非線形波動境界層流れの一計算

Non-Linear Effect on The Velocity Fields In Wave Boundary Layer

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Introduction

The magnitude and the predominant direction of the sediment transport near the bottom are known to be closely related to the flow in the wave induced boundary layer. The property that the magnitude of water particle velocity at the Crest phase differs from the one of the Trough phase due to the non-linearity of the waves plays an important role to determine the sediment movement. The non-linearity of the waves become predominant near and in the surf zone, where the sediment movement occurs vigorously. Therefore, in order to describe the flow near the bottom and relationship between the flow and the movement of the sediment, an analysis done by using non-linear boundary layer equation and the finite amplitude wave theory is essential to represent the velocity field. Many theoretical and experimental studies have been done by others on the basis of wave boundary layer.

In this study, a non-linear solution of wave boundary layer is obtained numerically based on two dimensional boundary equations and the method consists of assuming periodicity and representing variation with x by the Fourier expansion. This procedure enables us to reduce an independent variable x and to perform the calculation within the limited memory space by Disk Operating System of the Personal Computer, but we have to solve as much equation as the Fourier components.

Numerical Analysis and Calculation

A two-dimensional situation is considered, in which X is measured along the plane of the horizontal bottom boundary and Z is measured perpendicular to the horizontal bottom boundary. The velocity components are represented as u and w in the direction of x and z respectively. Assuming an incompressible homogeneous fluid of constant depth, the two-dimensional momentum equation and continuity equation are given as follows:

$$\frac{\partial \bar{u}}{\partial t}+\frac{\partial}{\partial x}\bar{u}^2+\frac{\partial}{\partial z}\overline{uw}=\frac{\partial U}{\partial t}+U\frac{\partial U}{\partial x}+\frac{\partial}{\partial z}\left(k\frac{\partial \bar{u}}{\partial z}\right) \quad (1), \qquad \frac{\partial \bar{u}}{\partial x}+\frac{\partial \bar{w}}{\partial z}=0 \quad (2)$$

The over bar denotes the Reynolds average. The nondimensionalized equations become

$$\frac{\partial \hat{u}}{\partial \hat{t}}+\alpha\left(\frac{\partial}{\partial \hat{u}}\bar{u}^2+\frac{1}{\beta}\frac{\partial}{\partial \xi}\hat{u}\hat{w}\right)=\sin(\hat{x}-\hat{t})-\frac{1}{2}\alpha\gamma\, \sin 2(\hat{x}-\hat{t})+\frac{1}{\beta}\frac{\partial}{\partial \xi}\left(\frac{\hat{k}}{\beta}\,\frac{\partial \hat{u}}{\partial \xi}\right) \quad (3), \qquad \frac{\partial \hat{u}}{\partial \hat{x}}+\frac{1}{\beta}\frac{\partial \hat{w}}{\partial \xi}=0,\ \beta=\eta_o\hat{\xi}e^{\xi} \quad (4)$$

Where, $\hat{k}$ is the nondimensional kinematic viscosity for laminar flow given by $\hat{k}=\nu\sigma/u_\infty^{\ 2}$ and nondimensional eddy viscosity for the turbulent boundary layer. For the latter case, $\hat{k}$ is related to the turbulent energy through eq. 5 and 6. (Spalding and Launder)

$$\hat{k}=C^{1/4}k(\hat{Z}+\hat{\delta}_r)\hat{E}^{1/2} \quad (5), \qquad \frac{\partial \hat{E}}{\partial \hat{t}}+\alpha\left(\frac{\partial}{\partial \hat{x}}(\hat{u}\hat{E})+\frac{1}{\beta}\frac{\partial}{\partial \xi}(\hat{w}\hat{E})\right)=\frac{K}{\beta^2}\left(\frac{\partial \hat{u}}{\partial \xi}\right)^2+\frac{1}{\beta}\frac{\partial}{\partial \xi}\left(\frac{\hat{k}}{\beta}\frac{\partial \hat{E}}{\partial \xi}\right)-\hat{\varepsilon} \quad (6)$$

$$\alpha=\frac{ku_\infty}{\sigma} \quad (7), \qquad u_\infty=\frac{a\sigma}{\sinh kh} \quad (8)$$

ν is the Kinematic viscosity of the fluid which is taken 0.01 cm^2/sec at 20° C. The initial condition is taken from the coefficient of exponential term of $\cos(\hat{x}-\hat{t})$. The time variable is replaced by a discrete sequence of time-instants. Assuming periodicity of the flow with $\hat{x}$, velocity $\hat{u}$ is expressed by eq. 10. The value of C_p is essential to find out the velocity field.

$$\gamma = 1 - \frac{3\alpha^2}{a^2 k^2} \quad (9), \qquad \hat{u}(\hat{x},\hat{z},\hat{t}) = \sum_{p=-2}^{2} C_p(\hat{z},\hat{t}) e^{ipt} \quad (i^2 = -1) \qquad (10)$$

Computational technique

The Forward time centered space (FTCS) scheme is used to perform the numerical analysis. To keep the stability one wave period is divided into six thousand time steps along the time axis. A non linear grid system is used by transferring the vertical z axis into ξ, where the fine spacing grids are taken near the bottom boundary to make the calculation more bright. ξ varies from zero to 1 in the vertical axis. Zero is taken at the bottom and One is taken at the upper boundary respectively. The study is done ξ_{max} is equal to 2. There were 40 division between the lower and upper boundary so $\Delta\xi$ becomes 0.025. When the upper boundary is according to the linear wave theory then we have considered γ equal to one.

Result and Discussion.

The calculation for laminar boundary layer is complected and the One equation model for the turbulent boundary layer is still under process. The calculation is done up to 6 cycles and 30 cycles. The data for the graph is taken at the interval of thirty degree. The velocity profile of two different calculated cycles have compared. It is found that after 3 cycles (When $\hat{t}=6\pi$) the calculation becomes effective. It is also found that mostly there is no difference between 6 and 30 cycle data. The velocity field based on the linear wave theory is shown in fig-1 and the velocity field based on the finite amplitude wave theory is shown in fig-2. While the calculation starts, it calculate from the lower boundary to upper boundary. The profile of upper boundary has been checked into linear wave theory and finite amplitude wave theory which shows the good agreement.

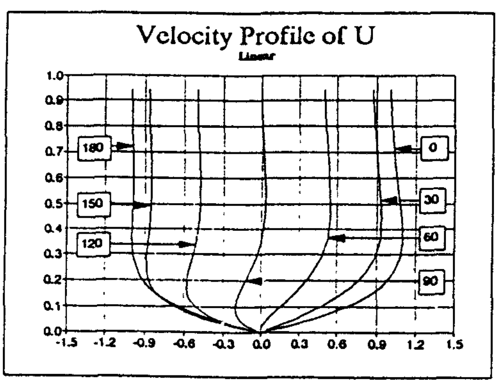

Fig - 1

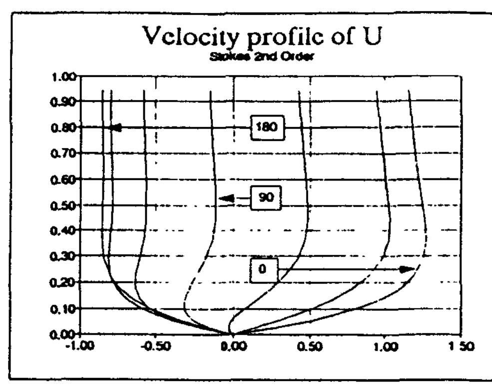

Fig - 2

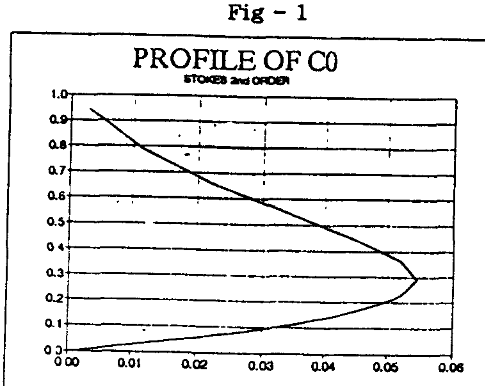

Fig - 3

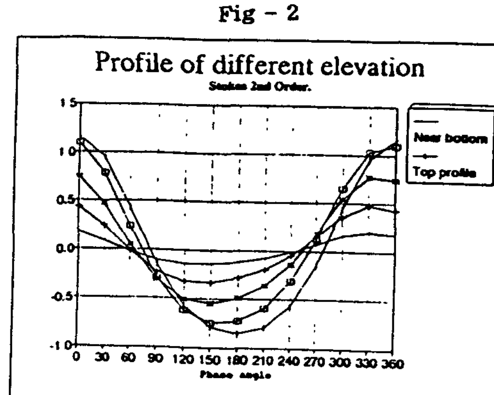

Fig - 4