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A DIRECT APPROACH TO WAVE-CURRENT INTERACTION

Md. Hasanat Zaman, SM. JSCE, Nagasaki University
Hiroyoshi Togashi, M. JSCE, Nagasaki University
Xiping Yu, M. JSCE, Nagasaki University

Introduction

Knowledge on the wave-current interaction over uneven sea bottoms has always been of great interest for the coastal engineers to design coastal structures and to predict their functional performance. Traditionally, the analysis of the wave-current interaction has been based on the concept of radiation stress proposed by Longuet-Higgins and Stewart (1961). Since both the effects of the current on wave and vice versa are significant, iteration is conceptually necessary for accurate solutions with this theory. To avoid the iteration, which is time consuming in numerical computations, we present a method for the description of the interacted waves and currents in this study. The model is developed by integrating the continuity equation and the equation of motion in the vertical direction and, as an application, is utilized to analyze the wave-current coexistent field over a submerged mound.

Development of Theory

Let us consider a two dimensional problem in the vertical plane. By the conventional assumption of invicid and incompressible fluid, the motion induced by waves and currents can be described by the following continuity equation and the equation of motion

$$\frac{\partial u}{\partial x}+\frac{\partial w}{\partial z}=0 \qquad (1) \qquad\qquad \frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+w\frac{\partial u}{\partial z}+\frac{\partial}{\partial x}\left(\frac{p_d}{\rho}\right)=0 \qquad (2)$$

where u and w are the velocity components in the x and z direction, respectively, ρ the water density and p_d the dynamic pressure. The kinematic boundary conditions on the free surface and at the bottom can be expressed as

$$\frac{\partial \eta'}{\partial t}+u^f\frac{\partial \eta'}{\partial x}-w^f=0 \qquad (3) \qquad\qquad u^b\frac{\partial h}{\partial x}+w^b=0 \qquad (4)$$

where the subscripts f and b represent the values at the free surface and at the bottom, respectively; h is the still water depth and η' the deviation of the free surface from the still water level.

Based on the exact solution of a linear wave over uniform current, we assume that the dynamic pressure p_d can be expressed by

$$p_d=\rho g\eta'\frac{\cosh k_I(h+z)}{\cosh k_I(h+\eta')} \qquad (5)$$

where k_I is the modified wave number and g the gravitational acceleration.

By Eq. (5), Eqs. (1) and (2) can be integrated from the bottom$(z=-h)$ to the free surface$(z=\eta')$. When Eqs. (3) and (4) are also considered, we obtain

$$\frac{\partial \eta'}{\partial t}+\frac{\partial Q}{\partial x}=0 \qquad (6) \qquad\qquad \frac{\partial Q}{\partial t}+\frac{\partial}{\partial x}\left(\frac{\beta Q^2}{h+\eta'}\right)+\left(\eta' g\kappa_1+C_I^2\right)\frac{\partial \eta'}{\partial x}+\eta' g\kappa_2\frac{\partial h}{\partial x}=0 \qquad (7)$$

where $Q=\int_{-h}^{\eta'}u\,dz$ is the discharge in the x direction, C_I the modified celerity, β the momentum correction factor and κ_1 and κ_2 are the relative water depth dependent parameters;

$$C_I^2=\frac{g}{k_I}\tanh k_I(h+\eta') \qquad (8) \qquad\qquad \beta=\frac{1}{2}\left[1+\frac{2k_I(h+\eta')}{\sinh 2k_I(h+\eta')}\right]\left[\frac{k_I(h+\eta')}{\tanh k_I(h+\eta')}\right] \qquad (9)$$

$$\kappa_1=\mathrm{sech}^2 k_I(h+\eta')-1 \qquad (10) \qquad\qquad \kappa_2=\mathrm{sech}^2 k_I(h+\eta')-\mathrm{sech}\, k_I(h+\eta') \qquad (11)$$

The modified dispersion relation can be obtained as

$$\frac{\sigma^2}{g}\left(1-\frac{k_I U}{\sigma}\right)=gk_I\tanh k_I(h+z) \qquad (12)$$

where σ is the angular frequency and U the current velocity.

Numerical Method

Eqs.(6) and (7) have been discretized following the finite difference scheme proposed by Dronkers (1967), where Q is defined at the integer grid points and fractional time steps and, on the other hand, η' is defined at the fractional grid points and integer time steps. Central difference has been adopted for both the temporal and spatial differentiations. The convective term in the equation of motion has been linearized and properly expressed by the defined values of the unknowns. The discretized equations form a tridiogonal system which can be solved by the double sweep method.

Results and Discussions

As an application of the model established, we consider the case where a wave propagates in a steady current. The sea bed is essentially flat and horizontal but a mound is assumed to be in presence, so the water depth is non-uniform in the domain concerned. The length of the mound is $150m$ and the maximum height $5m$. The domain for our computation is $450m$ long. The water depth over the flat bed is $10m$. In the computations, we take $\Delta t = 0.5s$ and $\Delta x = 7.5m$, which satisfy the CFL's stability condition. Forced currents of three different magnitudes (U=0.5, 0.75 and $1.0m/s$) have been considered while the incident wave height (H_0=0.62m), wave length (L_0=70.85m) and wave period (T_0=8s) are kept constant for all the cases. Figs. 1 and 2 show the variations of the mean water level Δ and the local wave height H (both normalized by the incident wave height H_0) with the magnitude of the forced currents (normalized by the incident wave celerity C_0). It is found that on the flat bottom the variation of the mean water level is almost the same in spite of the variable magnitude of the forced currents, because the incident wave conditions are kept constant. Over the mound, however, larger drop of the mean water level is observed when the forced current is strengthened. This can be explained by the principle of open channel hydraulics. It is also noted that the wave length over the mound is significantly shortened when the magnitude of the forced current increases. This could be interpreted by the larger decrease of the mean water depth over the mound for stronger forced current.

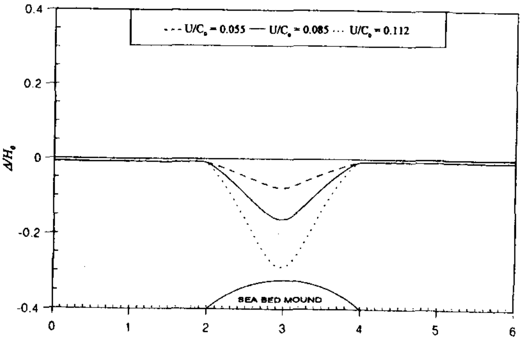

Fig.1 Mean water level for various currents

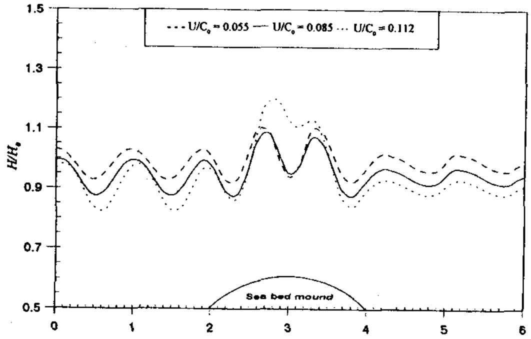

Fig.2 Wave height for various currents

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