# FLUID RESISTANCE DUE TO OSCILLATION OF COLUMNS IN STILL WATER

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**1. INTRODUCTION:** Accurate estimation of wave forces acting on various components of structures is one of the most important steps in any offshore design. The analytical methods such as diffraction theory ignore the fluid viscous effects and hence are not reliable when drag force is significant as in the case of slender members ($d/L<0.2$). Morison gave an empirical formula for calculating the wave forces on slender stationary members which is now widely employed in engineering practice. Brebbia generalised the Morison's equation for oscillating members by considering the relative motion.

The authors suggest another modification to the Morison's equation by treating the effects of fluid motion and motion of the members seperately. When waves act on any structure, owing to the loading actions of water particles it begins to oscillate. At the same time, the damping forces due to the very movement of the structure as well as due to the waves prevent the oscillations from growing. It might be a method to divide the mutual effects of the water and the structure into two portions: one is the loading force causing the structure to oscillate, another is the restoring force restraining the oscillations from growing. The former is called the wave force and the latter may be named as fluid resistance. The net effective wave force p on an oscillating structure is regarded as the algebraic sum of these two forces. The wave force F is represented by the Morison's equation where as the fluid resistance R by an equation similar to the Morison's equation. Mathematically:

$$F = C_m \rho \pi r_0^2 \frac{\delta u}{\delta t} + C_d \rho r_0\, u|u| \qquad R = C_m' \rho \pi r_0^2 \frac{d^2x}{dt^2} + C_d' \rho r_0 \frac{dx}{dt}\left|\frac{dx}{dt}\right|$$

In this paper the authors have investigated the fluid resistance component.

**2. THEORETICAL COMPUTATIONS:** The inertia component of the fluid resistance is computed by diffraction analysis. Fig.1 shows the definition sketch of the column. The fluid is assumed to be nonviscous, irrotational and incompressible. Airy wave theory is used. The differential equation for the velocity potential due to the diffracted wave field is solved using the boundary conditions at the sea surface, sea bottom and the column surface. It is then substituted in the Bernoulli's pressure equation which on integration gives the total horizontal force on the column. The coefficient $C_m'$ is shown against d/L ratio for different d/h ratios in Fig.2. For the case of slender members $C_m'$ approaches unity.

**3. EXPERIMENTS:** The experiments were carried out in a two dimensional water tank of height=1.2m, width=1m and length=30m. The depth of water in the tank was 0.75m. The test model was a hollow cylindrical column made of P.V.C. plastic. The dimensions were outer diameter=60mm, inner diameter=56.6mm and length=0.9m. The bottom was hinged to the floor of the tank and the oscillation was in the form of rigid rotational displacement. The frictional resistance at the hinge was neglected. The hinge was so designed that the column could oscillate in one direction only. (single degree of freedom system).

The column top was given initial displacement and then was released from this stationary displaced position. The free-vibration response was recorded using a 16mm cine camera.

**4. EQUATION OF MOTION:** Fig.3 is the schematic diagram of the column under oscillation. Using the principle of dynamic equilibrium, the equation of motion is:

$$\frac{ml^2}{3}\frac{d^2\beta}{dt^2} + \left[\frac{Bh}{2} - \frac{mgl}{2}\right]\beta = -C_m'\rho\pi r_0^2 \frac{h^3}{3}\frac{d^2\beta}{dt^2} - C_d'\rho r_0 \frac{h^4}{4}\frac{d\beta}{dt}\frac{d\beta}{dt}$$

On rearranging the terms and linearising the damping term, above equation can be written as:

$$\frac{d^2\beta}{dt^2} + 2h_0\omega_0\frac{d\beta}{dt} + \omega_0^2\beta = 0 \quad \text{where} \quad \omega_0 = \sqrt{\frac{3(Bh - mgl)}{2(ml^2 + C_m'\rho\pi r_0^2 h^3)}} \;;\; h_0 = \frac{C_d'\omega\beta_0\rho r_0 h^4}{\pi\omega_0(ml^2 + C_m'\rho\pi r_0^2 h^3)}$$

**5. DATA ANALYSIS AND RESULTS:** Fig.4 shows typical free-vibration response patterns observed in the experiments. The column oscillates about the neutral position with a constant circular frequency $\omega_0$ called the natural frequency. The magnitude of the response diminishes exponentially as the system damps out. The mean value of $\omega_0$ and the corresponding value of $C_m'$ are shown in table-1 along with their theoretical equivalents obtained by diffraction theory.

Fig.5 is a plot between the displacement x of the column top and the corresponding equivalent damping ratio $h_0$. The values of the damping ratio based on free vibration responses for small initial displacements are smaller in comparison with those based on the free vibration responses for larger initial displacements. When the amplitude of vibration is large, the fluid viscous forces are also large and this might be the reason for the increase in the value of the damping ratio. Fig.6 shows the values of the damping ratio based on the initial displacements. The damping ratio increases linearly with the initial displacement.

| | Experiment | Theory |
|---|---|---|
| $\omega_0$ (rad/sec) | 3.39 | 3.19 |
| $C_m'$ | 0.87 | 1.00 |

Table-1

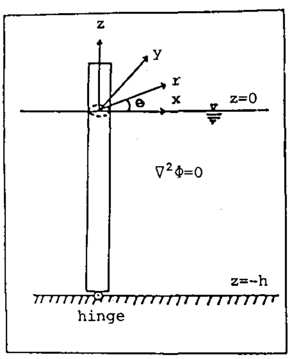


Fig.1-Definition Sketch

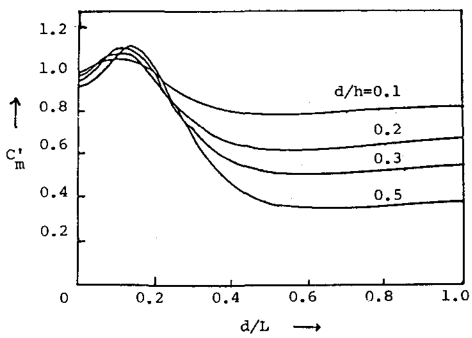


Fig.2-Added mass coefficient $C_m'$

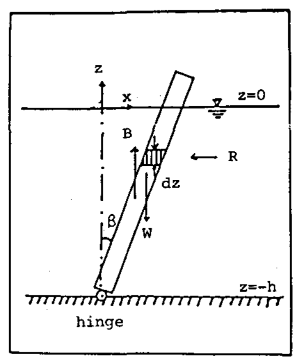


Fig.3-Forces on the column at equilibrium.

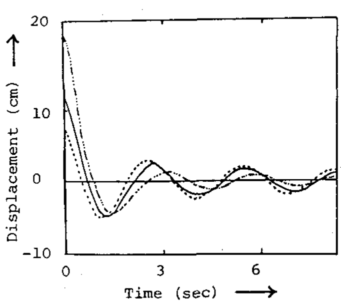


Fig.4

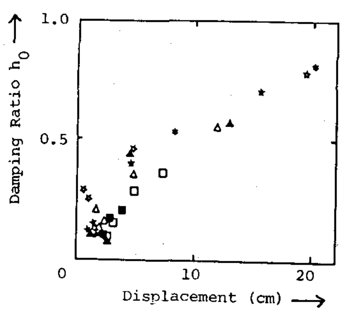


Fig.5

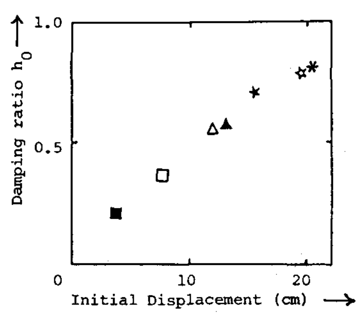


Fig.6

NOTATIONS: d=significant length of the member(=diameter of the column); L=wave length; $C_m$ & $C_d$=hydrodynamic coefficients; $C_m'$ & $C_d'$=fluid resistance coefficients; u & $\delta u/\delta t$=horizontal water particle velocity and acceleration at the centre position if the column were not present; x=horizontal displacement of the column; $\beta$= angular displacement of the column; $\beta_0$=amplitude of the angular displacement; B=buoyancy force on the column; h=depth of water.