STUDY ON OPTIMAL OPERATION OF HOKUZAN DAM/RESERVOIR USING STOCHASTIC DYNAMIC PROGRAMMING

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INTRODUCTION

Dynamic programming is an optimization algorithm which is widely used in making a sequence of decisions, when the problems can be divided into a number of stages with a decision required at each stage, so as to maximize overall effectiveness. In the past few decades, the dynamic programming has been successfully applied to water management. However, most of the applications were based on a deterministic approach, on the assumption that system inputs were known or could be replaced by their average or mean values. Nowadays, with a continuing increase in water demand due to high population growth as well as urban and industrial development while the available water resource is limited, more careful planning, design and operation of water resource systems are necessary. Therefore, the need has arisen for improved techniques to tackle the problems with uncertainties and random effects.

To cope with the uncertainties in hydrologic regime in the river basin and some other random factors, the so-called stochastic dynamic programming was developed and applied in a reservoir operation problem (Howard, 1960; Schweig, 1968; Schweig and Cole, 1968; Kottegoda, 1980; Loucks, et al, 1981; etc.). In this study, the stochastic dynamic programming model is applied to the Hokuzan reservoir located in Saga Prefecture, Japan. The objective is to determine the optimum release of water from the reservoir in each month in order to cope with the downstream water demand.

STOCHASTIC DYNAMIC PROGRAMMING RESERVOIR OPERATING MODEL

The stochastic dynamic programming model used in this study is to determine the optimum release from the reservoir in each month such that the total damage caused by water shortage over the whole study period is minimum. Similar to the model developed by Loucks, et al (1981), the storage volume of the reservoir at the end of each month or the beginning of the next month is considered as the state variable of this model. Given the initial storage volumes S_{kt}, the inflows Q_{it}, the evaporation and seepage losses E_{klt}, and final storage volumes $S_{l,t+1}$, the reservoir releases R_{kilt} are determined from:

$$R_{kilt} = S_{kt} + Q_{it} - E_{klt} - S_{l,t+1} \tag{1}$$

In this study, the backward-moving algorithm is used. It is assumed that the reservoir operation ends at the month $t = T$ in the future. If $f_t^n(k,i)$ is the total expected value of the damage to be caused by water shortage during the n-month period from the end, including the current month t, the recursive relationships can be written as:

$$f_t^n(k,i) = minimum[D_{kilt} + \sum_j P_{ij}^t f_{t+1}^{n-1}(l,j)] \qquad \forall\ k,i \tag{2}$$

where D_{kilt} is the damage caused by water shortage; P_{ij}^t is the probability of inflow Q_{jt} in the month t when the inflow in the month $t-1$ equals $Q_{i,t-1}$.

Water use in the areas downstream of the Hokuzan reservoir is for two main purposes, i.e., domestic water supply and agriculture. Let W_{dt} and W_{at} be the minimum water demands in the month t for domestic and agricultural purposes, respectively. The damage due to water shortage in the month t can be expressed as follows:

Let Z_{dt} and Z_{at} be the amount of water shortage for domestic and agricultural uses respectively in the month t. That is:

For domestic use: $Z_{dt} = 0$ when $R_{dt} \geq W_{dt}$ and $Z_{dt} = W_{dt} - R_{dt}$ when $W_{dt} > R_{dt}$ (3)

For agricultural use: $Z_{at} = 0$ when $R_{at} \geq W_{at}$ and $Z_{at} = W_{at} - R_{at}$ when $W_{at} > R_{at}$ (4)

Total damage in month t: $D_{kilt} = C_d Z_{dt}^p + C_a Z_{at}^q$ (5)

where R_{dt} and R_{at} are the amount of water available for domestic water supply and agricultural use, respectively. It should be noted that summation of R_{dt} and R_{at} should not be more than the released water R_{kilt}. The coefficients C_d and C_a and the parameters p and q depend on the severity of water shortage which is transformed to monetary values. Their values can be assigned for some different percentage of water shortage.

Allocation of the available water for domestic and agricultural uses should also be based on the policy of the local government. Usually, domestic use should have higher priority. However, for the Hokuzan reservoir, the farmers contributed lot of money for the dam construction. Therefore, in this study the 'trial and error' method is employed to determine the minimum damage provided that the reservoir release in the month t is R_{kilt} and summation of the allocated water R_{dt} and R_{at} equals the total water release R_{kilt}.

SYNTHETIC STREAMFLOW MODEL

The Thomas–Fiering seasonal model is used to generate the monthly inflows into the reservoir. This model is as follows:

$$X_{t,\tau} = \mu_\tau + \sigma_\tau \rho_\tau (X_{t-1,\tau-1} - \mu_{\tau-1}) / \sigma_{\tau-1} + \sigma_\tau \eta_t \tag{6}$$

in which

$$\rho_\tau = E[(X_{t,\tau} - \mu_\tau)(X_{t-1,\tau-1} - \mu_{\tau-1})] / \sigma_\tau \sigma_{\tau-1} \tag{7}$$

where μ_τ and σ_τ represent the mean and standard deviation respectively of the historical reservoir inflows in the calendar month τ. ρ_τ is the coefficient of correlation between the flows in months τ and $\tau-1$. η_t is an independent random variable with expected value equal to zero and $\mathrm{var}(\eta_t) = \sigma^2_{\eta,\tau} = 1 - \rho^2_\tau$. In this study, it is assumed that η_t has a normal distribution function.

In order to determine the values of probability P_{ij}^t, the reservoir inflow in each month is divided into a number of discrete values. For a given value of $Q_{i,t-1}$ in the month $t-1$ and various different values of Q_{jt} in the month t, the value of random variable η_t in Eq.(6) can be determined. Then, the probability that the inflow in the month t equal to Q_{jt} while the value in the month $t-1$ equal to $Q_{i,t-1}$ is computed from

$$P_{ij}^t = \int_a^b f(\eta_t)\, d\eta_t \tag{8}$$

where $f(\eta_t)$ is the probability density function of the random variable η_t. The limits a and b correspond to the values of η_t for the inflows $(Q_{j-1,t}+Q_{jt})/2$ and $(Q_{jt}+Q_{j+1,t})/2$, respectively.

MODEL APPLICATION AND CONCLUSION

The above mentioned model is applied to determine the optimum release from the Hokuzan reservoir. This model is considered useful for reservoir operation especially when the available water resource is limited. More careful study is needed to evaluate the damage coefficients C_{dt}, C_{at}, and parameters p and q. Other functions of the reservoir such as flood mitigation, hydropower generation, fishery, etc., should also be considered.

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