SEDIMENT-LADEN BUOYANT JET IN CROSS FLOW

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INTRODUCTION

This paper deals with sediment -laden jet and plume in horizontal cross flow. The smoke discharge from an industrial chimney is a typical example of pollutants released as buoyant jets. Buoyant jets are an integral part of many waste disposal systems including diffuser outfalls,smoke stacks etc.The objective of this study is to develop a mathematical model of a buoyant plume caused by sediment particles. When particles are discharged vertically into flow at the water surface, their behavior may be similar to that of negative buoyant plume in flow,since the sediment laden part of the fluid is heavier than the ambient one. In addition to this motion, the plume will be pulled down due to the settling of the particles. This is the unique point ofthe sediment laden plume.

The aim of this work is to investigate the effect of fall velocity on buoyant plume in a cross flow.The case is treated where the fall velocity of the particles is not so high compared with the uniform flow. Our intention is a brief and simple description of the phenomena. Assuming the similarity profiles of both for flow velocity and sediment concentration, the variations of plume properties along the plume axis are discussed.

BASIC EQUATIONS

It is assumed that the flow is steady, fully turbulent and fluid properties are constant. Using the boundary layer approximation and Boussinesq approximation,we have,

$$\frac{\partial u}{\partial s}+\frac{1}{r}\frac{\partial (r v)}{\partial r}=0 \tag{1}$$

$$\left(u\frac{\partial u}{\partial s}+v\frac{\partial u}{\partial r}\right)\sin\theta - \Delta\bar{\rho}cg-\frac{1}{r}\frac{\partial}{\partial r}\left(\overline{r u'v'}\right)\sin\theta+u^2\cos\theta/R \tag{2.1}$$

$$\left(u\frac{\partial u}{\partial s}+v\frac{\partial u}{\partial r}\right)\cos\theta = -u^2\cos\theta/R-\frac{1}{r}\frac{\partial}{\partial r}\left(\overline{r u'v'}\right)\cos\theta \tag{2.2}$$

$$\frac{\partial}{\partial s}\left\{\left(u+w_o\sin\theta\right)c\right\}+\frac{1}{r}\frac{\partial}{\partial r}\left\{r\left(v+w_o\cos\theta\right)c\right\}=\frac{1}{r}\frac{\partial}{\partial r}\left(\overline{r u'c'}\right) \tag{3}$$

$$\tan\theta'=\frac{u\sin\theta+w_o}{u\cos\theta}=\tan\theta+\frac{w_o}{u\cos\theta} \tag{4}$$

$$R=-ds/d\theta \tag{5}$$

$$\Delta\bar{\rho}-(\rho_s-\rho_o)/\rho_o \tag{6}$$

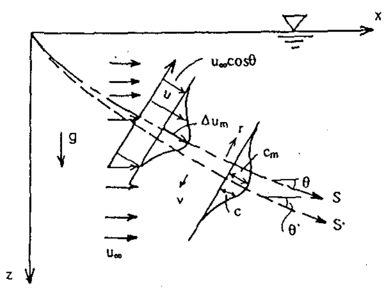

Fig.1. Definition sketch

in which s is the trajectory of the maximum velocity, r is the radial coordinate taken from the s - axis.(Fig.1). u and v are velocities in the s-and r direction respectively. The densities of sediment particles and that of ambient water are denoted by and respectively. is the fall velocity of sediment particles.Overbars in Eqs. 2 - 3 denote the average with respect to time.Substituting Eqs. 7-12 in Eqs. 1-6 we obtain Eqs. 13-16.

$$F_{dc}=u_\infty/\sqrt{\Delta\bar{\rho}\,c_o\,g\,l_b} \tag{7}$$

$$l_b=\rho_o B/u_\infty^3\ ,\ B=\frac{\pi}{4}D_o^2\,(w_{oo})\,g\,(\rho_s-\rho_o)\,c_o \tag{8}$$

$$u_*=\sqrt{(u_m-u_\infty\cos\theta)^2+(u_\infty\sin\theta)^2} \tag{9}$$

$$\left.\begin{array}{llll}\bar{u}=u/u_\infty\ , & \bar{r}=r/l_b\ , & \bar{b}=b/l_b\ , & \bar{s}=s/l_b \\ \overline{u_*}=u_*/u_\infty\ , & \bar{A}=A/l_b^2\ , & \overline{w_o}=w_o/u_\infty\ , & \bar{c}=c/c_o\end{array}\right\}\tag{10}$$

$$\bar{u}=(\Delta u_m e^{-r^2/b^2}+u_\infty\cos\theta)/u_\infty=\Delta\overline{u_m}e^{-r^2/b^2}+\cos\theta \tag{11}$$

$$\bar{c}=c_m e^{-r^2/a^2}/c_o=\overline{c_m}e^{-r^2/(\lambda b)^2}\quad ,\quad \lambda=a/b \tag{12}$$

$$\frac{d}{d\bar{s}}\left(\frac{\bar{v}_*\overline{b^2}}{2}\right)=E_o\,\bar{b}\,\bar{u}_*\qquad \overline{v_*}=\Delta\overline{u_m}+2\cos\theta \tag{13}$$

$$\frac{d}{d\bar{s}}\left(\frac{\bar{v}_*\overline{b^2}}{4}\right)=F_{dc}^{-2}\frac{(\lambda\bar{b})^2}{2}\overline{c_m}\sin\theta+\alpha E_o\bar{b}\bar{u}_*\cos\theta \tag{14}$$

$$\frac{d}{d\bar{s}}\int_{\bar{A}}\left(\Delta u_m e^{-r^2/b^2}+u_\infty\cos\theta+\overline{w_o}\sin\theta\right)\overline{c_m}e^{-r^2/a^2}\,d\bar{A}=0 \tag{15}$$

$$\frac{d\theta}{d\bar{s}}=\frac{\frac{1}{2}(\lambda\bar{b})^2\overline{c_m}\cos\theta-\alpha E_o\bar{b}^2\overline{u_*}^2\sin\theta-c_d\sqrt{2}\,\bar{b}\sin^2\theta}{\frac{1}{4}\left(\bar{b}^2u_*^2-E_o\bar{b}^2\overline{u_*}^2\right)} \tag{16}$$

By integrating the Eqs. 13-16, we obtain the behavior of sediment - laden negative buoyant plume.To get numerical solutions,the ordinary finite difference method was used.

EXPERIMENTAL FACILITY

To examine the validity of the theoretical investigation, the experiment was performed in a 0.5m wide, 1.5m deep and 5.0m long circulating tank, in which the flow velocity was set uniform. The suspension of a given concentration was released vertically downward into flow through a nozzle.The sediment particles were seive sized and whose fall velocity is 1.28 cm/s. The flow velocity was measured with electromegnatic flow velocity meter and the sediment concentration was obtained by siphoning.At the begining of each run of the experiment, photographs were taken to get the trajectory of the plume. Then the cross sectional distribution of both the velocity and sediment concentration were measured at several points along the plume trajectory. The experimental conditions are summarized in Fig.2.

RUN	NO 1	NO 2	NO 3
C_o(%)	4.89	8.41	9.64
u_∞(cm/s)	3.48	3.53	3.90
l_b(cm)	51.92	66.23	70.73
F_{dc} (-)	0.06	0.01	0.04
$\dot{W}_o$ (-)	0.36	0.36	0.32

Fig. 2 .Experimental Condition

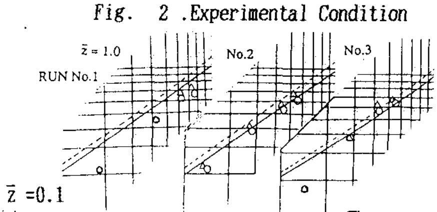

Fig.3. PLume trajectory

EXPERIMENTAL RESULTS

The experimental results are plotted in Figs. 3 - 6 together with theoretical predictions. Fig.7 shows the normalised distribution of flow velocity and sediment concentration. For the velocity profile, the upper part of this distribution is not represented by the Gaussian distribution.Except this part the experimental results are well predicted by the assumed distribution function, so that it can be said that the assumption of similarity profile is checked within our experimental conditions. However the condition of similiraty will not hold when $\overline{w_o}$ becomes large.From Figs.3-7 it can be concluded that the numerical model is useful for a simple description of the sediment - laden jet and plume in flow when w_o is small.

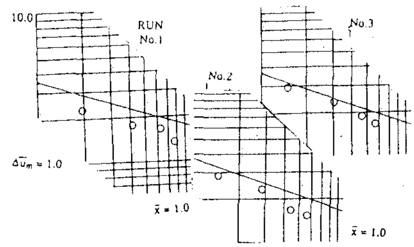

Fig.4. Decay of additive velocity , $\Delta\overline{u_m}$ along $\overline{x}$

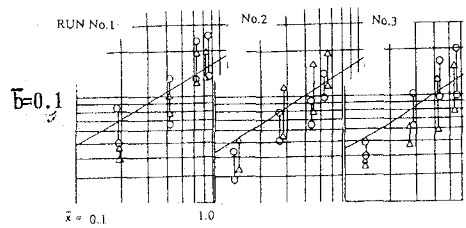

Fig.5. Growth of plume width

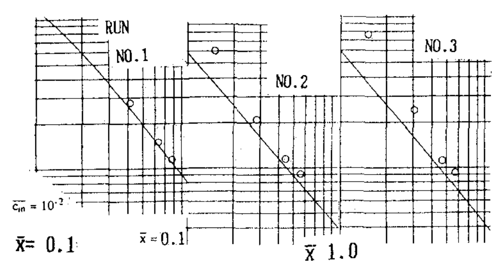

Fig.6. Decay of sediment concentration

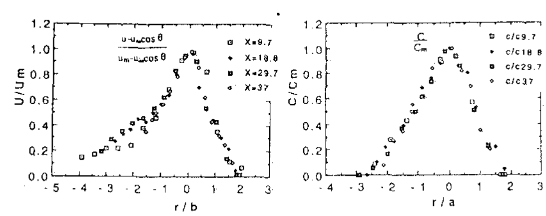

Fig.7. Similarity profile of velocity and sediment concentration

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