# SHAPE ANALYSIS BY COMBINATION OF AXIAL LINE ELEMENTS AND SOAP FILM ELEMENTS

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## 1. Introduction

In the case of pneumatic structure, the shape analysis is to find rational curved surfaces mechanically and geometrically. The perfect equilibrium solution with isotonic tensile can be obtained by shape analysis using tangent stiffness method. The solutions constitute the polyhedrons which can be similar in curved surfaces of soap film. When we evaluate the polyhedron equilibrium solution, generally the approximation performance is depending on their mesh density and mesh distribution.

This study proposes a new approach to obtain the polyhedron equilibrium solutions, combining the axial line element and triangular soap film element. By using this approach, the node distribution becomes more rational, and the large compulsory displacement can be adopted when the simultaneous control is used as the incremental method.

## 2. Analysis method

### 2.1 Tangent stiffness method

Let the vector of the element edge forces independent of each other be indicated by **N**, and let matrix of equilibrium which relates **N** to the general coordinate system by **J**. Then the nodal forces **U** expressed in the general coordinate follow the equation:

$$U = JN \tag{1}$$

The tangent stiffness equation is expressed as the deferential calculus of Eq.(1), as follow

$$\delta U = J\delta N + \delta JN = (K_0 + K_G)\delta u \tag{2}$$

In which, $K_0$ is the element stiffness which provide the element behavior in element (local) coordinate, and $K_G$ is the tangent geometrical stiffness.

### 2.2 Simultaneous control

It is a incremental method which can be adopted by tangent stiffness method. The technique controls both of compulsory displacements on a control point and the inner pressure derived from the shape in each iteration step at the same time, when it is applied to shape analyses under inner pressure. Also, the technique can find high-rise and large volume shapes, which are in the state over the maximum point of its p-ν path.

### 2.3 The axial force line elements located in tangent plane direction

#### 2.3.1 triangular soap film element

Element measure potential can be written as follows:

$$P = \sigma t A \tag{3}$$

The element edge force

$$N_i = \frac{\partial P}{\partial l_i} = \frac{1}{2}\sigma t r_i \qquad (i = 1,2,3) \tag{4}$$

In which $r_i$ is the distance from the orthocenter of triangle to each node.

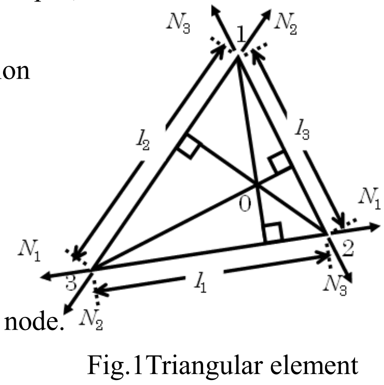



Fig.1Triangular element

#### 2.3.2 axial line element

Element measure potential

$$P = CL^n \tag{5}$$

The element edge force

$$N_i = \frac{\partial P}{\partial l_i} = nCL^{n-1} \tag{6}$$

In which, C is the line element proportional coefficient.

2.3.3 Combination of tangent geometrical stiffness

Tangent geometrical stiffness by a line element

$$K_L = \begin{bmatrix} k_l & -k_l \\ -k_l & k_l \end{bmatrix} \quad k_l = nCL^{n-2}\left[e+(n-2)\alpha\alpha^T\right] \qquad (7),(8)$$

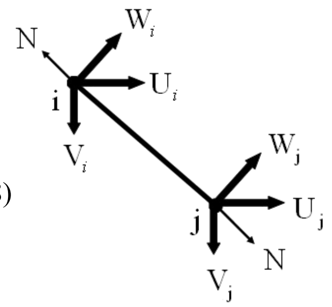


Fig.2 Axial line element

Tangent geometrical stiffness by a soap film element

$$K_G = \begin{bmatrix} k_{G2}+k_{G3} & -k_{G3} & -k_{G2} \\ -k_{G3} & k_{G3}+k_{G1} & -k_{G1} \\ -k_{G2} & -k_{G1} & k_{G1}+k_{G2} \end{bmatrix} \quad k_{Gi} = \frac{N_i}{L_i}(e-\alpha_i\alpha_i^T) \quad (i=1,2,3) \qquad (9),(10)$$

In which, **e** is unit matrix. Further, **α** is the component of cosine vector along side direction.

$$K = K_{GSoap} + K_{GLine} \qquad (11)$$

In which, $K_{GSoap}$ is the total stiffness matrix of triangular soap film element. $K_{GLine}$ is the total stiffness matrix of line element.

## 3. Analytical results

3.1 Setting the control point in arbitrary position.

Setting the control point A and control point B on the fig.3 and fig.4 respectively, the solution shape in the example is quite the same.

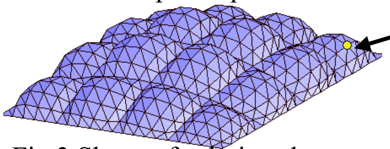


Fig.3 Shape of solutions by control point A

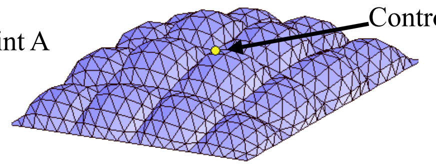
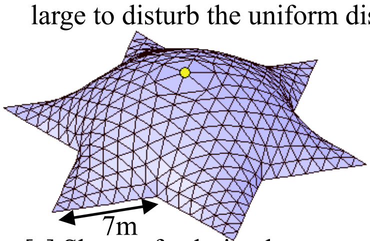


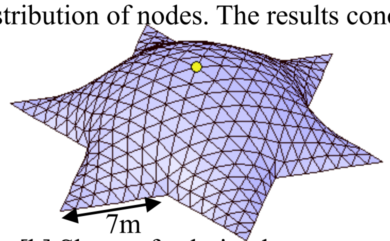
Fig.4 Shape of solutions by control point B

3.2 Combination of Triangular soap film element and axial line element

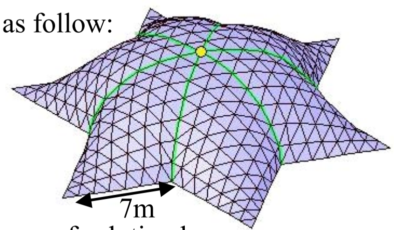
The hexagonal shape in fig.5[a,b] is meshed by triangular soap film elements, and the pitch of compulsory displacement is set 0.5m and 0.1m respectively. By contrast, the hexagonal shape in fig.5[c] is meshed by combination of Triangular soap film elements and axial line elements, and the pitch of compulsory displacement is set 1m. The fig.3[a] shows that because of adopting large compulsory displacement, the area of triangular elements, around the control point, become too large to disturb the uniform distribution of nodes. The results conclude as follow:



[a] Shape of solution by compulsory displacement=0.5m



[b] Shape of solution by compulsory displacement=0.1m



[c] Shape of solution by compulsory displacement=1m, n=2

Fig.5 The total displacement on control point=5m

(1)The large compulsory displacement can be adopted in shape analysis of combination of line element and triangular element.

(2)The fig.6 shows that the unbalanced force quickly converged by using the tangent stiffness method in the example of fig5[c].

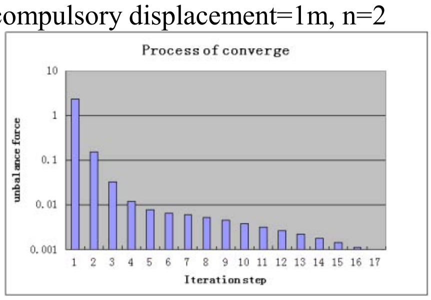


Fig.6 Convergent process of unbalanced force

## 4. Conclusions

In this study, the triangular soap film element and axial line element can be expected to determine composite structure of membrane and cable. We combined the triangular soap film element and axial line element to obtain the perfect equilibrium solution with isotonic tensile in the example. And also we set the two control points in different positions, however the solution shape in the example is quite the same.