STUDY ON HYDRODYNAMIC PHENOMENA IN THE ARIAKE SEA

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Introduction

The Ariake sea is located near the center of Kyushu island on the south of Japan. It covers approximately 1,700 sq.km. surrounded by four prefectures, namely Saga, Fukuoka, Kumamoto and Nagasaki. It is recognized as an inland sea with the largest tidal range. During spring tide, the tidal range varies from 3 m at the Hayasaki strait to 6 m at the Suminoe harbor which is located at the innermost part of the sea. There exist some meteorological and gaging stations located in or near the Ariake sea. These stations have been monitoring and recording climatic conditions, tidal current and water level in the sea. The observation data have been reported by some researchers, e.g., Watanabe (1988), Fujimoto (1988), and others.

In studying dispersion of pollutants and transport of sediment in the Ariake sea, more details about current pattern and tidal fluctuation are needed. This information can be obtained by using a simulation model. Several mathematical models have been developed and applied to simulate hydrodynamic circulation in the Ariake sea. Each model has some different advantages and disadvantages compared with others.

In this study, a hydrodynamic model is developed using the finite element method. The governing equations include the vertically averaged two-dimensional continuity and momentum equations. The Galerkin weighted residual method is used in model formulation. The developed model is verified and applied to simulate hydrodynamic phenomena in the Ariake sea.

Continuity and Momentum Equations

The vertically averaged two-dimensional continuity and momentum equations (Pritchard, 1971) are used as the basic governing equations in model development. These equations are

$$\frac{\partial(h-h_o)}{\partial t}+\frac{\partial(hu)}{\partial x}+\frac{\partial(hv)}{\partial y}=0 \tag{1}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+g\frac{\partial(h-h_o)}{\partial x}+\frac{1}{\rho}\frac{\partial P_a}{\partial x}-fv-\frac{\tau_{wx}}{h\rho}+\frac{\tau_{bx}}{h\rho}=0 \tag{2}$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+g\frac{\partial(h-h_o)}{\partial y}+\frac{1}{\rho}\frac{\partial P_a}{\partial y}+fu-\frac{\tau_{wy}}{h\rho}+\frac{\tau_{by}}{h\rho}=0 \tag{3}$$

where h is total water depth, h_o is averaged water depth, u and v are flow velocities, g is gravitational acceleration, P_a is atmospheric pressure, ρ is water density, f is coriolis factor, τ_{wx} and τ_{wy} are surface shear stresses, τ_{bx} and τ_{by} are bottom shear stresses.

From these equations, the Galerkin weighted residual method is applied. With this method, water depth and flow velocities, which are dependent variables in the governing equations, are expressed in terms of the values at the nodal points identified in the study domain. The original equations which are in the form of partial differential equations are transformed to a set of algebraic equations in which water depths and flow velocities at the nodal points are unknown variables. The continuity and momentum equations can be written in the finite element form as

$$M\frac{\partial H}{\partial t}+E_h(U,V).H+B_q=0 \tag{4}$$

$$M\frac{\partial U}{\partial t}+E_u(U,V)+M_x.H=0 \tag{5}$$

$$M\frac{\partial V}{\partial t}+E_v(U,V)+M_y.H=0 \tag{6}$$

Solution Techniques

In this study the *separated time* technique is used in solving the time dependent finite element equations, i.e., the water level matrix H and velocity matrices U and V are computed at different halves of the time step. The time derivatives of H, U and V are approximated by using the trapezoidal rule. The water depth at time $t+\Delta t/2$ can be computed from

$$H^{t+\frac{\Delta t}{2}} = A^t H^{t-\frac{\Delta t}{2}} + B^t \tag{7}$$

where A^t and B^t are functions of U and V. These equations are solved by using the Gauss elimination method providing that water levels at the open boundary nodes are specified.

Similarly, the velocities U and V at time $t+\Delta t$ can be computed from

$$U^{t+\Delta t} = P\ U^t + Q_u \tag{8}$$

and

$$V^{t+\Delta t} = P\ V^t + Q_v \tag{9}$$

where Q_u and Q_v are functions of U and V at time t and H at time $t+\Delta t/2$.

In solving the velocity matrices, the velocities normal to the boundary are usually specified. So the velocities U and V at the boundary nodes must be transformed to the normal and tangential velocities U_n and U_s. This can be done by merging Eqs.(8) and (9) which will result in the following equation:

$$U_{uv}^{t+\Delta t} = P_{uv}\, U_{uv}^{t} + Q_{uv} \tag{10}$$

where

$$U_{uv} = \{U_1\ V_1\ U_2\ V_2\ ....U_n\ V_n\} \tag{11}$$

$$P_{uv} = \begin{vmatrix} P_{11} & 0 & P_{12} & 0 & .... & P_{1n} & 0 \\ 0 & P_{11} & 0 & P_{12} .... & 0 & P_{1n} \\ . & . & . & . & .... & . & . \\ \\ . & . & . & . & .... & . & . \\ P_{n1} & 0 & P_{n2} & 0 & .... & P_{nn} & 0 \\ 0 & P_{n1} & 0 & P_{n2} .... & 0 & P_{nn} \end{vmatrix} \tag{12}$$

and

$$Q_{uv} = \{Q_{u1}\ Q_{v1}\ Q_{u2}\ Q_{v2}\ ....Q_{un}\ Q_{vn}\} \tag{13}$$

The velocity matrix U_{uv} is transformed to the matrix U_{ns} by replacing the velocity component at the boundary nodes U_k and V_k by their normal and tangential components U_{nk} and U_{sk} , i.e.

$$U_{uv} = A_k\, U_{ns} \tag{14}$$

where

$$U_{ns} = \{U_1\ V_1\ U_2\ V_2\ ...U_{nk}\ U_{sk}\ ...U_n\ V_n\} \tag{15}$$

and

$$A_k = \begin{vmatrix} 1 & 0 & 0 & 0 ... & 0 & 0 ... & 0 & 0 \\ 0 & 1 & 0 & 0 ... & 0 & 0 ... & 0 & 0 \\ 0 & 0 & 1 & 0 ... & 0 & 0 ... & 0 & 0 \\ 0 & 0 & 0 & 1 ... & 0 & 0 ... & 0 & 0 \\ \\ 0 & 0 & 0 & 0 ... & & ... & 0 & 0 \\ 0 & 0 & 0 & 0 ... & \alpha_k & ... & 0 & 0 \\ \\ 0 & 0 & 0 & 0 ... & 0 & 0 ... & 1 & 0 \\ 0 & 0 & 0 & 0 ... & 0 & 0 ... & 0 & 1 \end{vmatrix} \tag{16}$$

in which

$$\alpha_k = \begin{vmatrix} \cos\theta_k & -\sin\theta_k \\ \\ \sin\theta_k & \cos\theta_k \end{vmatrix} \tag{17}$$

where θ_k is an angle between the normal direction and the x-axis.

The velocity matrix U_{uv} in Eq.(10) is then replaced by Eq.(14). With specified normal velocities at the boundary nodes, the velocity matrix U_{ns} at time $t+\Delta t$ can be computed provided that U_{ns} at time t and H at time $t+\Delta t/2$ are known.

Model Application

The developed model is verified and applied to study the hydrodynamic circulation in the Ariake sea. The Ariake sea is divided into 194 triangular elements with 133 nodal points as shown in Figure 1. Data on mean water depths at the nodal points are fed as input data together with some other parameters. The fluctuating water level at the Hayasaki strait and the normal flow velocities along the shoreline are considered as the boundary conditions. The results obtained from the model are plotted to show the current patterns and profiles of water levels in the sea.

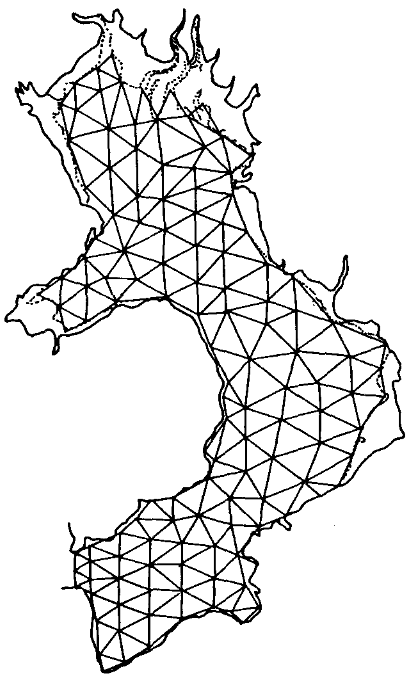

Figure 1. Element configuration of the Ariake sea.

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