A 3-D Degenerate Beam Element for Large Displacement Analysis

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1. INTRODUCTION

When structural analyses are conducted by the finite element method, two classes of approach are possible: the classical and the degeneration approaches[1)]. In the former, formulation is based on the structural theory specific to the type of structure under consideration. On the other hand, the latter treats a structural member as a special case of continuum, and formulation is entirely based on the theory of continuum mechanics. The assumptions in the structural theory are utilized in this approach as well, but in quite a different way. Owing to the simplicity of the governing equations in the theory of continuum mechanics, the degeneration approach results in simpler discretized governing equations in general. The objective of this research is to propose a finite element formulation for large displacement analysis of three-dimensional beams, for which the classical approach inevitably leads to very complicated formulation due to the involvement of rotational degrees-of-freedom[2)].

2. FORMULATION

2.1 Assumptions

The following assumptions are employed in this study: 1) a plane cross section remains plane after deformation; 2) a cross section is not distorted; and 3) the constitutive laws are given by

$$S_{XX} = E\,E_{XX}\,,\quad S_{XY} = 2G\,E_{XY}\,,\quad S_{ZX} = 2G\,E_{ZX} \tag{1}$$

where X-, Y- and Z-axes form a Cartesian coordinate system with the X-axis being set along the beam axis; S_{IJ} and E_{IJ} are the second Piola-Kirchhoff stress and the Green strain, respectively; and E and G are respectively Young's modulus and the shear modulus. The stress components other than those in Eq.(1) are assumed zero. It should be noted that these assumptions have been employed in the theory of Timoshenko beam and anything but special.

2.2 Discretized Governing Equations

From the equilibrium equations and the strain-displacement relationships developed in the theory of continuum mechanics together with appropriate constitutive equations, we can derive the discretized governing equations, using the principle of virtual work and the finite element method. For the present formulation of a three-dimensional beam, Eq.(1) must be used as the constitutive relationships. Moreover, Assumption 2) is embedded by means of the penalty method in this derivation. To deal with large displacement, a total Lagrangian formulation is employed herein.

The discretized governing equations thus derived are nonlinear. The Newton-Raphson technique is utilized to solve these equations. To this end, the consistent linearization is performed to yield the linearized equations, which takes the following form:

$$\mathbf{K_T}\,\Delta\mathbf{U} = \mathbf{R} - \mathbf{K} \tag{2}$$

where $\mathbf{K_T}$ = the tangent stiffness matrix; $\Delta\mathbf{U}$ = the nodal displacement increment vector; $\mathbf{R}$ = the nodal external force vector; and $\mathbf{K}$ = the nodal internal force vector. The element tangent stiffness matrix $\mathbf{K}_\mathbf{T}^{ba}$ and the element nodal internal force vector $\mathbf{K}^a$ can be expressed as

$$\mathbf{K}_\mathbf{T}^{ba} = \int_{V^e} \left(\mathbf{B}^b\right)^T \mathbf{D}\left(\mathbf{B}^a\right) dV + \int_{V^e} \left(\mathbf{A}^b\right)^T \mathbf{S}\left(\mathbf{A}^a\right) dV \cdot \mathbf{I}\,,\qquad \mathbf{K}^b = \int_{V^e} \left(\mathbf{B}^b\right)^T \mathbf{S}\, dV \tag{3}$$

where $\mathbf{B}^b$ relates the Green strain to the nodal displacements, $\mathbf{A}^b$ is a matrix of the derivatives of shape functions and $\mathbf{I}$ is the identity matrix. $\mathbf{D}$ and $\mathbf{S}$ are given by

$$
\mathbf{D} = \begin{bmatrix} E & & & & & \\ & \lambda & & & \mathbf{0} & \\ & & \lambda & & & \\ & & & 2G & & \\ & \mathbf{0} & & & \lambda & \\ & & & & & 2G \end{bmatrix}, \quad \mathbf{S} = \begin{bmatrix} S_{XX} \\ \lambda E_{YY} \\ \lambda E_{ZZ} \\ S_{XY} \\ \lambda E_{YZ} \\ S_{ZX} \end{bmatrix} \tag{4}
$$

where λ is the Penalty constant.

2.3 Beam Element

Due to Assumption 1) together with the fact that three points define a plane completely, only three nodes need be assigned in a cross section. Employing quadratic shape functions in the direction of the beam axis, a 9-node element shown in Fig.1 is developed in the present study. Out of 9 nodes, three nodes located on the beam axis are called reference nodes and the others are relative nodes. The nodal quantities at a relative node are displacements relative to a reference node in the same cross section. No rotational degrees-of-freedom are involved, but the relative displacements serve as a measure of the rotation of a cross section. Note that the avoidance of the rotational degrees-of-freedom has considerably simplified the large displacement formulation for three-dimensional beam analysis

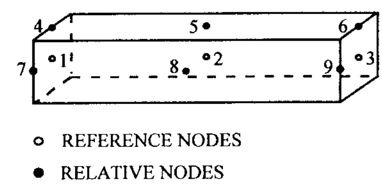

Fig.1 9-Node Beam Element

3. NUMERICAL EXAMPLE

A cantilever 45-degree bend subjected to a concentrated load at the free end, as shown in Fig.2, is analyzed. The bend is modeled by 4 proposed elements. The material is assumed linear elastic and the 2x2x2 Gauss scheme is employed for numerical integration. The numerical result together with the solution due to Bathe et al.[2] is presented in Fig.3. A close agreement is observed and the effectiveness of the present approach is confirmed.

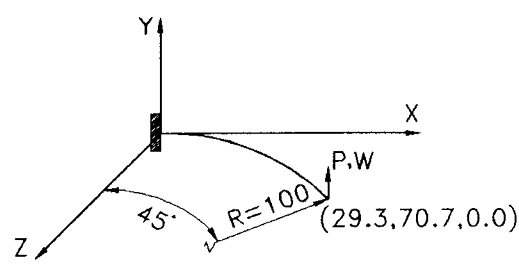

Fig.2 45-Degree Circular Bend

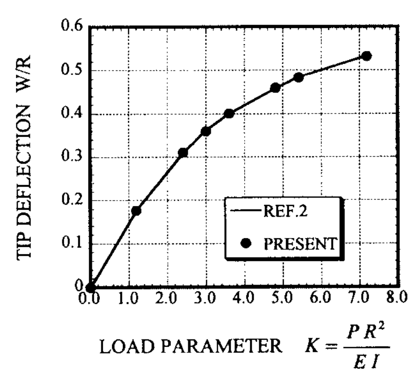

Fig.3 Load-Deflection Curve of the Bend

4. CONCLUDING REMARKS

A large displacement formulation for three-dimensional beam analysis was presented. It is based on the degeneration approach, so that the employment of rotational degrees-of-freedom has been avoided. The governing discretized equations thus derived are rather simple. Nevertheless, a good result can be obtained by the proposed method, as the numerical example has confirmed.

REFERENCES

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2) K.J. Bathe and S. Bolourchi, Int. J. Numer. Meth. Eng., Vol.14, pp.961-986, 1979.