Highway bridge dynamic characteristics estimation used by ERA,ERA/DC, ARE methods for ambient vibration

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1.<u>Introduction</u>: In this study, highway bridge dynamic characteristics (frequency, damping, and vibration mode) were estimated by using ERA, ERA/DC, ARE methods for ambient vibration. The process was repeated for various level of noisy ambient vibration and the successive deviation of estimation error are recorded. The performance of three methods was observed regarding computational speed as well as for estimation error.

2.<u>Basic formulation</u>: Discrete time, linear and time invariant dynamic system can be represented by the state-variable equations;

$$\mathbf{x(k+1) = Ax(k) + Bu(k)} \quad \text{and} \quad \mathbf{y(k) = Cx(k) + Du(k)} \tag{1}$$

where, $\mathbf{x} \in \mathbf{R}^{\mathbf{n}}$, $\mathbf{u} \in \mathbf{R}^{\mathbf{m}}$, $\mathbf{y} \in \mathbf{R}^{\mathbf{l}}$ and $\mathbf{A} \in \mathbf{R}^{\mathbf{n \times n}}$

Impulse response function can be found from the Eq.(1) i.e.

$$\mathbf{Y(k)} = \begin{cases} \mathbf{D} & \mathbf{k = 0} \\ \mathbf{CA^{p-1}B} & \mathbf{k = 1, \cdots, p} \end{cases} \tag{2}$$

The constant matrixes in this sequence are same as covariance matrix obtained by ambient vibration of bridges.

3.<u>Realization theory</u>:

(1) ERA method (Eigensystem Realization Algorithm):

Introducing Hankel matrix from covariance matrix introduced in Eq.(2), i.e.

$$\mathbf{H(k-1)} = \begin{bmatrix} \mathbf{Y_{(1)}} & \cdots & \mathbf{Y_{(\beta-1)}} \\ \vdots & & \vdots \\ \mathbf{Y_{(\alpha-1)}} & \cdots & \mathbf{Y_{(\alpha+\beta-2)}} \end{bmatrix} = \mathbf{P_\alpha A^{k-1} Q_c} \tag{3}$$

Factorization of Hankel matrix, Eq. (3), for $k=1$ and $k=2$, using singular value decomposition;

$$\mathbf{H(0) = P_\alpha Q_\beta = USV^T = US^{1/2}S^{1/2}V^{\mathit{T}}} \quad \text{where, } \mathbf{P_\alpha = U^{1/2}S^{1/2}} \quad \mathbf{Q_\beta = S^{1/2}V^T} \tag{4}$$

$$\mathbf{H(1) = P_\alpha AQ_\beta} \ ; \ \mathbf{A = U_n S_n^{-1/2} H(1) S_n^{1/2} V_n^T = P_\alpha^{-1} H(1) Q_\beta^{-1}} \qquad \mathbf{C = E_m^T U_n S_n^{1/2}} \tag{5}$$

where, $\mathbf{E_m^T}$ assumed as a $\mathbf{m}$ observation point.

(2) ERA/DC method（ERA with Data Correlations）:

From covariance matrix stated in Eq.(2);

$$\mathbf{R_{hh}(k-1) = H(k-1)H^T(0)} = \begin{bmatrix} \sum_{i=1}^{\beta} \mathbf{Y_{k+i}Y_i} & \cdots & \sum_{i=1}^{\beta} \mathbf{Y_{k+i}Y_{\alpha+i-1}^T} \\ \vdots & & \vdots \\ \sum_{i=1}^{\beta} \mathbf{Y_{k+\alpha+i-1}Y_i^T} & \cdots & \sum_{i=1}^{\beta} \mathbf{Y_{k+\alpha+i-1}Y_{\alpha+i-1}^T} \end{bmatrix} = \mathbf{P_\alpha AQ_c} \tag{6}$$

where, $\mathbf{Q_c = Q_\beta Q_\beta^T P_\alpha^T}$ .Similar to the ERA method, Eq. (6) for $k=1$, $k=2$ becomes, $\mathbf{R_{hh}(0) = H(0)H^T(0) = P_\alpha Q_c}$; $\mathbf{R_{hh}(1) = H(1)H^T(0) = P_\alpha AQ_c}$ (7)

Execution of singular value decomposition same as to ERA method;

$$\mathbf{R}_{hh}(0) = \mathbf{P}_\alpha \mathbf{Q}_c = \mathbf{USV}^T = \mathbf{US}^{1/2}\mathbf{S}^{1/2}\mathbf{V}^T \tag{8}$$

$$\mathbf{A = S_n^{-1/2} U_n^T R_{hh}(1) V_n S_n^{-1/2} = P_\alpha^{-1} R_{hh}(1) Q_\beta^{-1}} \ , \quad \mathbf{C = E_m^T U_n S_n^{1/2}} \tag{9}$$

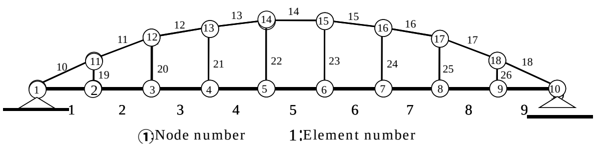

Fig.-1: Model bridge

Table-1: Model dimensions

Bridge characteristics	Units	Numerical value
Effective span	L (m)	58.995
Ridge of the bridge	f(m)	9.36
Density of steel	D (t/m^3)	7.85
Young's Modulus	E (t/m^2)	2.1*10^7
Total mass of the bridge	Ton	19.3
Total weight of bridge	Ton	189.17
Sectional area	A1 (m^2)	0.0224
	A2 (m^2)	0.0123
	A3 (m^2)	0.0137
	A4 (m^2)	0.006015
Moment of inertia	I1 (m^4)	0.00066
	I2 (m^4)	0.00001
	I3 (m^4)	0.000001

Table-2: Natural frequencies. (Hz)

Order	Natural frequency (Hz)
1st	1.749
2nd	2.893
3rd	5.69
4th	7.375
5th	9.416
6th	11.179
7th	14.236
8th	15.615

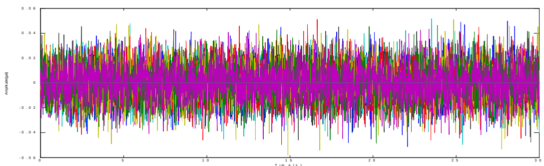
Fig.-2: White noise

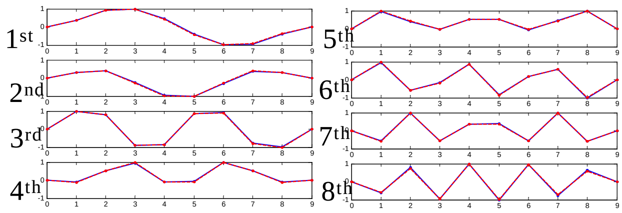

Fig.-3: Vibration mode

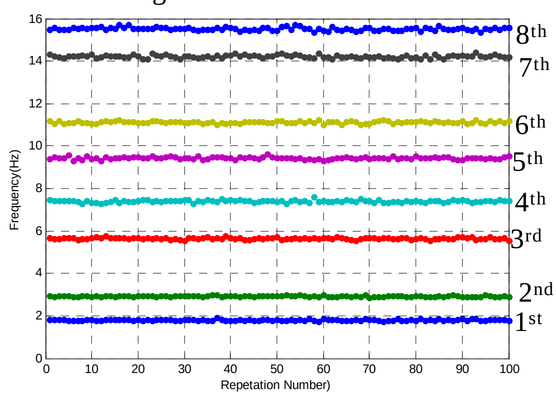

Fig.-4: Frequencies (Hz)

(3) ARE method (Algebric Riccatti Equation): In view of Eq. (4) the controllability and observability matrices can be expressed by the following equations; $\mathbf{P}_\alpha = \mathbf{U}_n^{1/2}\mathbf{S}_n^{1/2}$ and $\mathbf{Q}_\beta = \mathbf{S}_n^{1/2}\mathbf{V}_n^T$ (10) Define $\mathbf{P}_\alpha^{\downarrow m}$ as the matrix formed by deleting the first m rows of $\mathbf{P}_\alpha$ and $\mathbf{P}_\alpha^{\uparrow m}$ as the matrix formed by deleting the last m rows of $\mathbf{P}_\alpha$. The system matrix can be then found from the relation; $\mathbf{U}_n^{\downarrow m}\mathbf{S}_n^{1/2}\mathbf{A} = \mathbf{U}_n^{\uparrow m}\mathbf{S}_n^{1/2}$; $\mathbf{A} = \mathbf{S}_n^{-1/2}\left[\mathbf{U}_n^{\downarrow m}\right]^{+}\mathbf{U}_n^{\uparrow m}\mathbf{S}_n^{1/2}$ (11)

4.<u>Calculation of dynamic characteristics</u>: System matrix $\mathbf{A}$ estimated by three methods yields eigenvector and eigenvalue. Frequencies ω_k and damping h_k can be then calculated from the real X_{Re} and imaginary X_{Im} part of eigenvalue, by using the next expressions;

$$\omega_k\sqrt{1-h_k^2} = (1/\Delta)\tan^{-1}(X_{Im}/X_{Re}) \qquad (12)$$

$$h_k\omega_k = (-1/\Delta)\ln\sqrt{X_{Re}^2 + X_{Im}^2} \qquad (13)$$

whereas, vibration mode can be found from eigenvector.

5.<u>Numerical simulation</u>: Langer steel bridge (in **Fig.**-1) has modeled by using FEM method that's properties are indicated in **Table**-1. Natural frequencies (in **Table**-2) and vibration mode are calculated from the eingemvalue analysis. For dynamic analysis damping constant is assumed to 0.02 and applying white noise (in **Fig.**-2) on 8-points (2~8) in model bridge. The velocity response (ambient vibration) then calculated which has been used as an input force of realization methods. Dynamic characteristics are then estimated by using ERA, ERA/DC, and ARE methods for 0%, 10%, 20%, and 50% noisy ambient vibration. This paper shows only the result from 0% noisy data and the result from others data could be shown at the time of final presentation.

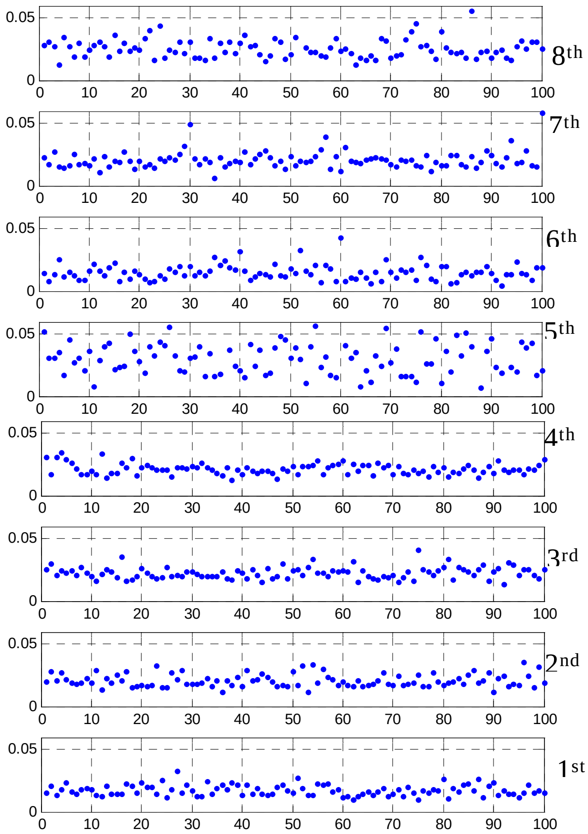

Fig.-5: Damping

Table-3: Estimated frequency. and damping.

	Frequency(Hz)					Damping Constant				
	Analy.	Mean	Err(%)	Std.	CV(%)	Analy.	Mean	Err(%)	Std.	CV(%)
ERA	1.723	1.800	4.46	0.0275	1.528	0.02	0.032	60.00	0.015	46.856
ERA/DC		1.800	4.46	0.0275	1.528		0.032	60.00	0.015	46.856
ARE		1.906	10.62	0.0373	1.958		0.1057	428.50	0.0221	21.00
ERA	2.864	2.920	1.95	0.025	0.858	0.02	0.015	25.000	0.0063	42.169
ERA/DC		2.920	1.95	0.025	0.858		0.015	25.000	0.0063	42.169
ARE		2.956	3.21	0.0203	0.688		0.0568	184.00	0.0103	18.149
ERA	5.625	5.645	0.35	0.0417	0.739	0.02	0.0207	3.50	0.0071	34.20
ERA/DC		5.645	0.35	0.0417	0.739		0.0207	3.50	0.0071	34.20
ARE		5.675	0.88	0.031	0.546		0.0554	177.00	0.0083	15.00
ERA	7.375	7.396	0.28	0.0494	0.668	0.02	0.0259	29.50	0.0084	32.462
ERA/DC		7.396	0.28	0.0494	0.668		0.0259	29.50	0.0084	32.462
ARE		7.397	0.30	0.0369	0.499		0.0522	161.00	0.008	15.386
ERA	9.491	9.431	0.63	0.052	0.551	0.02	0.0173	13.50	0.0043	24.993
ERA/DC		9.431	0.63	0.052	0.551		0.0173	13.50	0.0043	24.993
ARE		9.349	1.50	0.3034	3.245		0.0515	157.50	0.0274	53.18
ERA	11.06	11.120	0.54	0.0524	0.471	0.02	0.0208	4.00	0.0051	24.638
ERA/DC		11.120	0.54	0.0524	0.471		0.0208	4.00	0.0051	24.638
ARE		11.094	0.30	0.3037	2.738		0.0454	127.00	0.0053	11.686
ERA	14.099	14.221	0.86	0.0698	0.491	0.02	0.0226	13.00	0.0047	21.00
ERA/DC		14.221	0.86	0.0698	0.491		0.0226	13.00	0.0047	21.00
ARE		14.056	0.30	0.5162	3.672		0.0443	121.50	0.0048	11.00
ERA	15.411	15.520	0.70	0.0708	0.456	0.02	0.0213	6.50	0.0043	20.40
ERA/DC		15.520	0.70	0.0708	0.456		0.0213	6.50	0.0043	20.40
ARE		15.449	0.24	0.2393	1.549		0.0463	131.50	0.0053	11.548

6.<u>Discussion on estimated result</u>: The numerical values of dynamic characteristics were obtained from ambient vibration. **Fig.**-3, 4, 5 shows the vibration mode, frequencies and damping constant. Results of three methods from 0% noisy ambient vibration are presented in **Table**-3. The maximum estimation error for frequency is less than 5% for ERA, ERA methods and around 10% for ARE methods. The estimation error for damping constant is 60% for ERA, ERA/DC method while 428% for ARE methods. The addition of 10%, 20%, and 50%, noise does not shows so much deviation as compared to the percentage of noise present in input data. **Table**-4 shows the performance of used methods regarding calculation speed and estimation error.

Table-4: Performance comparison

Method	calculation speed	Estimation error	
ERA	○	○	○
ARA/DC	◎	○	○
ARE	△	△	△

7.<u>Conclusion</u>: Frequency, damping and vibration mode of highway bridge were estimated. Estimation error for ERA, ERA/DC method is almost same and less than ARE. Computation speed for ERA/DC is higher than other two methods. So, it can be conclude that ERA/DC exhibited better performance than other two methods regarding calculation speed and estimation accuracy.

Reference: [1] Jer-Nan Juang: An applied system Identification, Prentice Hall PTR, 1994