Prediction of sunspots time series

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1. Introduction

An unusual climate pattern is being experienced all over the world in recent time. One of the reasons that should be taken account is that the sunspot numbers which are in high level in present decade. Sunspot numbers is a quantitative coefficient of the activity of the sun. The effects of it on the climate of the earth, on the behavior of human beings and so on, have been demonstrated for a long time. The studies concerning the chaotic behavior of sunspot time series have been reported[1][2].

The information that we can easily get for the activity of the sun is the time series of sunspot numbers, which contaminates stochastic random component. It seems, however, that the stochastic component is not so sensitive to the nonlinearity of mathematical models, which will be discussed in the present paper. Therefore in the present paper, the stochastic component is removed from the observed sunspot time series, the discussion on reconstructing system equations from time series is made, and then the prediction of the time series is carried out by using the Extended Kalman Filter.

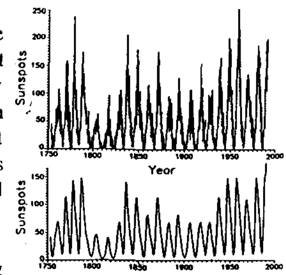

Figure 1. The process of ***sunspot time series (above:*** **original, down: smoothed)**

2. Smoothing calculation of sunspot time series

The data used are the monthly sunspot numbers from 1753 to 1990, resulting in a time series consisting 2856 points. The process of it is shown in Figure 1. In order to remove the stochastic component, a noise reduction method[3], which can remove noise in a time series with nonlinear time evolution is applied. The smoothed sunspot time series is also shown in Figure 1. The portrait of it in phase shift space (lag $\tau=10$) is shown in Figure 2. It is clear that there exists an attractor so that the chaotic characteristic of sunspot time series is confirmed.

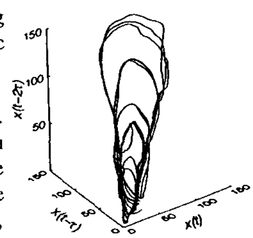

Figure 2. The phase shift portrait of smoothed sunspot time series

3. Reconstructing system equations from a time series

It is a general situation in practice that only partial information of a system(e.g. one observed time series) is available. The problem is to find out system equations which is a continuous system with multiple dimensions from only one discretely observed time series. More over it is not sure whether observed variable is a fundamental variable of the system or not. Therefore it will be difficult to get exact original system equations(if exist), and reconstructing an equivalent system equations in an artificial phase space is desired.

Through the previous studies[5], when reconstructing system equations the following four parts need to be clarified: one is the number of dimensions, second is the structure of equations, third is the values of parameters in system equations, and fourth is observed time series. The more we have the information on these parts, the better we can get the reconstructed system equations. The concept of reconstructing system equations from an observed time series is shown in Figure 3.

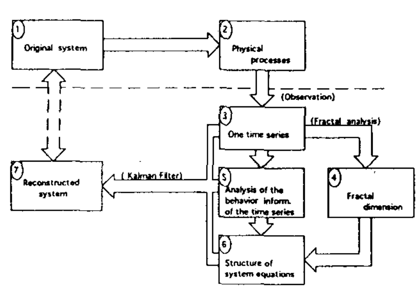

Figure 3. The concept of reconstructing system equations from an observed time series

4. Prediction of smoothed sunspots time series

As it is seen in section 2, the smoothed time series of sunspot numbers is a chaotic time series. Using it as an observed time series, the frame of prediction work is: assuming structure of differential equations, choosing proper calculation condition and initial values of parameters, identifying parameters of the system equations by Extended Kalman Filter and predicting future development.

(1) Structure of system equations

If a system is a three dimensional one, there are three system variables: x, y and z. After the functions of differentials $\dot{x}$, $\dot{y}$, and $\dot{z}$ are expanded in Taylor series and the terms up to second order are taken the system equations are shown as equation (1).

$$\begin{cases} \dot{x}=a_{11}+a_{12}x+a_{13}y+a_{14}z+a_{15}xy+a_{16}xz+a_{17}yz+a_{18}x^2+a_{19}y^2+a_{110}z^2 \\ \dot{y}=a_{21}+a_{22}x+a_{23}y+a_{24}z+a_{25}xy+a_{26}xz+a_{27}yz+a_{28}x^2+a_{29}y^2+a_{210}z^2 \\ \dot{z}=a_{31}+a_{32}x+a_{33}y+a_{34}z+a_{35}xy+a_{36}xz+a_{37}yz+a_{38}x^2+a_{39}y^2+a_{310}z^2 \end{cases} \quad (1)$$

Equation (1) includes 30 parameters. It is usually difficult to identify 30 parameters at same time based on only one observed time series so that the equation (1) need to be simplified again. Rössler equations is a set of nonlinear differential equations with three dimension. What we are interesting in is that the behavior of time series z(t) of Rössler equations is somehow similar to sunspot time series: (1) the values of two time series are not negative, (2) there is no exact period but pseudoperiod and (3) possess chaotic characteristics (exist attractor). Therefore it is possible to refer to Rössler equations to reconstruct system equations of sunspots time series. It is clear that we can not use Rössler equations as the system equations for sunspots directly. In order to keep that the time series z(t) of the Rössler equations synchronizes with sunspots time series, the interval Δt that is used to generate time series by Rössler equation takes the value of 0.047. Then the linear transformation of that $z'=\gamma z$ is used to change the amplitude of z(t). According to the maximums of smoothed sunspot numbers and z(t), the value of γ takes 30.0.

(2) Application of Extended Kalman Filter

We have reported that Kalman Filter is effective in identifying parameters based on observe time series even though the time series possess chaotic characteristics[5]. In this paper, when the Extended Kalmam Filter is applied to identify parameters and predict the future development of sunspots time series, Equation (1) is used as system equations. The system vector X include 33 components.

$$X=[x_1,x_2,x_3,\ldots\ldots,x_{33}]^T=[x,y,z,a_{11},a_{12},a_{13},\ldots\ldots,a_{310}]^T \qquad (2)$$

According to the analysis in section 4.(1), the initial values of parameters: a_{11}, a_{12}, ..., a_{310} take the values of the coefficients of linear transformed Rössler equations, that is, $a_{13}=-1$, $a_{14}=-1/30.0$, $a_{22}=1$, $a_{23}=0.398$, $a_{31}=2.0/30.0$, $a_{34}=-4.0$, $a_{36}=1.0$, and others equal to zero. The initial values of variables x and y are 1.0 and 1.0, same as that Rössler time series are generated. The initial value of z takes real value of smoothed sunspots data, $z_0=32$. Observed data is the monthly sunspots time series.

(3) Prediction results

As mentioned above, applying the Extended Kalman Filter to smoothed sunspots time series, three steps(three months) ahead, and six steps(six months) ahead prediction results are shown in Figure 4 and 5 as follows. From Figure 4 and 5 it is clear that the prediction results are reasonable and the prediction results become worse as the prediction time increase. Up to three months ahead predictions are quite good. Relative prediction errors are less then 5%. Six month ahead predictions are also good in smooth part of the sunspots time series. The relative prediction errors are about 5%. The prediction errors are bigger in sharply changeable parts. Correspondently the relative prediction errors are about 10%.

As the parameters of system equations are identified and future development of sunspots time series are predicted, the processes of other two system variables x and y are also obtained. Generally speaking, they converge to continue processes under the calculation condition given. The prediction of z always converges and corresponds to the observed time series.

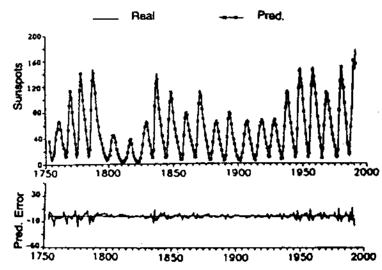

Figure 4. Three month ahead prediction of smoothed sunspot time series

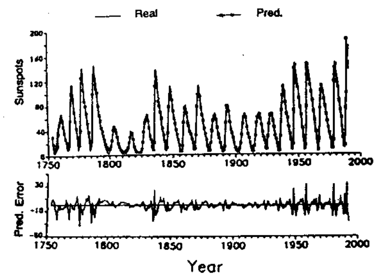

Figure 5. Six month ahead prediction of smoothed sunspot time series

5. Conclusion

(1) In order to stress main behavior and reduce affection of stochastic component smoothing calculation of sunspots time series is done. The calculation results reveal the fact that there is an attractor in sunspots time series.

(2) In practice we usually have only one observed time series and system equation are unknown beforehand, so that it is necessary to reconstruct a set of equivalent system equation instead.

(3) The Extended Kalman Filter is an effective tool to identify parameters and make prediction of future development of system. Because available information of system is not sufficient and system has some change in observed period, the update prediction capability of the Extended Kalman Filter should be stressed.

(4) The prediction results of smoothed sunspots time series are reasonable. The studies of longer step ahead prediction and of how to treat the stochastic component is the future work.

References: [1] Mundt D. M. et al. Chaos in the sunspot cycle: analysis and prediction, Journal of Geophysical Research. Vol.96, No.A2, 1991, pp1705–1716. [2] Kurths J. An attractor in a solar time series, Physica, 25D, 1987, pp165–172. [3] Schreiber T., An extremely simple nonlinear noise reduction method. Neis Bohr Institute, Denmark, 1992. [4] Gouesbet G. Reconstruction of the vector field of continuous dynamical from numerical scalar time series. Physica Review A, Vol.43, No.10, 1991,pp5321–5331. [5] Xu S., Jinno K., Kawamura A., et al. Application of the Extended Kalman Filter for reconstructing system from chaotic numerical time series. Proceedings of Hydraulic Engineering, JSEC Vol. 37(1993), pp853–856.