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SIMULATION ON HYDRODYNAMIC CIRCULATION AND SUBSTANCE DISPERSION IN KARATSU BAY

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INTRODUCTION

Rapid growth in population and urban development in the areas around the Karatsu Bay during the past few decades has resulted in deterioration in water quality. For water quality management purpose, besides regular monitoring of some water quality parameters at a number of locations, it is beneficial to understand the pollutant dispersion phenomena in the bay which is influenced by tidal current. In order that the substance dispersion phenomena can be clearly understood, it is necessary that hydrodynamic circulation within the bay is investigated together with the dispersion phenomena.

In this study, the hydrodynamic circulation and substance dispersion patterns in the Karatsu Bay are investigated by using mathematical models. The developed models are based on the vertically averaged continuity, momentum and mass balance equations. The finite element technique with linear triangular element is used in model formulation. The results obtained from the hydrodynamic model which are current velocities and water depths at the identified nodal points in the study area are fed as input data of the dispersion model. The water quality parameter used in this study is the chemical oxygen demand (COD) which is a measure of organic matter concentration in water.

GOVERNING EQUATIONS

The vertically averaged two-dimensional continuity, momentum and mass balance equations are the basic governing equations of the developed models. By neglecting variation in sea water density, these equations are as follow (Pritchard, 1971):

$$\frac{\partial \eta}{\partial t} + \frac{\partial (hu)}{\partial x} + \frac{\partial (hv)}{\partial y} = 0 \tag{1}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + g\frac{\partial \eta}{\partial x} + \frac{1}{\rho}\frac{\partial P_a}{\partial x} - fv - \frac{\tau_{wx}}{\rho h} + \frac{gu(u^2+v^2)^{1/2}}{hC_h^2} = 0 \tag{2}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + g\frac{\partial \eta}{\partial y} + \frac{1}{\rho}\frac{\partial P_a}{\partial x} + fu - \frac{\tau_{wy}}{\rho h} + \frac{gv(u^2+v^2)^{1/2}}{hC_h^2} = 0 \tag{3}$$

$$\frac{\partial c}{\partial t} + \frac{\partial (uc)}{\partial x} + \frac{\partial (vc)}{\partial y} - \frac{1}{h}\left\{\frac{\partial}{\partial x}\left(hD_x\frac{\partial c}{\partial x}\right) + \frac{\partial}{\partial y}\left(hD_y\frac{\partial c}{\partial y}\right)\right\} + R_c - S_c = 0 \tag{4}$$

where η is water surface elevation; h is total water depth; u and v are vertically averaged flow velocities in the x- and y-directions, respectively; ρ is water density; f is coriolis factor; g is gravitational acceleration; P_a is atmospheric pressure; τ_{wx} and τ_{wy} are surface shear stresses in the x- and y-directions, respectively; C_h is Chezy's coefficient; c is vertically averaged COD concentration; D_x and D_y are the effective' diffusion coefficients; R_c and S_c are sink and source of COD, respectively.

As regards boundary conditions, the domain boundary is divided into two types, i.e., (1) *shoreline boundary*, where normal flow velocities and discharge flux are specified, and (2) *ocean boundary*, where water depths and COD concentrations are specified.

FINITE ELEMENT MODELS

In this study, the finite element method (FEM) is used in model formulation. The unknown variables are expressed in terms of values at nodal points in the study domain. The Galerkin's weighted residual method is employed to obtain a set of algebraic equations with nodal flow velocities, water depths, and COD concentrations as the system variables. The system equations can be written in a compact form as follow:

$$\boldsymbol{M}\frac{\partial \boldsymbol{H}}{\partial t} + \boldsymbol{E}_h.\boldsymbol{H} + \boldsymbol{B}_q = 0 \tag{5}$$

$$\boldsymbol{M}\frac{\partial \boldsymbol{U}}{\partial t} + \boldsymbol{M}_x.\boldsymbol{H} + \boldsymbol{E}_u = 0 \tag{6}$$

$$\boldsymbol{M}\frac{\partial \boldsymbol{V}}{\partial t} + \boldsymbol{M}_y.\boldsymbol{H} + \boldsymbol{E}_v = 0 \tag{7}$$

$$\boldsymbol{M}\frac{\partial \boldsymbol{C}}{\partial t} + \boldsymbol{E}_c.\boldsymbol{C} + \boldsymbol{D}_c = 0 \tag{8}$$

SOLUTION TECHNIQUE

First, the hydrodynamic model (Eqs. 5,6 and 7) is run to compute the values of flow velocities and water depths at various nodal points. The obtained results are then fed as input data of the dispersion model (Eq. 8). In the hydrodynamic model, Eqs. 6 and 7 are merged together, so that at the nodal points along the shoreline boundary the flow velocities in the x– and y– directions can be transformed to the normal and tangential directions (because the normal flow velocities are specified along the shoreline). The *split time* technique is used in solving these time dependent equations, i.e., the water depth matrix $\boldsymbol{H}$ and the flow velocity matrix $\boldsymbol{F}$, which is obtained from merging of the matrices $\boldsymbol{U}$ and $\boldsymbol{V}$, are solved at different halves of the time step. The time integration scheme used is the explicit scheme with iteration for the hydrodynamic model and the trapezoidal rule for the substance dispersion model.

MODEL APPLICATION

The developed models are applied to study hydrodynamic circulation and COD distribution in the Karatsu Bay. The Karatsu Bay is divided into a number of triangular elements as shown in Figure 1. For the hydrodynamic model, the no–flow condition with water depth measured at the mean sea level is considered as the initial condition, and water depth fluctuations at the nodal points along the ocean boundary are specified as the ocean boundary condition while normal flow velocity is set equal to zero at the nodal points along the shoreline boundary. For the dispersion model, the average value of the available monitored COD values at some different points in the bay is used as the initial condition, and the COD concentrations at those points located near the ocean boundary of the study domain are used as the ocean boundary condition. The COD discharge loadings along the shoreline boundary are estimated based on the number of population, livestocks, existing wastewater treatment systems, as well as present land use patterns.

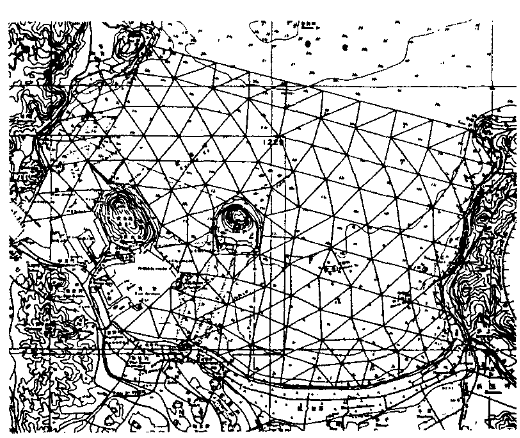

Figure 1. Karatsu Bay divided into elements.

REFERENCE

Pritchard, D.W., Hydrodynamic Models. In TRACOR, Inc. *Estuarine Modeling: An Assessment*, for the Water Quality Office, U.S. Environmental Protection Agency, February, 1971.