OPTIMIZATION MODEL FOR WATER QUALITY MANAGEMENT WITH CARBONACEOUS AND NITROGENOUS BOD LOADING CONTROL

○*Kouji Yamamoto; Winai Liengcharernsit; Hiroyuki Araki; Kenichi Koga*
Department of Civil Engineering, Saga University, Saga 840

INTRODUCTION

Water quality management in a river concerns primarily with control of discharge of pollutants into that river. Excessive pollutant discharge will result in deterioration in water quality which inhibits water use for various purposes. However, it is not economical to completely avoid the discharge of pollutants and the river itself has some self-purification. The common question being asked is how much we can discharge wastewater into the river without causing any problem.

MASS BALANCE EQUATION

In this study, a simple one-dimensional dispersion model is considered. A river is divided into N segments as shown in Figure 1. From the law of conservation of mass, the mass balance equation can be written as follows:

$$\frac{dC_i}{dt} = \frac{1}{V_i}\left[C_{i-1}Q_{i-1} - C_iQ_i - \frac{A_{i-1}D_x(C_i - C_{i-1})}{0.5(L_i + L_{i-1})} + \frac{A_iD_x(C_{i+1} - C_i)}{0.5(L_{i+1} + L_i)}\right] - k_{1i}C_i + \frac{S_i}{V_i} \tag{1}$$

where C_i is substance concentration in segment i; V_i is volume of segment i; L_i is length of segment i; A_i is cross-sectional area at the downstream end of segment i; Q_i is the outflow from segment i; D_x is longitudinal dispersion coefficient, k_{1i} is the first-order substance decaying rate; and S_i is the substance discharge loading into segment i. Eq.(1) can be written in a compact form as:

$$\frac{dC_i}{dt} = A_{i,i-1}C_{i-1} + A_{i,i}C_i + A_{i,i+1}C_{i+1} + E_i \tag{2}$$

Eq.(2) can be applied to segments *2* to *N–1*, while additional assumptions are needed for segments *1* and *N*. Finally, the mass balance equations for all the N river segments can be written in the matrix form as:

$$\frac{dC}{dt} = \mathbf{A}.\mathbf{C} + \mathbf{E} \tag{3}$$

At steady state, Eq.(3) is reduced to

$$\mathbf{A}.\mathbf{C} + \mathbf{E} = 0 \tag{4}$$

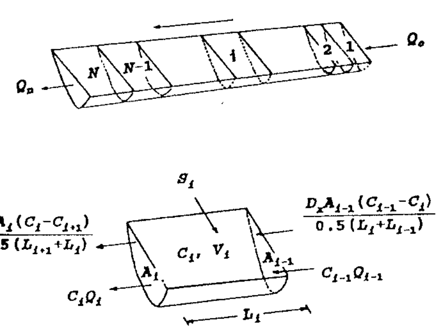

Figure 1. A river divided into N segments.

BOD–DO EQUATIONS

Using the similar configuration, the steady-state carbonaceous BOD, nitrogenous BOD, and DO mass balance equations can be written as:

$$\mathbf{A}_b\mathbf{B}_c - \mathbf{K}_{1c}\mathbf{B}_c + \mathbf{S}_{cc}\mathbf{P}_c + \mathbf{S}_{cu} = 0 \tag{5}$$

$$\mathbf{A}_b\mathbf{B}_n - \mathbf{K}_{1n}\mathbf{B}_n + \mathbf{S}_{nc}\mathbf{P}_n + \mathbf{S}_{nu} = 0 \tag{6}$$

$$\mathbf{A}_d\mathbf{D} - \mathbf{K}_{1c}\mathbf{B}_c - \mathbf{K}_{1n}\mathbf{B}_n + \mathbf{S}_d - \mathbf{R}_d = 0 \tag{7}$$

where $\mathbf{A}_b$ and $\mathbf{A}_d$ are coefficient matrices; $\mathbf{B}_c$ and $\mathbf{B}_n$ are matrices of carbonaceous and nitrogenous BOD concentrations; $\boldsymbol{D}$ is matrix of DO concentrations; S_{cc} and S_{cu} are matrices of controllable and uncontrollable carbonaceous BOD loadings per unit volume of the river segment; S_{nc} and S_{nu} are matrices of controllable and uncontrollable nitrogenous BOD loadings per unit volume of the river segment; $\boldsymbol{P}_c$ and $\boldsymbol{P}_n$ are matrices of ratios of remaining carbonaceous and nitrogenous BOD loadings after treatment; $\boldsymbol{K}_{1c}$ and $\boldsymbol{K}_{1n}$ are matrices of the first-order decaying rates of carbonaceous and nitrogenous BOD; $\boldsymbol{S}_d$ and $\boldsymbol{R}_d$ are matrices of source and sink terms of DO per unit volume of the river segment.

OPTIMIZATION MODEL

Objective Function In water quality management optimization models, the common objective function may be either (1) to maximize the waste loading that can be discharged into the receiving water, or (2) to minimize the total waste treatment cost, such that water quality in the receiving water body is still maintained within the specified standards. These objective functions can be expressed as follow:

Case 1: $$\text{Max. } Z = \sum (P_{ci}S_{cci} + P_{ni}S_{nci})V_i \tag{8}$$

Case 2: $$\text{Min. } Z = \sum \{f_{ci}(S_{cci},V_i,P_{ci}) + f_{ni}(S_{nci},V_i,P_{ni})\} \tag{9}$$

where $f_{ci}(S_{cci},V_i,P_{ci})$ is cost for carbonaceous BOD removal at segment i; and $f_{ni}(S_{nci},V_i,P_{ni})$ is cost for nitrogenous BOD removal at segment i;

Decision Variables In this model, P_{ci} and P_{ni} (i=1,2,..,N) are the decision variables.

Constraints The constraints of this optimization model are:

(1) Total BOD concentrations in the river are equal to or lower than the specified limits, i.e., $\boldsymbol{B}_c+\boldsymbol{B}_n \leq \boldsymbol{B}^*$. From Eqs.(5) and (6), the matrices $\boldsymbol{B}_c$ and $\boldsymbol{B}_n$ can be expressed in terms of $\boldsymbol{P}_c$ and $\boldsymbol{P}_n$. Then, the BOD constraint inequality can be written as:

$$[\boldsymbol{A}_b-\boldsymbol{K}_{1c}]^{-1}[-\boldsymbol{S}_{cc}\boldsymbol{P}_c-\boldsymbol{S}_{cu}] + [\boldsymbol{A}_b-\boldsymbol{K}_{1n}]^{-1}[-\boldsymbol{S}_{nc}\boldsymbol{P}_n-\boldsymbol{S}_{nu}] \leq \boldsymbol{B}^* \tag{10}$$

(2) Dissolved oxygen concentrations are equal to or higher than the specified limits, i.e., $\boldsymbol{D} \geq \boldsymbol{D}^*$. From Eqs.(5), (6) and (7), the matrix $\boldsymbol{D}$ can be expressed in terms of $\boldsymbol{P}_c$ and $\boldsymbol{P}_n$. Then, the DO constraint inequality can be written as:

$$\begin{aligned}\boldsymbol{A}_d^{-1}\{ &\boldsymbol{K}_{1c}[\boldsymbol{A}_b-\boldsymbol{K}_{1c}]^{-1}[-\boldsymbol{S}_{cc}\boldsymbol{P}_c-\boldsymbol{S}_{cu}] \\ &+ \boldsymbol{K}_{1n}[\boldsymbol{A}_b-\boldsymbol{K}_{1n}]^{-1}[-\boldsymbol{S}_{nc}\boldsymbol{P}_n-\boldsymbol{S}_{nu}] -\boldsymbol{S}_d+\boldsymbol{R}_d \} \geq \boldsymbol{D}^*\end{aligned} \tag{11}$$

(3) The values of P_{ci} and P_{ni} are in the range of 0–1.0. That is

$$0 \leq \boldsymbol{P}_c \leq 1 \quad ; \quad 0 \leq \boldsymbol{P}_n \leq 1 \tag{12}$$

SOLUTION TECHNIQUE AND MODEL APPLICATION

It should be noted that all the constraint inequalities mentioned above are linear. If the first case of the objective function which is also a linear function is considered, this optimization problem will be a linear programming model, of which solutions can be obtained by using the well known 'Simplex' method. If the second case of the objective function is considered, the problem will be in the form of non-linear programming. Approximate solutions can be obtained by reducing the problem to a linear programming problem. This can be done by approximating the cost items by a piecewise linear function. Otherwise, the methods developed for solving non-linear programming problems must be employed. This optimization model is applied to the Kase River as a case study, and only the first case of the objective function is considered.