Interactions of Wave with Equivalent Uniform Current

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1. Introduction:

Many coastal problems involve interactions of waves with currents. In the nearshore region river discharge, tidal currents and nearshore currents modify wave parameters such as amplitude, length, frequency, direction, etc. Thus wave propagation in presence of preexisting current field bears considerable practical interest in coastal engineering and related disciplines. In this study a numerical model based on Madsen *et al.*(1992) type extended Boussinesq equations is presented to study wave-current interactions on a slowly varying topography and numerical results are compared with experimental results and analytical solution. Effects of depth varying current on wave-current interactions are adopted by using equivalent uniform current theory (Hedges *et al.*, 1992).

2. Theoretical development:

The 1D depth-integrated Boussinesq equations (Madsen *et al.*, 1992) can be written as follows:

$$S_t + P_x = 0 \qquad (1)$$

$$P_t + \left(P^2/d\right)_x + gdS_x - h^2(B+1/3)P_{xxt} - Bgh^3S_{xxx} = h_x\left(hP_{xt}/3 + 2Bgh^2S_{xx}\right) \qquad (2)$$

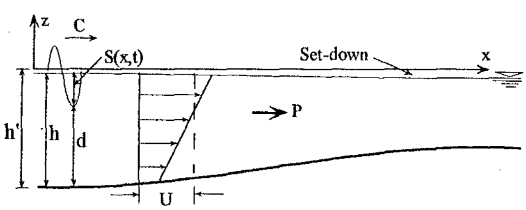


Fig. 1: Definition sketch of a wave-current field.

where d = total water depth, h = water depth, S = surface elevation, P = depth-integrated velocity (flux), g = acceleration due to gravity, and B = a curve fitting parameter (=1/15).

The combined wave-current velocity potential can be written as Eq. (3). Equation (3) satisfies Laplace equation, surface and bottom boundary conditions. Equation (4) is obtained by substituting the velocity potential, Φ into the dynamic surface boundary condition.

$$\Phi = Ux + A\cosh k(h'+z)\cos(kx-\omega t) + U^2 t/2 \qquad (3) \qquad S = (A/g)\{\omega\cosh kh\sin(kx-\omega t) + Uk\cosh kh\sin(kx-\omega t)\} \qquad (4)$$

in which ω is the absolute wave angular frequency ($2\pi/T$), T is the absolute wave period, k is the wave number, U is the current velocity in x-direction (equivalent uniform current), and A is a constant. In Eq. (3) the last term is included for mathematical simplification.

The water particle velocity (U_P) can be expressed by the superposition of particle velocity associated with the current field (current part, U) and particle velocity associated with the wave motion (wave part, u). After including the current component the non-linear term of the momentum equation becomes as Eq. (6).

$$U_p = U + u \qquad (5) \qquad \left(P^2/d\right)_x = \left\{U^2(S+h)\right\}_x + \left\{2uU(S+h)\right\}_x + \left\{u^2(S+h)\right\}_x = 2UP_x - U^2(S+h)_x \qquad (6)$$

Now combining Eqs. (1), (2), (4) and (6), and assuming the water depth, h and the current velocity, U vary slowly in x-direction, and neglecting product of derivatives yield the following dispersion relation (Eq. 7) for the wave-current field. If $U=0$, Eq. (7) reduces to Madsen *et al.* (1992) improved dispersion relation.

$$-\omega^3\left\{1+k^2h^2(B+1/3)\right\} + \omega^2Uk\left\{1-k^2h^2(B+1/3)\right\} + \omega k^2\left\{U^2 + gh\left(1+Bk^2h^2\right)\right\} - Uk^3\left\{U^2 - gh\left(1+Bk^2h^2\right)\right\} = 0 \qquad (7)$$

3. Numerical computations:

Equations (1) and (2) are numerically integrated by finite difference technique (Dronkers scheme). A gradually varying topography, as in Fig. (2), is adopted to predict wave characteristics in presence of preexisting current field. At any new time level momentum equation forms a tridiagonal matrix and this matrix is solved by the Thomas algorithm. The initial condition is stated as (a) the surface elevation, S is equal to zero; (b) the depth-integrated velocity, P is equal to the discharge per unit width of the domain. As an incident boundary, the flux boundary is imposed and surface elevation is defined as a function of time. For the outgoing boundary, radiation boundary condition with extended domain is considered in such a way that reflection from the outgoing boundary could not reach at the last point of the domain of interest.

4. Results and discussions:

Incident wave heights for numerical simulations are calculated from the root mean square of measured surface elevations at gage station 1 (Fig. 2). The curve fitting parameter, B (=1/15) also gives satisfactory results for wave-current field. Here, Jonsson *et al.* (1970) mean energy level concept is adopted for analytical solution to the combined field. Comparative results show that wave height increases for wave propagation on adverse currents and wave height decreases for wave propagation on following currents. And also, in wave-current combined field set-down results from the combination of steady current and wave motion.

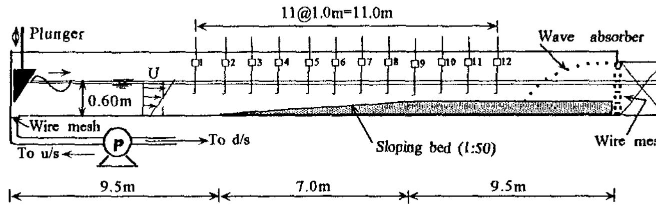
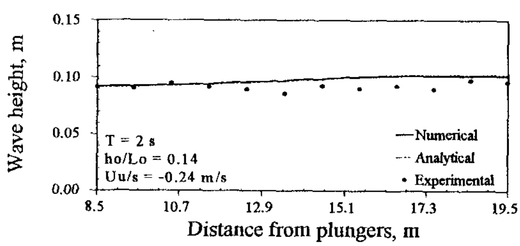
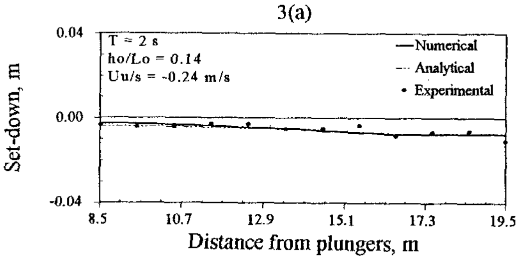


Fig. 2: Sketch of experimental set-up (not to scale).

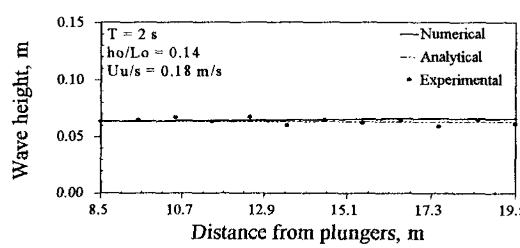


3(a)

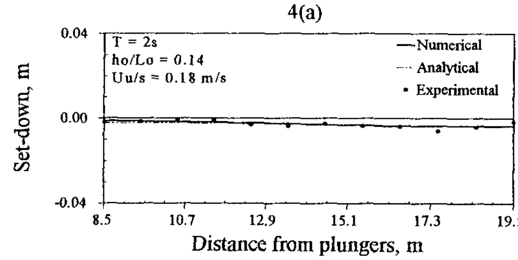


3(b)

Fig. 3: Waves on adverse currents (a) Comparison of wave height; (b) Comparison of set-down.



4(a)



4(b)

Fig. 4: Waves on following currents (a) Comparison of wave height; (b) Comparison of set-down.

5. Conclusions:

A non-linear model is presented for a wave-current coexistence field. Both adverse and following currents are considered for wave-current interactions. In numerical simulations, concept of equivalent uniform current is adopted for combined field. The model is verified with laboratory experiments and analytical theory. Although there are some limitations in experiments and numerical computations, comparative studies show that numerical simulations of wave-current phenomena are reasonably well.

6. References:

[1] Hedges, T. S. and Lee, B. W. (1992). "The equivalent uniform current in wave-current computations.", *Coast. Engrg.*, 16, 301-311. [2] Jonsson, I. G., Skougaard, C. and Wang, J. D. (1970). "Interaction between waves and currents.", *Proc. 12th Conf. on Coast. Engrg.*, Washington D. C., ASCE, 489-507. [3] Madsen, P. A. and Sørensen, O. R. (1992). "A new form of the Boussinesq equations with improved linear dispersion characteristics. Part 2. A slowly-varying bathymetry.", *Coast. Engrg.*, 18, 183-204.