Basic study on fracture mechanics analysis using finite element with drilling DOFs

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１．Introduction

In fracture mechanics analyses by finite element method, finite element mesh generation for complex crack shape is a big labor. On the other hand, free mesh method (FMM) developed by Yagawa[1)] can calculate an auto crack propagation analyses. However, FMM cannot use a quadric element with the middle position node in local element generation algorithm. Therefore, highly accurate FMM using finite element with drilling degree of freedoms which is one of the generalized element is proposed[2), 3)].

In this paper, a basic research of the fracture mechanics analysis which used the finite element with drilling degree of freedoms is discussed. As a numerical example, the stress intensity factor is calculated by the displacement method.

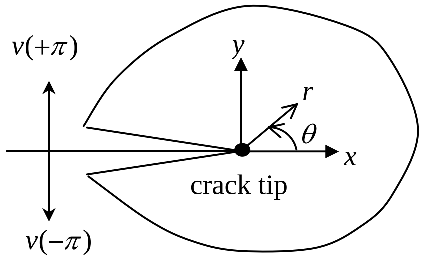

Fig1. Polar coordinate system at crack tip

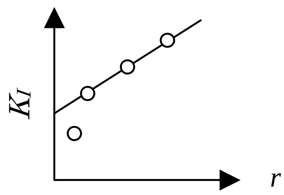

Fig2. Stress intensity factor calculated by displacement method

２．Calculate of stress intensity factor using displacement method

This section explains the calculating method of the stress intensity factor which uses the displacement method[4)]. Considering a polar coordinate system as shown in Fig. 1, stress intensity factor can be computed by

$$K_I = \frac{\{v(+\pi) - v(-\pi)\}}{\sqrt{r}} \frac{\sqrt{2\pi} G}{\kappa + 1} \qquad (1)$$

Where $v(+\pi)$, $v(-\pi)$ are nodal displacement at crack line, K_I is the stress intensity factor of mode I, r is a distance from the crack tip, G is a shear modulus. κ can be calculated by $\kappa = 3 - 4\nu$ for the plane strain and $\kappa = (3-\nu)/(1+\nu)$ for the plane stress , ν is poisson's ratio. In order that accuracy may improve more the stress intensity factor computed from the eq.(1), a least-squares m-

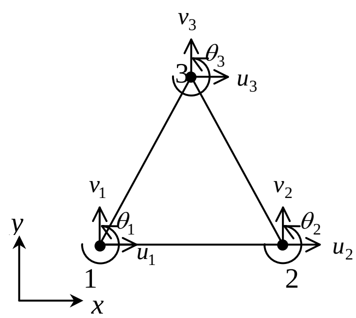

Fig3. Finite element with drilling DOFs

ethod as shown in Fig.2 is used.

３．Triangular element with drilling DOFs

Sekiguchi-Kikuchi assumed displacement fields of the finite element with drilling degree of freedoms is expressed as

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$$\begin{cases} u = \sum_{i=1}^{3} (u_i - (y - y_i)\theta_i) N_i \\ v = \sum_{i=1}^{3} (v_i + (x - x_i)\theta_i) N_i \end{cases} \quad (2)$$

where, u_i, v_i are nodal displacement, x_i, y_i are coordinate of triangular element at top, θ_i is drilling degree of freedoms, N_i is shape function of linear triangular finite element. Thus, Sekiguchi-kikuchi's finite element stiffness matrix $[k]$ obtained by eq.(3).

$$[k] = t\int_0^1 \int_0^{1-L_1} [B]^T [D] [B] \det [J] dL_1 dL_2 \quad (3)$$

where $[J]$ is Jacobian matrix, $[B]$ is strain - displacement matrix, $[D]$ is elasticity matrix.

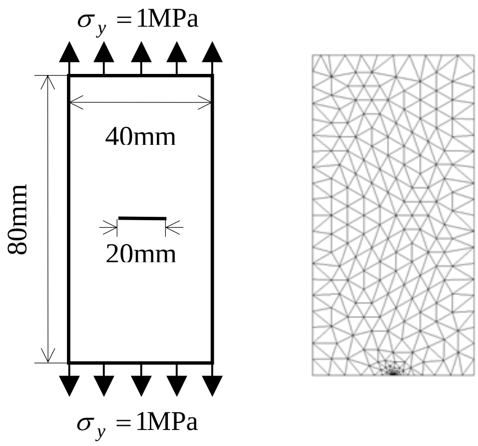
４．Numerical examples

Problems of center cracked under tensile stress loading in Fig.4(a) are solved. The finite element division is shown in Fig.4(b). Here, Young's modulus and Poisson's ratio are set to be 210GPa and 0.3.

The results for the stress intensity factor and crack opening displacement at crack center are present in Tab.1 and Tab.2, respectively. Comparisons of the error of stress intensity factor at each element are: liner element is 4.3%, quadric element is within 1% and present is 2.3%. In addition, comparison of the crack opening displacement are : liner element is 5.1%, quadric element is within 1% and present is 2.3%, respectively.

５．Conclusions

In this paper, the stress intensity factor was calculated by displacement method as basic study of application to the fracture mechanics analyses using the finite element with drilling degree of freedoms. As a numerical result, present method calculation of stress intensity factor and crack opening displacement was better than constant strain triangle element. In addition, present method is possible the high accuracy of the fracture mechanics analysis of free mesh method.



(a) Analysis model (b) Element division(1/4 model)

Fig4. Center cracked subject to tension

Tab1. Result of stress intensity factor

Element	K_I	Error(%)
Liner	6.348	4.326
Quadric	6.692	0.868
Pressent	6.484	2.267
Theory	6.635	---

Tab2. Result of crack opening displacement at crack center

Element	COD	Error(%)
Linear	2.15E-04	5.112
Quadric	2.25E-04	0.767
Pressent	2.22E-04	2.289
Theory	2.27E-04	---

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