NEW DEVELOPMENT OF CONSOLIDATION THEORY FOR DOUBLE-LAYERED GROUND WITH VERTICAL IDEAL DRAINS

Saga University SM.JSCE X.W.Tang
Saga University M.JSCE K.Onitsuka

1. INTRODUCTION

The analytical consolidation solutions for double-layered ground with vertical ideal drains have been developed by Horne(1964) and Xie(1994).However,because of real characteristic roots and imaginary characteristic roots and corresponding trigonometric functions and hyperbolic functions,the soluions are very complicated. In order to solve even more complicated consolidation problems, for examples, three-layered ground with vertical ideal drains and double-layered ground with vertical non-ideal drains, etc., the consolidation problem of double-layered ground with vertical ideal drains must be further studied. Therefore, this paper presents the consolidation theory unified of real roots and imaginary roots and trigonometric functions and hyperbolic functions under Euler's Formula. Besides, the paper proves that the time factors of total system are genuine characteristic values, but not the characteristic roots defined by Horne (1964) and Xie (1994).

2. MATHEMATICAL BASE

Based on Euler's Formula, the Eq.(2.1)--Eq.(2.4) are given:

$\mathrm{Sin}(xi) = i\,\mathrm{Sinh}(x)$ (2.1) $\mathrm{Sinh}(xi) = i\,\mathrm{Sin}(x)$ (2.2)

$\mathrm{Cos}(xi) = i\,\mathrm{Cosh}(x)$ (2.3) $\mathrm{Cosh}(xi) = i\,\mathrm{Cos}(x)$ (2.4)

where x is complex number,especially, in this paper,x is pure real number or pure imaginary number. The software, MATHEMATICA, can realize the transformation between Eq.(2.1) and Eq.(2.2) and between Eq.(2.3) and Eq.(2.4).

3. MATHEMATICAL MODELLING

The double-layered system with vertical ideal drains is shown in Figure 1. In the figure,H is the total thickness of the system ,r_w and r_e, respectively,are the radius of the vertical drains and the radius of soil influenced by drains. In each layer i , there are the thickness h_i , the oedometric modulus E_{si},the horizontal and vertical coefficients of permeability, k_{hi} and k_{vi}, q_0 is the uniform load. The drainage condition is that the top surface of the system is pervious, and the bottom is impervious.
Based on Terzaghi's basic assumptions and Barron's equal vertical strain condition,the simultaneous partial differential equations for the consolidation of the system are:

$$\frac{\partial \bar{u}_{rzi}}{\partial t}=\frac{E_{si}k_{vi}}{r_w}\frac{\partial^2 \bar{u}_{rzi}}{\partial z^2}+\frac{E_{si}k_{hi}}{r_w}\left(\frac{\partial^2 u_{rzi}}{\partial r^2}+\frac{1}{r}\frac{\partial u_{rzi}}{\partial r}\right) \quad (3.1)$$

$$u_{rzi}=\frac{f(r)}{F}\bar{u}_{rzi} \quad , \quad r_w \le r \le r_e \quad , \quad i=1,2,\cdots \quad (3.2)$$

$$f(r)=\ln\left(\frac{r}{r_w}\right)-\frac{3n^2-1}{4n^2} \quad (3.3) \qquad F=\frac{n^2}{n^2-1}\ln(n)-\frac{3n^2-1}{4n^2} \quad (3.4)$$

where, r_w is the desity of water, u_{rzi} is the excess pore water pressure at any point (r,z) in layer i, $\bar{u}_{rzi}$ is the average pore pressure at any depth z.
The condition for the solutions of Eq.(3.1) are:

$$(1).\ z=0, u_{rz1}=0; (2).\ z=H, \frac{\partial u_{rz2}}{\partial z}=0; (3).\ z=h_1,\ k_{v1}\frac{\partial u_{rz1}}{\partial z}=k_{v2}\frac{\partial u_{rz2}}{\partial z};$$

$$(4).\ z=h_1, u_{rz1}=u_{rz2}; \qquad (5).\ r=r_w, u_{rz1}=u_{rz2}=0$$

$$(6).\ r=r_e, \frac{\partial u_{rz1}}{\partial r}=\frac{\partial u_{rz2}}{\partial r}=0; \qquad (7).\ t=0, u_{rz1}=u_{rz2}=u_0=q_0$$

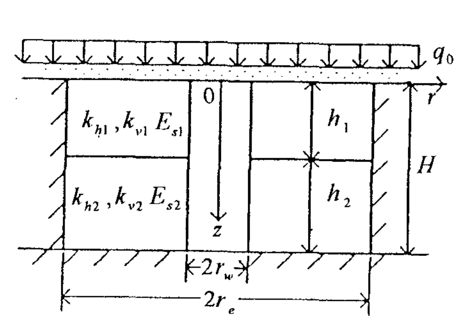

Fig. 1 A double-layered ground with vertical ideal drains

4. SOLUTION OF EQUATIONS AND PROOF OF ORTHOGONAL RELATION

4.1 SOLUTION OF EQUATIONS

The solution of Eq.(3.1) is formally developed by a separation of variables,such that:

$$U_{rzi} = Z(r,z)T(t) \quad (4.1) \qquad \frac{-\frac{E_{si}k_{vi}}{\gamma_w}Z_i'' + \frac{E_{si}k_{hi}}{\gamma_w}\frac{2}{r_e^2F}Z_i}{Z_i} = -\frac{T'_i}{T_i} = \beta_{rzi} \quad (4.2)$$

for any values of t, $u_{rz1}(r,h_1,t) = u_{rz2}(r,h_1,t)$ (4.3), so, $\beta_{rz1} = \beta_{rz2} = \beta_{rz}$ (4.4)

The solution for the average pore pressure is:

$$\bar{u}_{rzi} = \sum_{m=0}^{\infty} g_{mi}(A_{mi}e^{-\beta_{rzm}t}) \quad (4.5)$$

$$g_{m1}(z) = \mathrm{Sin}(\lambda_{m1}\frac{z}{H}), \quad g_{m1}(z) = \mathrm{Sinh}(\lambda_{m1}\frac{z}{H}) \quad (4.6) \qquad g_{m2}(z) = b_{m2}\mathrm{Cos}(\lambda_{m2}\frac{H-z}{H}), \quad g_{m2}(z) = b_{m2}\mathrm{Cosh}(\lambda_{m2}\frac{H-z}{H}) \quad (4.7)$$

$$\lambda_{mi}^2 = (\beta_{rzm} - \frac{E_{si}k_{hi}}{\gamma_w}\frac{2}{r_e^2F}) \Big/ \frac{E_{si}k_{vi}}{\gamma_w} \quad (4.7) \qquad b_{m2} = \frac{g_{m1}(h)}{g_{m2}(h)} \quad (4.8) \qquad \frac{k_{v2}}{k_{v1}}\frac{g_{m1}(h)}{g'_{m1}(h)}\frac{g'_{m2}(h)}{g_{m2}(h)} = 1 \quad (4.9)$$

Substituting Eq.(4.5)–Eq.(4.8) into Eq.(4.9),the constant β_{rzm} can be determined. The functions $g_{mi}(z)$ selected any combination of Eq.(4.6) and E.(4.7), β_{rzm} calculated by MATHEMICA are the same.

4.2 PROOF OF ORTHOGONAL RELATION

In Eq.(4.2), let:

$$-\frac{E_{si}k_{vi}}{\gamma_w}Z_{mi}'' + \frac{E_{si}k_{hi}}{\gamma_w}\frac{2}{r_e^2F}Z_{mi} = \beta_{rzm}Z_{mi} \quad (4.10) \qquad -\frac{E_{si}k_{vi}}{\gamma_w}Z_{ni}'' + \frac{E_{si}k_{hi}}{\gamma_w}\frac{2}{r_e^2F}Z_{ni} = \beta_{rzn}Z_{ni} \quad (4.11)$$

subtracting multipling Eq.(4.11) by Z_{mi}/E_{si} from multipling Eq.(4.10) by Z_{ni}/E_{si} and integrate with respect to z , then:

$$\frac{\beta_{rzm} - \beta_{rzn}}{E_{s1}}\int_0^h Z_{m1}Z_{n1}dz + \frac{\beta_{rzm} - \beta_{rzn}}{E_{s2}}\int_h^H Z_{m2}Z_{n2}dz = 0 \quad m \neq n \quad (4.12)$$

Eq.(4.12) is the orthogonal relation. So:

$$A_m = \frac{\frac{1}{E_{s1}}\int_0^h u_0 g_{m1}dz + \frac{1}{E_{s2}}\int_h^H u_0 g_{m2}dz}{\frac{1}{E_{s1}}\int_0^h g_{m1}^2 dz + \frac{1}{E_{s2}}\int_h^H g_{m2}^2 dz} \quad (4.13)$$

5. VERIFICATION OF THEORY

The theory was verified by comparing u_{rzi}, β_{rzm} and λ_{mi} with the results presented by Xie (1994). u_{rzi} and β_{rzm} are the same. Under Euler's Formula, λ_{mi} is the same.

6. CHARACTERISTIC VALUES OF SYSTEM

The following points can be drawn from comparing β_{rzm} with λ_{mi} :

(1). β_{rzm} has definite physical meaning.$\beta_{rzm}$is the time factor. However, λ_{mi} has not definite physical meaning.

(2). β_{rzm} belogs to total system.However, λ_{mi} only belongs to each layer,not total system.

(3). For ideal drains,based on β_{rzm} or λ_{mi},the orthogonal relation can be proved.However,for non-ideal drains, it will be proved that, based on λ_{mi},the orthogonal relation can not be proved.

7. CONCLUSIONS

From this study the main conclusions can be summarized as follows:

(1).The consolidation theory unified of real roots and imaginary roots and trigonometric functions and hyperbolic functions has been developed.

(2).The time factors of total system are genuine characteristic values.

REFERENCES

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