Numerical simulation of flood flow in river networks by graph theory

宮崎大学工学部　　○学生会員　NGUYEN LE TUAN　　正会員　杉尾哲

1. Introduction

The well-known St. Venant equations mathematically describing a gradually varied unsteady flow in open-channels are usually used for the subject of flood routing with the finite difference method. The common numerical methods for implicit finite difference equations are double-sweep method and Newton's iteration method. In the case of a river network, it needs some modifications as attempts to avoid solving complicated and extremely large coefficient matrices resulting from the full-system approach. In this study, an improvement of simultaneous solution based on the double-sweep method is introduced and applied for flood routing in river networks. From this improvement, the simultaneous solution can be applied for any branched network regardless of its complexity.

2. Governing equations and simultaneous solution

Gradually varied unsteady flows in open-channel described by the St. Venant equations for one-dimensional free-surface flow with continuity equation and the momentum equation as follows:

$$\frac{\partial y}{\partial t}+\frac{1}{b_s}\frac{\partial Q}{\partial x}=0 \qquad (1)$$

$$\frac{\partial Q}{\partial t}+\frac{\partial}{\partial x}\left(\frac{\alpha Q^2}{A}\right)+gA\frac{\partial y}{\partial x}+gAS_f=0 \qquad (2)$$

The St. Venant equations can be solved numerically by approximating them with a set of finite difference algebraic equations. In this study, the widely-used Preissmann implicit finite difference scheme is applied. A convergent discretized form of the basic relationships, together with appropriate boundary conditions, furnishes a system of non-linear algebraic equations in terms of unknown flow variables at the next time. The double-sweep method is usually used because of its advantages. As its name, the double-sweep method includes two "sweeps" along a river reach. The first sweep is forward sweep or elimination sweep and the second sweep is backward sweep or substitution sweep. In other words, there are two steps in calculation process. In the first step, the coefficients are calculated for all cross sections (computational points) of the reach and then the unknown values of Q and y are obtained by back substitution in the second step.

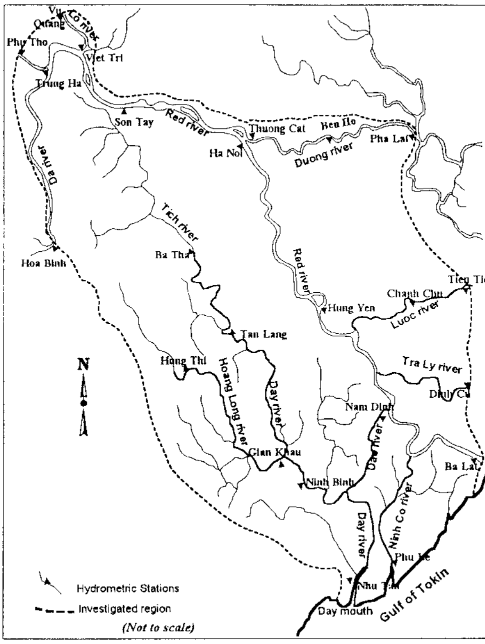

Fig. 1 Map of the investigated region

3. Modeling the river network for double-sweep solution

The double-sweep method can be applied for dendritic or branched river networks with some modifications. For branched networks, the main problem is determining the order for elimination in forward sweep. That is, the network will be rearranged so that the double-sweep method can be applied as in the case of a single reach. The river network is modeled by a basic element of river reach or branch (i.e., link in graph theory). Each reach has two nodes at two ends. The reach's index and node's index are independently numbered as integer numbers. There is no strict criterion for numbering river reaches and nodes, except the number of a reach (or a node) must not be coincided with others. As double-sweep method is applied only to subcritical flows, it is not necessary to make distinction between upstream and downstream boundary in the elimination process. Thus, the "positive" flow direction in a reach from one node to the other is supposed firstly. When the computation completed, if the value of discharge is negative, it means that the real flow direction in the reach is opposite with

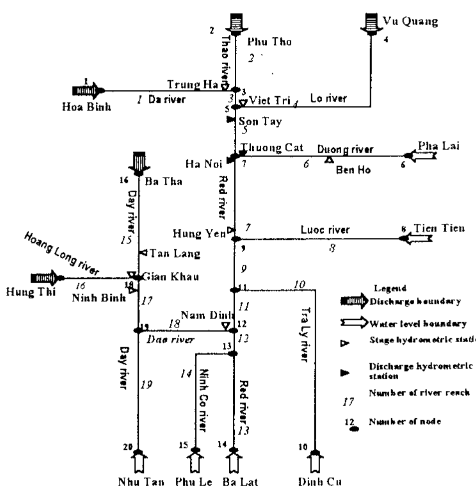

Fig. 2 Schematic of investigated river network

the supposed one. By this way, the river network can be described by a set of river reaches and then the respective nodes. Mathematically, it can be described by two vectors or two lists. The first list is the list of river reach's indexes (*ListR*) and the second one is the list of node's indexes (*ListN*). The second depends upon the first and has a relationship with the first: if the *ListR* has *n* elements, the *ListN* will have *2n* elements.

The two lists are set up simply. At first, the *ListR* is initiated with free order. From the *ListR* and respective supposed flow direction in each reach, the *ListN* is constructed. The procedure to look for the order of elimination now will work with these two lists. Its substance is the alternative progress of searching unique nodes in the *ListN* and removing the respective reach from *ListR* until *ListN* is empty. The orders of reaches that are eliminated successively, as well as the direction of elimination in each reach are both formed by this procedure. They are *ListROrder* and *ListNOrder*, respectively. A problem can occur when the last appeared element in *ListNOrder* (last node) is not a boundary node. In this case, the solution at this last node must be resolved theoretically by solving the equations written for this node. This should lead a complexity in computation, so the two lists (*ListROrder* and *ListNOrder*) need to be modified and rearranged so that the last node always is the boundary condition node. At last, the new *ListROrder* and new *ListNOrder* are received. Combining these two lists by mixing the elements with the form as **node** → *river reach* → **node** → **node** → *river reach* → ... → **node**, the final order for computation is found out, named *FinalList*. It controls the elimination sweep.

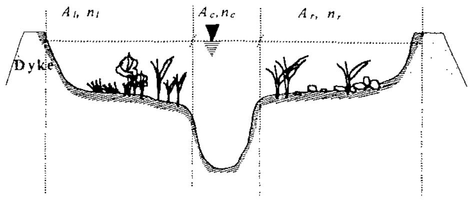

Fig. 3. Typical cross section in the investigated network

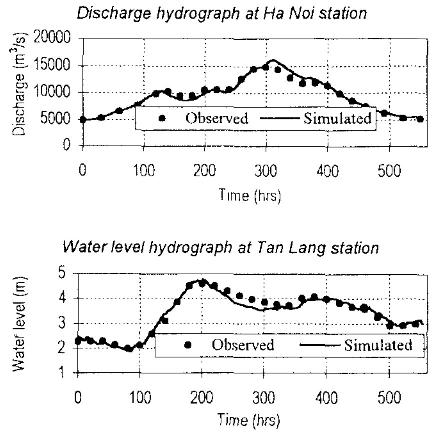

Fig. 4 Results from the model applied for a part of the Red river network.

4. Application and results

The model is applied for a part of the Red river basin, a significant basin in Vietnam. The map of this part is shown in **Fig. 1**. The investigated river network is described by reaches and nodes as shown in **Fig. 2**. The total length of the river reaches using in computation is about 900 km with 262 cross sections. The distance between a cross section to adjoining one varies from 400-500 m to 3-4 km, even to 6 km. The typical shape of cross sections in the network is the form of compound section (**Fig. 3**). The network is virtually dyked with extensive floodplains about 200-300 m; especially 2-3 km at some places. The floodplain is cultivated area. The cover conditions of floodplain vary strongly in space and time. The flow regime in the region is affected by both river flow and tide from the sea and the bottom slope of riverbed is mild (about 0.5–0.01%). The flood from 9 to 31, August 1996 is taken into calibration process. This flood was a large flood in the Red river basin. Results of calibration are shown in **Fig. 4**. The error in discharge is 4-15%. The maximum error in peak water level is 42 cm and the maximum error in time of the peak is 10 hours. From these results, it can be said that the model with the improvement of automatically determining the order for double- sweep can more conveniently use in flood routing in branched river network.

5. Conclusions

An improvement of simultaneous solution for dynamic flood routing in branched network by Preissmann implicit finite difference scheme is introduced in this study. With the consideration that a river reach is the basic element for river network modeling, a procedure of determining the order for double-sweep algorithm is developed. By this procedure, the model can be applied to complex branched networks regardless the number of reaches at a node. This permits to "complicate" river network without any modification of the model, excepting input data. The model developed from the improvement is constructed and applied for routing a flood in a part of the Red river basin. The calculated values have good agreements with observed ones.

References

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