Consolidation characteristics of double-layered ground with vertical drains

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INTROUCTION

Vertical drains are usually installed in subsoil consisting of several layers. The closed-form solution of consolidation of double-layered ground with vertical drains has been obtained by Tang (1997). There are many different characteristics between homegeneous ground and stratified ground. Some consolidation characteristics of double-layered ground with vertical drains are presented.

SOLUTION OF SYSTEM

The average pore pressure of soil at any depth are:

$$\overline{u}_{1} = \sum_{m=0}^{\infty} A_{m} g_{m1}(z) e^{-\beta_{m}t} \quad (1) \qquad \overline{u}_{2} = \sum_{m=0}^{\infty} A_{m} g_{m2}(z) e^{-\beta_{m}t} \quad (2)$$
where:
$$A_{m} = \frac{m_{v1} \int_{0}^{h_{1}} u_{0} g_{m1}(z) dz + m_{v2} \int_{h_{1}}^{H} u_{0} g_{m2}(z) dz}{m_{v1} \int_{h_{1}}^{h_{1}} g_{m1}^{2}(z) dz + m_{v2} \int_{h_{2}}^{H} g_{m2}^{2}(z) dz}$$

$$g_{m1}(z) = \left(1 + \frac{\lambda_{m1}^2}{\alpha}\right) \sin\left(\lambda_{m1} \frac{z}{H}\right) + c_{m1} \left(1 - \frac{\xi_{m1}^2}{\alpha}\right) \sinh\left(\xi_{m1} \frac{z}{H}\right)$$

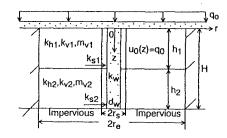


Fig. 1 Analysis scheme

$$g_{m2}(z) = b_{m2} \left(1 + \frac{\lambda_{m2}^2}{\varphi_2} \right) \cos \left[\lambda_{m2} \left(1 - \frac{z}{H} \right) \right] + d_{m2} \left(1 - \frac{\xi_{m1}^2}{\varphi_2} \right) \cosh \left[\xi_{m2} \left(1 - \frac{z}{H} \right) \right]$$

$$\varphi_{i} = (n^{2} - 1) \frac{2}{F_{i}} \frac{k_{hi}}{k_{w}} \frac{H^{2}}{r_{e}^{2}}, \quad \lambda_{mi} = H \sqrt{\frac{\Xi_{mi} + \sqrt{\Xi_{mi}^{2} - 4\Lambda_{i}\Theta_{mi}}}{2\Lambda_{i}}}, \quad \xi_{mi} = H \sqrt{\frac{-\Xi_{mi} + \sqrt{\Xi_{mi}^{2} - 4\Lambda_{i}\Theta_{mi}}}{2\Lambda_{i}}}$$

$$\Lambda_{i} = \frac{k_{vi}}{m_{vi}\gamma_{w}}, \quad \Xi_{mi} = -\left\{\frac{k_{hi}}{m_{vi}\gamma_{w}} \frac{2}{r_{o}^{2}F_{i}} \left[1 + \frac{k_{v}}{k_{w}}(n^{2} - 1)\right] - \beta_{mi}\right\}, \quad \Theta_{mi} = -(n^{2} - 1)\frac{2}{r_{o}^{2}F_{i}} \frac{k_{hi}}{k_{w}} \beta_{mi}$$

$$F_{i} = \left(\ln\frac{n}{s} + \frac{k_{hi}}{k_{si}}\ln s - \frac{3}{4}\right)\frac{n^{2}}{n^{2} - 1} + \frac{s^{2}}{n^{2} - 1}\left(1 - \frac{k_{hi}}{k_{si}}\right)\left(1 - \frac{s^{2}}{4n^{2}}\right) + \frac{k_{hi}}{k_{si}}\frac{1}{n^{2} - 1}\left(1 - \frac{1}{4n^{2}}\right), n = \frac{r_{e}}{r_{w}}, s = \frac{r_{s}}{r_{w}}$$

By virtue of continuous conditions at $z = h_1$, we obtain:

$$\mathbf{S} \mathbf{X}^{\mathrm{T}} = \mathbf{0} \quad (3) \qquad \text{where:} \quad \mathbf{S} = \begin{bmatrix} \mathbf{s}_{11} & \mathbf{s}_{12} & \mathbf{s}_{13} & \mathbf{s}_{14} \\ \mathbf{s}_{21} & \mathbf{s}_{22} & \mathbf{s}_{23} & \mathbf{s}_{24} \\ \mathbf{s}_{31} & \mathbf{s}_{32} & \mathbf{s}_{33} & \mathbf{s}_{34} \\ \mathbf{s}_{41} & \mathbf{s}_{42} & \mathbf{s}_{43} & \mathbf{s}_{44} \end{bmatrix} \quad (4) \qquad \mathbf{X} = \begin{bmatrix} 1 & c_{m1} & b_{m2} & d_{m2} \end{bmatrix} \quad (5)$$

$$\rho = \frac{n_1}{H}, \ \mathbf{s}_{11} = \sin(\lambda_{m1}\rho) \ , \ \mathbf{s}_{12} = \sinh(\xi_{m1}\rho) \ , \qquad \mathbf{s}_{13} = -\cos[\lambda_{m2}(1-\rho)] \ , \qquad \mathbf{s}_{14} = -\cosh[\xi_{m2}(1-\rho)]$$

$$\mathbf{s_{21}} = \left(1 + \frac{1}{\varphi_1} \lambda_{m1}^2\right) \mathbf{s_{11}} , \qquad \mathbf{s_{22}} = \left(1 - \frac{1}{\varphi_1} \xi_{m1}^2\right) \mathbf{s_{12}} , \qquad \mathbf{s_{23}} = \left(1 + \frac{1}{\varphi_2} \lambda_{m2}^2\right) \mathbf{s_{13}} , \qquad \mathbf{s_{24}} = \left(1 - \frac{1}{\varphi_2} \xi_{m2}^2\right) \mathbf{s_{14}} ,$$

$$\mathbf{s_{31}} = \lambda_{m1} \cos(\lambda_{m1} \rho) \; , \quad \mathbf{s_{32}} = \xi_{m1} \cosh(\xi_{m1} \rho) \; , \quad \mathbf{s_{33}} = -\lambda_{m2} \sin[\lambda_{m2} (1 - \rho)] \quad , \quad \mathbf{s_{34}} = \xi_{m2} \sinh[\xi_{m2} (1 - \rho)]$$

$$\mathbf{s_{41}} = k_{vl} \left(1 + \frac{1}{\varphi_1} \lambda_{ml}^2 \right) \mathbf{s_{31}} , \mathbf{s_{42}} = k_{vl} \left(1 - \frac{1}{\varphi_1} \xi_{ml}^2 \right) \mathbf{s_{32}}, \quad \mathbf{s_{43}} = k_{v2} \left(1 + \frac{1}{\varphi_2} \lambda_{m2}^2 \right) \mathbf{s_{33}}, \quad \mathbf{s_{44}} = k_{v2} \left(1 - \frac{1}{\varphi_2} \xi_{m2}^2 \right) \mathbf{s_{34}}$$

In order to get unequal zero solutions of X, ordering S=0, so, β_m is obtained. Then, substituting β_m into Eq.(3), c_{m1} , b_{m2} and d_{m2} are obtained.

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The average consolidation degree for any layer is:

$$\overline{U}_1 = 1 - \frac{1}{h_1} \int_0^{h_1} \frac{\overline{u}_1(z)}{u_0} dz \quad (6) \qquad \overline{U}_2 = 1 - \frac{1}{h_2} \int_{h_1}^{H} \frac{\overline{u}_2(z)}{u_0} dz \quad (7)$$

The overall average consolidation degree for whole soil layer defined by excess pore water pressure and that defined by total settlement is:

$$\overline{U}_{p} = \frac{1}{H} \left(h_{1} \overline{U}_{1} + h_{2} \overline{U}_{2} \right) (8) \quad \overline{U}_{s} = \frac{h_{1} m_{v_{1}} \overline{U}_{1} + h_{2} m_{v_{2}} \overline{U}_{2}}{h_{1} m_{v_{1}} + h_{2} m_{v_{2}}} (9)$$

CALCULATION AND DISCUSSION

It can be clearly seen in Fig. 2 that there is difference between the overall average consolidation of whole thickness of soil defined by excess pore water pressure and that defined by total settelment. There are three results drawn from Fig. 3:

- a). Using average parameters of soils of two layers to evaluate the overall average consolidation degree of whole thickness of soil, in this figure, before about 50% consolidation degree, there are two possible conditions that evaluating value is bigger or smaller than actual value, after about 50% consolidation degree, evaluating value is bigger than actual value.
- b). For Case A, $\overline{U}_p > \overline{U}_s$. For Case B, $\overline{U}_p < \overline{U}_s$.
- c). There is a cross point of two curves of \overline{U}_P of Case A and Case B. There is a cross point of two curves of \overline{U}_s of Case A and Case B too. In the early stage, the overall average consolidation degree of Case B, which has higher coefficient of consolidation of soil of the first layer, namely, $c_{h1} > c_{h2}$, is bigger than that of Case A, which has smaller coefficient of consolidation of soil of the first layer, namely, $c_{h1} < c_{h2}$. In the late stage, the result is opposite.

Fig. 4 can give out some analyses of Fig. 3.

- d). In the early consolidation stage, $\overline{U}_1(B) > \overline{U}_1(C)$ and $\overline{U}_2(B) \approx \overline{U}_2(C)$, so, $\overline{U}_p(B) > \overline{U}_p(C)$ and $\overline{U}_s(B) > \overline{U}_s(C)$.
- e). In the early stage, $\overline{U}_1(B) \overline{U}_1(A) > \overline{U}_2(A) \overline{U}_2(B)$, so, the overall average consolidation degree of Case B, is bigger than that of Case A. In the late stage, $\overline{U}_2(A) \overline{U}_2(B) > \overline{U}_1(B) \overline{U}_1(A)$, so, the overall average consolidation degree of Case B, is smaller than that of Case A.

CONCLUSIONS

- 1). It is difficult to determine which one is bigger, \overline{U}_p or \overline{U}_s .
- 2). If using average parameters of soils of two layers to evaluate the overall average consolidation degree, during the late stage, evaluating value is bigger than actual value. But, during the early stage, there are two possible conditions that evaluating value is bigger or smaller than actual value.

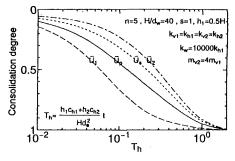


Fig. 2 Comparison of \overline{U}_p , \overline{U}_s , \overline{U}_1 and \overline{U}_2

A). k_{h1}=1, k_{h2}=4, m_{v1}=1, m_{v2}=0.5, k_w=10000 B). k_{h1}=4, k_{h2}=1, m_{v1}=0.5, m_{v2}=1, k_w=10000 C). k_{h1}=k_{h2}=2.5, m_{v1}=m_{v2}=0.75, k_w=10000

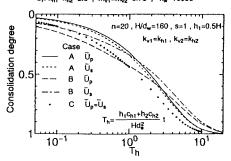


Fig. 3 Comparison of \overline{U}_p and \overline{U}_s with the same average mechanical parameters

A). k_{h1} =1, k_{h2} =4, m_{v1} =1, m_{v2} =0.5, k_{w} =10000 B). k_{h1} =4, k_{h2} =1, m_{v1} =0.5, m_{v2} =1, k_{w} =10000 C). k_{h1} = k_{h2} =2.5, m_{v1} = m_{v2} =0.75, k_{w} =10000

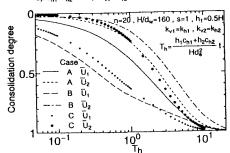


Fig. 4 Comparison of \vec{U}_1 and \vec{U}_2 with the same average mechanical parameters

3). If two cases with the same average parameters, in the early stage, the overall average consolidation degree of the case with higher coefficient of consolidation of soil of the first layer is bigger than that of the case with lower coefficient of consolidation of soil of the first layer, in the late stage, the result is opposite.

REFERENCE

1). Tang, X.W., Onitsuka, K.(1997): "Consolidation solution for double-layered ground with vertical drains," Proc. 52th Ann. Conf. of JSCE. 3-(A), pp.350-351.