Generalized two-phase mixture model and its application to improved ground

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1. Introduction

There are many kinds of composite materials. Improved ground with cement-treated soil columns is one of them and it is important to evaluate average behavior of such composite ground. Generalized two-phase mixture model is proposed based on consideration of stress distribution. This method is applied to the improved ground and average elastic modulus of the improved ground is discussed.

2. Generalized two-phase mixture model

The authors have been proposed a two-phase mixture model for obtaining average elastic moduli of composite materials with spherical inclusions or pile shaped inclusions. This model is generalized for mixtures with ellipsoidal inclusions based on consideration of relationship between stress distribution parameter and Eshelby's tensor. Ellipsoid with radiuses of a_1, a_2 and a_3 changes into various shapes, for example, sphere ($a_1=a_2=a_3=0$), cylinder ($a_1=\infty$) and plate ($a_1=a_2=\infty$). Eshelby gave an exact solution regarding distributions of stress and strain by existence of an ellipsoidal inclusion in infinite body. Relationship between stress distribution parameter and Eshelby's tensor shows in Fig.1. Stress distribution parameter is represented as a power function of stiffness ratio for any of the mixtures. On the other hand, Eshelby's tensor S_{ijkl} is represented using radiuses of ellipsoid and Poisson's ratio of matrix. As shown in Fig.1, a power, n, of stress distribution parameter is related to Eshelby's tensor (in direction of appllied stress σ_i) as follows.

$$n = 1 - S_{iiii} \qquad (1)$$

Average elastic modulus of mixtures with ellipsoidal inclusions is therefore obtained by substituting Eq.(1) into the following equation.

$$E = \frac{(b-1)f_s + 1}{\dfrac{f_s b}{E_s} + \dfrac{(1-f_s)}{E^*}} \qquad (2)$$

where $b=(E_s/E^*)^n$, f_s is volume content of inclusion, E_s and E^* are elastic moduli of inclusion and matrix. Eshelby's method is not suited for mixtures with large volume content of inclusions, but it is possible to apply to mixtures with inclusions of various shape. The previous method by authors is suited for mixtures with large content of inclusions, but its application has been limited to some mixtures. The present proposed method has both advantage of these methods.

3. Application to improved ground with cement-treated soil columns

Improved ground is made by the deep mixing method(DMM) is composite ground with pile-shaped columns as shown in Fig.2. There are many types of improved ground by DMM and a type of cement-treated soils with a range of columns is often used in practice. Figure 3 shows an example of cement-treated soils with a range of three columns. The cement-treated soils may be approximated to ellipsoidal inclusions with radiuses of a_1 and a_2. In this case, Eshelby's tensor S_{iiii} is represented as follows.

$$S_{1111} = \frac{1}{2(1-\nu^*)}\left\{\frac{1+2^* a_1/a_2}{(1+a_1/a_2)^2} + \frac{(1-2\nu^*)}{1+a_1/a_2}\right\}$$

$$S_{2222} = \frac{1}{2(1-\nu^*)}\left\{\frac{1+2^* a_2/a_1}{(1+a_2/a_1)^2} + \frac{(1-2\nu^*)}{1+a_2/a_1}\right\}$$

$$S_{3333} = 0$$

Average elastic modulus of the improved ground for each direction is obtained by substituting these equations into Eqs. (1) and (2).

Figure 4 shows relationship between normalized average modulus of mixtures and content of inclusion obtained from the proposed method. Average modulus of mixture increases with increase in content of inclusion and the tendency depends on the radius ratio a_1/a_2 of ellipsoidal inclusion remarkably. It is considered that this elastic modulus is important for evaluating average horizontal elastic modulus of improved ground with a range of cement-treated soil columns.

4. Conclusions

In order to evaluate elastic modulus of improved ground, generalized two-phase mixture model was proposed. This method may be applied to improved ground with a range of treated soil columns.

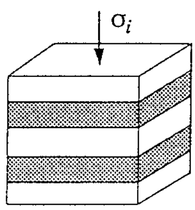
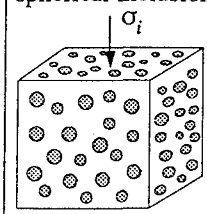
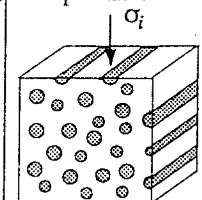
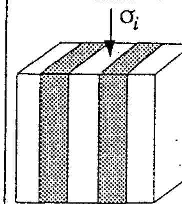
Type of mixtures	Horizontal laminate 	Mixture with spherical inclusions 	Mixture with pile shaped inclusions 	Vertical laminate 
Assumption	Constant Stress	Constant Strain energy 1)	Approximation based on numerical analysis 3)	Constant strain
Stress distribution parameter b	$\left(\frac{E_s}{E^*}\right)^n = \left(\frac{E_s}{E^*}\right)^0 = 1$ where, n = 0	$\left(\frac{E_s}{E^*}\right)^{\frac{1}{2}}$ where, n = 1/2	$\left(\frac{E_s}{E^*}\right)^{\frac{1}{3}\sim\frac{1}{4}}$ where, n = 1/3~1/4 (for ν^*=0.35)	$\left(\frac{E_s}{E^*}\right)^1$ where, n = 1
Eshelby's tensor *Siiii*	1	7/15 ~ 9/15 (for ν^*=0~0.5)	5/8 ~ 3/4 (for ν^*=0~0.5)	0
1 - *Siiii*	0	nearly 1/2	3/8 ~ 1/4	1

Fig.1 Relationship between stress distribution parameter and Eshelby's tensor

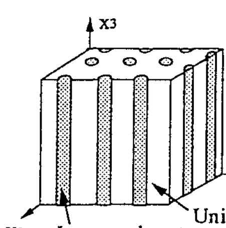


Fig.2 Improved ground with pile-shaped columns

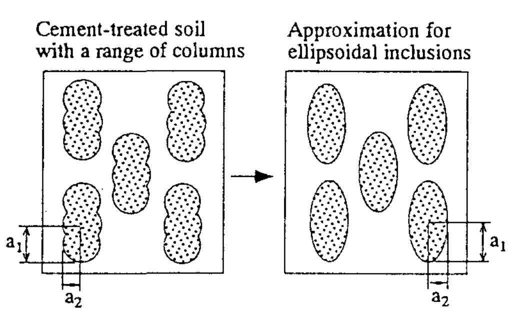


Fig.3 Approximation of cement-treated soil columns

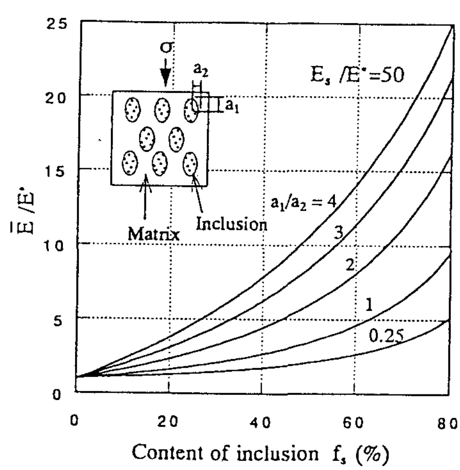


Fig.4 Relationship between E/E^* and f_s of mixtures with ellipsoidal inclusions

References 1) Omine, K. and Ochiai, H. (1992) : Stress-strain relationship of mixtures with two different materials and its application to one-dimensional compression property of sand-clay mixed soils, Proc. of JSCE, No.448/III-19, pp.121-130(in Japanese). 2) Mura, T. (1982) : Micromechanics of defects in solids, Martinus Nijhoff Publishers. 3) Adams, D.F. and Doner, D.R. (1967) : Transverse normal loading of a unidirectional