r =DK^-1 qj + MK^-1 pj
    s =qj
    for i =1,2,….,j do
    tij =qi^T r
     r := r - qj tij
     s := s - pj tij
        end for
        tj+l j = ||r||2
        if tj+l j = 0, breakdown
        else  %
        qj+l = r / tj+1 j
        pj+l = s / tj+1 j
        end if
    end for
```

5. Results

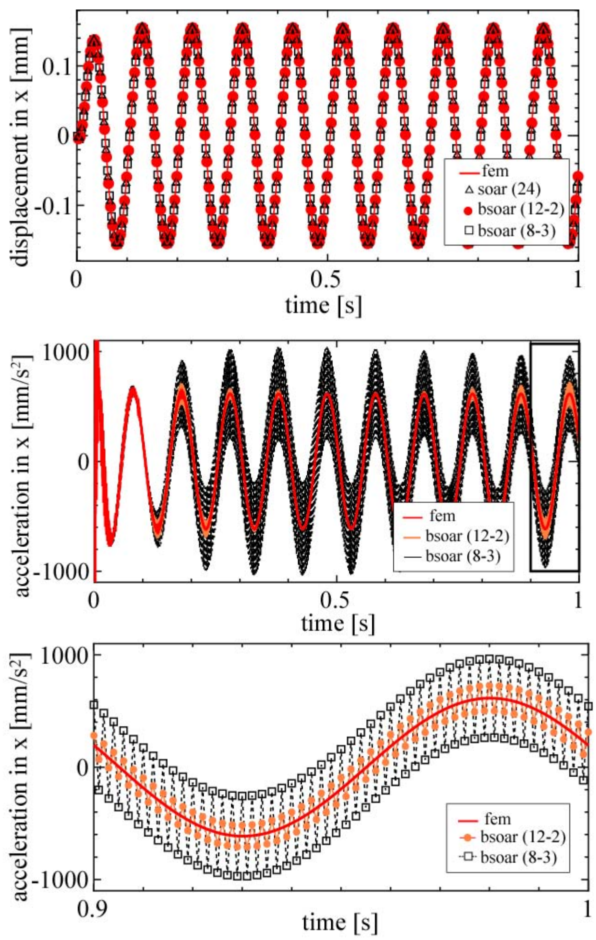

Figure 1: Displacement and Acceleration Time History for 24 bases

Results show in Figure 1 that MOR has shown very good tendency and an accurate result for all displacement and velocity compare to conventional FEM method. In acceleration, the mean show similarity but there are small oscillation occur in MOR system due to high frequency acceleration that create high frequency wave. It has proved in literature that MOR is not best fit for high frequency system. Increasing basis to 48 have correct the high frequency effect occurs in acceleration. Deformation of structure by stress analysis is shown in Figure 2. Thus, both have same behaviour and prove that MOR has preserve moment matching property of original.

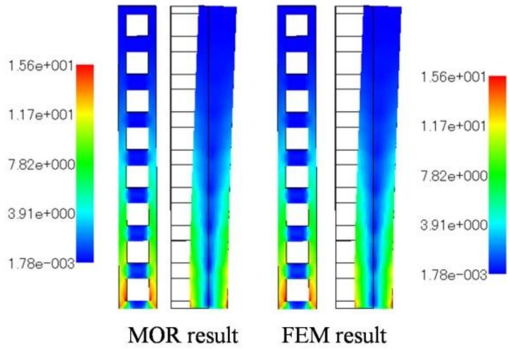

Figure 2: Von Mises Stress of both models

Table 1 shows summary result of numerical time cost for this model. With a model of 152,019 DOF, it shows an accurate result by projecting to as low as 12 bases with a computational time as 2.35 % from FEM method.

No of basis	Time MOR (sec)	Time FEM (sec)	Percentage Different (%)
12	495	21101	2.35
24	598		2.83
48	987		4.68

Table 1: Numerical Time cost

6. Conclusion

Using Multi Input for handling DBC in MOR shows an accurate result as conventional method. MOR can reduce time as fast as 42 times compare to FEM for accurate results using direct solver. Solution with iterative solver for large scale system is further study.

Reference

(1) Z.Bai, Su Yan Fen, Second Order Krylov Subspace and Arnoldi Method, Journal of Shanghai University(English Edition), 8(4):pp378-390,2004

(2) Y.Lin, L.Bao, Moel Order reduction of large –scale second order MIMO dynamical systems via block second order Arnoldi method, Int. Journal of Comp. Math., Vol 84, No 7, July 2007, 1003-1019