

二方向連続等方性矩形板の安定

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緒言 連続矩形平板の安定に関する既往の研究として一方角連続板に関しては、それぞれ一対辺単純支持および一対辺自由の場合を取り扱った倉田氏⁽¹⁾や著者らのものがあるが、二方向連続矩形板の安定問題を扱った論文は未だ見あたらない。本文は前記著者らの研究を、任意境界条件をもつ二方向連続矩形板に拡張し、かかる板が図-1のごとく互いに垂直な二方向からの周辺直圧力を受ける場合の安定問題の厳密解法を提示するものである。

1. たわみ角一端モーメント関係式の誘導

図-2のごとく二辺の長さが a , b なる等方性等断面矩形板が単位中あたりに N_x , N_y なる中立面内の等分布圧縮力を受ける場合の微分方程式は次式で表わされる。

$$\frac{\partial^2 w}{\partial x^2} + 2 \frac{\partial^2 w}{\partial x \partial y} + \frac{\partial^2 w}{\partial y^2} + \frac{1}{D} (N_x \frac{\partial^2 w}{\partial x^2} + N_y \frac{\partial^2 w}{\partial y^2}) = 0 \quad (1)$$

ここに D は板剛度で板厚を t , 材料のヤング率およびボアソン比をそれぞれ E , ν とすれば

$$D = \frac{E t^3}{12(1-\nu^2)} \quad \text{である。}$$

いま式(1)を満足するたわみ w の一般解が次式で表わされるものとする。

$$w = \sum_{n=1}^{\infty} X \sin \frac{n \pi y}{b} + \sum_{m=1}^{\infty} Y \sin \frac{m \pi x}{a} \quad (2)$$

ここに X , Y はそれぞれ x , y のみの函数であり式(2)を式(1)に代入して解けば X , Y がそれぞれ次式のごとく求められる。

$$X = C_{n1} \cosh \pi \lambda_1 \xi + C_{n2} \sinh \pi \lambda_1 \xi + C_{n3} \cosh \pi \lambda_2 \xi + C_{n4} \sinh \pi \lambda_2 \xi \quad (3)$$

$$Y = D_{m1} \cosh \pi \mu_1 \eta + D_{m2} \sinh \pi \mu_1 \eta + D_{m3} \cosh \pi \mu_2 \eta + D_{m4} \sinh \pi \mu_2 \eta$$

ただし $C_{n1} \sim C_{n4}$, $D_{m1} \sim D_{m4}$ は積分定数。

$$\lambda_1 = \frac{\pi}{b} \sqrt{\left\{ n^2 - \left(\frac{\pi}{a}\right)^2 \frac{Q}{2} \right\}} + \sqrt{\left\{ n^2 - \left(\frac{\pi}{a}\right)^2 \frac{Q}{2} \right\}^2 - n^2 \{ n^2 - Q \}}$$

$$\lambda_2 = \frac{\pi}{b} \sqrt{\left\{ n^2 - \left(\frac{\pi}{a}\right)^2 \frac{Q}{2} \right\}} - \sqrt{\left\{ n^2 - \left(\frac{\pi}{a}\right)^2 \frac{Q}{2} \right\}^2 - n^2 \{ n^2 - Q \}}$$

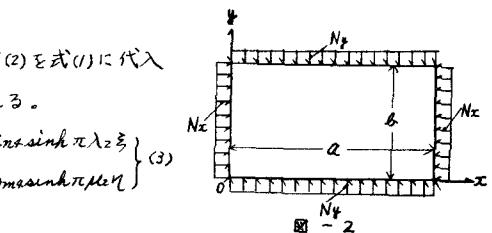
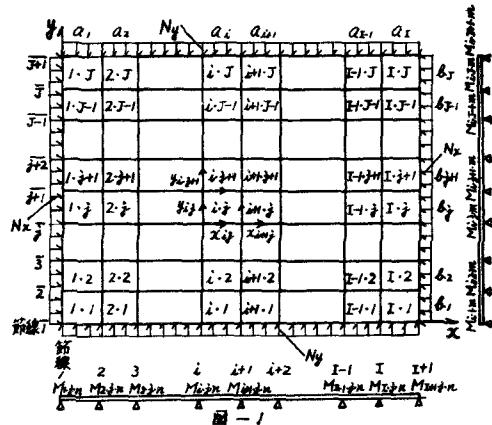
$$\xi = \frac{x}{a}, \eta = \frac{y}{b}, R = \frac{N_x a^2}{D \pi^2}, Q = \frac{N_y b^2}{D \pi^2}$$

以上の式(2), (3)より結局式(1)の一般解は次式で表わされることになる。

$$w = \sum_{n=1}^{\infty} (C_{n1} \cosh \pi \lambda_1 \xi + C_{n2} \sinh \pi \lambda_1 \xi + C_{n3} \cosh \pi \lambda_2 \xi + C_{n4} \sinh \pi \lambda_2 \xi) \sin n \pi \eta \\ + \sum_{m=1}^{\infty} (D_{m1} \cosh \pi \mu_1 \eta + D_{m2} \sinh \pi \mu_1 \eta + D_{m3} \cosh \pi \mu_2 \eta + D_{m4} \sinh \pi \mu_2 \eta) \sin m \pi \xi \quad (4)$$

いま $x = 0$, a および $y = 0$, b における境界条件を正弦フーリエ級数を用いて次式にて一般表示するものとする。

$$(w)_{x=0} = \sum_{n=1}^{\infty} \delta_{n1} \sin n \pi \eta, -D \left(\frac{\partial^2 w}{\partial x^2} + \nu \frac{\partial^2 w}{\partial y^2} \right)_{x=0} = \sum_{n=1}^{\infty} M_{n1} \sin n \pi \eta, (w)_{y=0} = \sum_{m=1}^{\infty} \delta_{m1} \sin m \pi \xi, -D \left(\frac{\partial^2 w}{\partial y^2} + \nu \frac{\partial^2 w}{\partial x^2} \right)_{y=0} = \sum_{m=1}^{\infty} M_{m1} \sin m \pi \xi \\ (w)_{x=a} = \sum_{n=1}^{\infty} \delta_{n2} \sin n \pi \eta, -D \left(\frac{\partial^2 w}{\partial x^2} + \nu \frac{\partial^2 w}{\partial y^2} \right)_{x=a} = \sum_{n=1}^{\infty} M_{n2} \sin n \pi \eta, (w)_{y=b} = \sum_{m=1}^{\infty} \delta_{m2} \sin m \pi \xi, -D \left(\frac{\partial^2 w}{\partial y^2} + \nu \frac{\partial^2 w}{\partial x^2} \right)_{y=b} = \sum_{m=1}^{\infty} M_{m2} \sin m \pi \xi$$



式(4)と式(5)に代入すれば積分定数 $C_{n1} \sim C_{n4}$ および $\theta^* D_{m1} \sim D_{m4}$ が次のとく決定される。

$$\left. \begin{aligned} C_{n1} &= -\frac{1}{(\lambda_1^2 - \lambda_2^2)} \left(\frac{\alpha}{\pi} \right)^2 \frac{M_{an}}{D} - \frac{\lambda_1^2 - \nu \left(\frac{mb}{\alpha} \right)^2}{\lambda_1^2 - \lambda_2^2} \delta_{an} \\ C_{n2} &= \frac{1}{(\lambda_1^2 - \lambda_2^2) \sinh \pi \lambda_1} \left(\frac{\alpha}{\pi} \right)^2 \left(\frac{M_{an}}{D} \coth \pi \lambda_1 - \frac{M_{bn}}{D} \right) + \frac{\lambda_1^2 - \nu \left(\frac{mb}{\alpha} \right)^2}{(\lambda_1^2 - \lambda_2^2) \sinh \pi \lambda_1} (\delta_{an} \coth \pi \lambda_1 - \delta_{bn}) \\ C_{n3} &= \frac{1}{(\lambda_1^2 - \lambda_2^2)} \left(\frac{\alpha}{\pi} \right)^2 \frac{M_{an}}{D} + \frac{\lambda_1^2 - \nu \left(\frac{mb}{\alpha} \right)^2}{\lambda_1^2 - \lambda_2^2} \delta_{an} \\ C_{n4} &= -\frac{1}{(\lambda_1^2 - \lambda_2^2) \sinh \pi \lambda_2} \left(\frac{\alpha}{\pi} \right)^2 \left(\frac{M_{an}}{D} \coth \pi \lambda_2 - \frac{M_{bm}}{D} \right) - \frac{\lambda_1^2 - \nu \left(\frac{mb}{\alpha} \right)^2}{(\lambda_1^2 - \lambda_2^2) \sinh \pi \lambda_2} (\delta_{an} \coth \pi \lambda_2 - \delta_{bm}) \\ D_{m1} &= -\frac{1}{(\mu_1^2 - \mu_2^2)} \left(\frac{\alpha}{\pi} \right)^2 \frac{M_{cm}}{D} - \frac{\mu_1^2 - \nu \left(\frac{mb}{\alpha} \right)^2}{\mu_1^2 - \mu_2^2} \delta_{cm} \\ D_{m2} &= \frac{1}{(\mu_1^2 - \mu_2^2) \sinh \pi \mu_1} \left(\frac{\alpha}{\pi} \right)^2 \left(\frac{M_{cm}}{D} \coth \pi \mu_1 - \frac{M_{dm}}{D} \right) + \frac{\mu_1^2 - \nu \left(\frac{mb}{\alpha} \right)^2}{(\mu_1^2 - \mu_2^2) \sinh \pi \mu_1} (\delta_{cm} \coth \pi \mu_1 - \delta_{dm}) \\ D_{m3} &= \frac{1}{(\mu_1^2 - \mu_2^2)} \left(\frac{\alpha}{\pi} \right)^2 \frac{M_{cm}}{D} + \frac{\mu_1^2 - \nu \left(\frac{mb}{\alpha} \right)^2}{\mu_1^2 - \mu_2^2} \delta_{cm} \\ D_{m4} &= -\frac{1}{(\mu_1^2 - \mu_2^2) \sinh \pi \mu_2} \left(\frac{\alpha}{\pi} \right)^2 \left(\frac{M_{cm}}{D} \coth \pi \mu_2 - \frac{M_{dm}}{D} \right) - \frac{\mu_1^2 - \nu \left(\frac{mb}{\alpha} \right)^2}{(\mu_1^2 - \mu_2^2) \sinh \pi \mu_2} (\delta_{cm} \coth \pi \mu_2 - \delta_{dm}) \end{aligned} \right\} \quad (6)$$

次いで $x=0, \alpha$ および $y=0$, 及びおけるたわみ角および反力も正弦フーリエ級数を用いて次式のとく表わされるものとする。

$$\left. \begin{aligned} \left(\frac{\partial w}{\partial x} \right)_{x=0} &= \sum_{n=1}^{\infty} \theta_{an} \sin n \pi \alpha \\ \left(\frac{\partial w}{\partial x} \right)_{x=\alpha} &= \sum_{n=1}^{\infty} \theta_{an} \sin n \pi \alpha \\ -D \left[\frac{\partial^3 w}{\partial x^3} + (2-\nu) \frac{\partial^3 w}{\partial x^2 \partial y^2} \right]_{x=0} &= \sum_{n=1}^{\infty} V_{an} \sin n \pi \alpha \\ -D \left[\frac{\partial^3 w}{\partial x^3} + (2-\nu) \frac{\partial^3 w}{\partial x^2 \partial y^2} \right]_{x=\alpha} &= \sum_{n=1}^{\infty} V_{an} \sin n \pi \alpha \\ \left(\frac{\partial w}{\partial y} \right)_{y=0} &= \sum_{n=1}^{\infty} \theta_{cm} \sin n \pi \alpha \\ \left(\frac{\partial w}{\partial y} \right)_{y=\beta} &= \sum_{n=1}^{\infty} \theta_{cm} \sin n \pi \alpha \\ -D \left[\frac{\partial^3 w}{\partial y^3} + (2-\nu) \frac{\partial^3 w}{\partial y^2 \partial x^2} \right]_{y=0} &= \sum_{n=1}^{\infty} V_{cm} \sin n \pi \alpha \\ -D \left[\frac{\partial^3 w}{\partial y^3} + (2-\nu) \frac{\partial^3 w}{\partial y^2 \partial x^2} \right]_{y=\beta} &= \sum_{n=1}^{\infty} V_{cm} \sin n \pi \alpha \end{aligned} \right\} \quad (7)$$

式(4)を式(7)に代入して解けば材端に付帯するたわみ角係数 $\theta_{an}, \theta_{bn}, \theta_{cm}, \theta_{dm}$ および反力係数 $V_{an}, V_{bn}, V_{cm}, V_{dm}$ と曲げモーメント係数 $M_{an}, M_{bn}, M_{cm}, M_{dm}$ およびたわみ係数 $\delta_{an}, \delta_{bn}, \delta_{cm}, \delta_{dm}$ との関係式が次のとく導かれる。

たわみ角一端モーメント関係式

$$\left. \begin{aligned} \theta_{an} &= A_n \frac{M_{an}}{DTC} + B_n \frac{M_{bn}}{DTC} \alpha + C_n \frac{\delta_{an}}{\alpha} + D_n \frac{\delta_{bn}}{\alpha} + \sum_{m=1}^{\infty} [E_{mn} \frac{\partial}{\partial \alpha} (M_{cm} - (-1)^m M_{dm}) + F_{mn} (\frac{\delta_{cm}}{\alpha} - (-1)^m \frac{\delta_{dm}}{\alpha})] \\ \theta_{bn} &= -B_n \frac{M_{an}}{DTC} \alpha - A_n \frac{M_{bn}}{DTC} - C_n \frac{\delta_{an}}{\alpha} + \sum_{m=1}^{\infty} (-1)^m [E_{mn} \frac{\partial}{\partial \alpha} (M_{cm} - (-1)^m M_{dm}) + F_{mn} (\frac{\delta_{cm}}{\alpha} - (-1)^m \frac{\delta_{dm}}{\alpha})] \\ \theta_{cm} &= G_m \frac{M_{cm}}{DTC} \alpha + H_m \frac{M_{dm}}{DTC} \alpha + I_m \frac{\delta_{an}}{\alpha} + J_m \frac{\delta_{bn}}{\alpha} + \sum_{n=1}^{\infty} [K_{mn} \frac{\partial}{\partial \alpha} (M_{an} - (-1)^m M_{bn}) + L_{mn} (\frac{\delta_{an}}{\alpha} - (-1)^m \frac{\delta_{bn}}{\alpha})] \\ \theta_{dm} &= -H_m \frac{M_{cm}}{DTC} \alpha - G_m \frac{M_{dm}}{DTC} \alpha - J_m \frac{\delta_{cm}}{\alpha} - I_m \frac{\delta_{dm}}{\alpha} + \sum_{n=1}^{\infty} (-1)^m [K_{mn} \frac{\partial}{\partial \alpha} (M_{an} - (-1)^m M_{bn}) + L_{mn} (\frac{\delta_{an}}{\alpha} - (-1)^m \frac{\delta_{bn}}{\alpha})] \end{aligned} \right\} \quad (8)$$

反力一端モーメント関係式

$$\left. \begin{aligned} -\frac{\partial^2}{\partial x^2} V_{an} &= R_n \frac{M_{an}}{DTC} \alpha + S_n \frac{M_{bn}}{DTC} + T_n \frac{\delta_{an}}{\alpha} + U_n \frac{\delta_{bn}}{\alpha} + \sum_{m=1}^{\infty} [V_{mn} \frac{\partial}{\partial \alpha} (M_{cm} - (-1)^m M_{dm}) + W_{mn} (\frac{\delta_{cm}}{\alpha} - (-1)^m \frac{\delta_{dm}}{\alpha})] \\ -\frac{\partial^2}{\partial x^2} V_{bn} &= -S_n \frac{M_{an}}{DTC} \alpha - R_n \frac{M_{bn}}{DTC} - U_n \frac{\delta_{an}}{\alpha} - T_n \frac{\delta_{bn}}{\alpha} + \sum_{m=1}^{\infty} (-1)^m [V_{mn} \frac{\partial}{\partial \alpha} (M_{cm} - (-1)^m M_{dm}) + W_{mn} (\frac{\delta_{cm}}{\alpha} - (-1)^m \frac{\delta_{dm}}{\alpha})] \\ -\frac{\partial^2}{\partial x^2} V_{cm} &= M_m \frac{M_{cm}}{DTC} \alpha + N_m \frac{M_{dm}}{DTC} \alpha + P_m \frac{\delta_{an}}{\alpha} + Q_m \frac{\delta_{bn}}{\alpha} + \sum_{n=1}^{\infty} [X_{mn} \frac{\partial}{\partial \alpha} (M_{an} - (-1)^m M_{bn}) + Y_{mn} (\frac{\delta_{an}}{\alpha} - (-1)^m \frac{\delta_{bn}}{\alpha})] \\ -\frac{\partial^2}{\partial x^2} V_{dm} &= -N_m \frac{M_{cm}}{DTC} \alpha - M_m \frac{M_{dm}}{DTC} \alpha - Q_m \frac{\delta_{cm}}{\alpha} - P_m \frac{\delta_{dm}}{\alpha} + \sum_{n=1}^{\infty} (-1)^m [X_{mn} \frac{\partial}{\partial \alpha} (M_{an} - (-1)^m M_{bn}) + Y_{mn} (\frac{\delta_{an}}{\alpha} - (-1)^m \frac{\delta_{bn}}{\alpha})] \end{aligned} \right\} \quad (9)$$

次に

$$\left. \begin{aligned} A_n &= \frac{1}{\lambda_1^2 - \lambda_2^2} (\lambda_1 \coth \pi \lambda_1 - \lambda_2 \coth \pi \lambda_2) & B_n &= \frac{-1}{\lambda_1^2 - \lambda_2^2} (\lambda_1 \cosech \pi \lambda_1 - \lambda_2 \cosech \pi \lambda_2) \\ C_n &= \frac{\pi}{\lambda_1^2 - \lambda_2^2} \{ \lambda_1 (\lambda_1^2 - \nu \left(\frac{mb}{\alpha} \right)^2) \coth^{-1} \lambda_1 \lambda_1^2 - \nu \left(\frac{mb}{\alpha} \right)^2 \coth \pi \lambda_2 \} & D_n &= \frac{-\pi}{\lambda_1^2 - \lambda_2^2} \{ \lambda_1^2 (\lambda_2^2 - \nu \left(\frac{mb}{\alpha} \right)^2) \cosech \pi \lambda_1 - \lambda_2 (\lambda_1^2 - \nu \left(\frac{mb}{\alpha} \right)^2) \cosech \pi \lambda_2 \} \\ E_{mn} &= \frac{2}{\pi} \frac{\partial}{\partial \alpha} \frac{m n'}{(\mu_1^2 + m^2)(\mu_2^2 + n'^2)} & F_{mn} &= \frac{2\pi}{\alpha} \frac{m n'}{(\mu_1^2 + m^2)(\mu_2^2 + n'^2)} \{ \mu_1^2 + \mu_2^2 + m^2 - \nu \left(\frac{mb}{\alpha} \right)^2 \} \\ G_m &= \frac{1}{\mu_1^2 - \mu_2^2} (\mu_1 \coth \pi \mu_1 - \mu_2 \coth \pi \mu_2) & H_m &= \frac{-1}{\mu_1^2 - \mu_2^2} (\mu_1 \cosech \pi \mu_1 - \mu_2 \cosech \pi \mu_2) \\ I_{mn} &= \frac{\pi}{\mu_1^2 - \mu_2^2} \{ \mu_1 (\mu_1^2 - \nu \left(\frac{mb}{\alpha} \right)^2) \coth \pi \mu_1 - \mu_2 (\mu_1^2 - \nu \left(\frac{mb}{\alpha} \right)^2) \coth \pi \mu_2 \} & J_m &= \frac{-\pi}{\mu_1^2 - \mu_2^2} \{ \mu_1 (\mu_2^2 - \nu \left(\frac{mb}{\alpha} \right)^2) \cosech \pi \mu_1 - \mu_2 (\mu_2^2 - \nu \left(\frac{mb}{\alpha} \right)^2) \cosech \pi \mu_2 \} \\ K_{mn} &= \frac{2}{\pi} \frac{\partial}{\partial \alpha} \frac{m n'}{(\lambda_1^2 + m^2)(\lambda_2^2 + n'^2)} & L_{mn} &= \frac{2\pi}{\alpha} \frac{m n'}{(\lambda_1^2 + m^2)(\lambda_2^2 + n'^2)} \{ \lambda_1^2 + \lambda_2^2 + m^2 - \nu \left(\frac{mb}{\alpha} \right)^2 \} \\ R_n &= \frac{1}{\lambda_1^2 - \lambda_2^2} \{ \lambda_1 (\lambda_1^2 - (2-\nu) \left(\frac{mb}{\alpha} \right)^2) \coth \pi \lambda_1 - \lambda_2 (\lambda_2^2 - (2-\nu) \left(\frac{mb}{\alpha} \right)^2) \coth \pi \lambda_2 \} & S_n &= \frac{-\pi}{\lambda_1^2 - \lambda_2^2} \{ \lambda_1 (\lambda_1^2 - \nu \left(\frac{mb}{\alpha} \right)^2) \cosech \pi \lambda_1 - \lambda_2 (\lambda_2^2 - \nu \left(\frac{mb}{\alpha} \right)^2) \cosech \pi \lambda_2 \} \end{aligned} \right.$$

$$\begin{aligned}
P_m &= \frac{\pi}{\mu_1^2 - \mu_2^2} [\mu_1 (\mu_1^2 - (2-v) \frac{m^2}{\alpha^2})^2 \{(\mu_2^2 - v(\frac{m^2}{\alpha^2})^2) \coth \pi \mu_1 - \mu_2 (\mu_1^2 - (2-v) \frac{m^2}{\alpha^2})^2 \} (\mu_1^2 - v(\frac{m^2}{\alpha^2})^2) \coth \pi \mu_2] \\
T_n &= \frac{\pi}{\lambda_1^2 - \lambda_2^2} [\lambda_1 (\lambda_1^2 - (2-v) \frac{m^2}{\alpha^2})^2 \{(\lambda_2^2 - v(\frac{m^2}{\alpha^2})^2) \coth \pi \lambda_1 - \lambda_2 (\lambda_1^2 - (2-v) \frac{m^2}{\alpha^2})^2 \} (\lambda_2^2 - v(\frac{m^2}{\alpha^2})^2) \coth \pi \lambda_2] \\
U_n &= \frac{-\pi}{\lambda_1^2 - \lambda_2^2} [\lambda_1 \{ \lambda_1^2 - (2-v) \frac{m^2}{\alpha^2} \} \{ \lambda_2^2 - v(\frac{m^2}{\alpha^2})^2 \} \coth \coth \pi \lambda_1 - \lambda_2 \{ \lambda_2^2 - (2-v) \frac{m^2}{\alpha^2})^2 \} \{ \lambda_1^2 - (2-v) \frac{m^2}{\alpha^2})^2 \} \coth \coth \pi \lambda_2] \\
V_{mn} &= \frac{-2}{\pi} \frac{\alpha}{\lambda_1^2 + m^2} \frac{m n}{(\lambda_1^2 + m^2)(\lambda_2^2 + m^2)} \{ m^2 + (2-v) \frac{m^2}{\alpha^2} \}, \quad X_{mn} = \frac{-2}{\pi} \frac{\alpha}{\lambda_1^2 + m^2} \frac{m n}{(\lambda_1^2 + m^2)(\lambda_2^2 + m^2)} \{ m^2 + (2-v) \frac{m^2}{\alpha^2} \} \\
W_{mn} &= \frac{-2b}{\alpha} \frac{m n}{(\lambda_1^2 + m^2)(\lambda_2^2 + m^2)} \{ m^2 (\mu_1^2 + \mu_2^2) - (2-v) \frac{m^2}{\alpha^2} \mu_1^2 \mu_2^2 + m^2 (1 - v(2-v)) - v(\frac{m^2}{\alpha^2})^2 m^2 \} \\
Y_{mn} &= \frac{-2a}{\alpha} \frac{m n}{(\lambda_1^2 + m^2)(\lambda_2^2 + m^2)} \{ m^2 (\lambda_1^2 + \lambda_2^2) - (2-v) \frac{m^2}{\alpha^2} \lambda_1^2 \lambda_2^2 + m^2 (1 - v(2-v)) - v(\frac{m^2}{\alpha^2})^2 m^2 \} \\
M_m &= \frac{1}{\mu_1^2 - \mu_2^2} [\mu_1 (\mu_1^2 - (2-v) \frac{m^2}{\alpha^2})^2 \coth \pi \mu_1 - \mu_2 (\mu_1^2 - (2-v) \frac{m^2}{\alpha^2})^2 \coth \pi \mu_2], \quad N_m = \frac{-1}{\mu_1^2 - \mu_2^2} [\mu_1 (\mu_1^2 - (2-v) \frac{m^2}{\alpha^2})^2 \coth \coth \pi \mu_1 - \mu_2 (\mu_1^2 - (2-v) \frac{m^2}{\alpha^2})^2 \coth \coth \pi \mu_2] \\
Q_m &= \frac{-\pi}{\mu_1^2 - \mu_2^2} [\mu_1 (\mu_1^2 - (2-v) \frac{m^2}{\alpha^2})^2 \{ \mu_2^2 - v(\frac{m^2}{\alpha^2})^2 \} \coth \pi \mu_1 - \mu_2 (\mu_1^2 - (2-v) \frac{m^2}{\alpha^2})^2 \{ \mu_2^2 - v(\frac{m^2}{\alpha^2})^2 \} \coth \pi \mu_2]
\end{aligned}$$

2. 連続条件および端辺条件

図-2 のごとき座標を図-1 の二方向連続板の各ブロックの矩形板ごとに設け、 $x_{i,j}=a_i$, $y_{i,j}=b_j$ なる節線がそれぞれ $x_{i+1,j}=0$, $y_{i+1,j}=0$ と一致するとして定めれば各ブロックごとに材端のためみ角係数 $\theta_{i,j}, \theta_{i,j}, \theta_{i,j}, \theta_{i,j}, \theta_{i,j}, \theta_{i,j}, \theta_{i,j}, \theta_{i,j}$ 端モーメント係数 $M_{i,j}, A_{i,j}, M_{i,j}, B_{i,j}, M_{i,j}, C_{i,j}, M_{i,j}, D_{i,j}$ 反力係数 $V_{i,j}, V_{i,j}, V_{i,j}, V_{i,j}, V_{i,j}, V_{i,j}, V_{i,j}, V_{i,j}$ $V_{i,j}, D_{i,j}$ ためみ係数 $\delta_{i,j}, \delta_{i,j}, \delta_{i,j}, \delta_{i,j}, \delta_{i,j}, \delta_{i,j}, \delta_{i,j}, \delta_{i,j}$ が前記1の内容で定義される。ブロック1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 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1332, 1332, 1333, 1333, 1334, 1334, 1335, 1335, 1336, 1336, 1337, 1337, 1338, 1338, 1339, 1339, 1340, 1340, 1341, 1341, 1342, 1342, 1343, 1343, 1344, 1344, 1345, 1345, 1346, 1346, 1347, 1347, 1348, 1348, 1349, 1349, 1350, 1350, 1351, 1351, 1352, 1352, 1353, 1353, 1354, 1354, 1355, 1355, 1356, 1356, 1357, 1357, 1358, 1358, 1359, 1359, 1360, 1360, 1361, 1361, 1362, 1362, 1363, 1363, 1364, 1364, 1365, 1365, 1366, 1366, 1367, 1367, 1368, 1368, 1369, 1369, 1370, 1370, 1371, 1371, 1372, 1372, 1373, 1373, 1374, 1374, 1375, 1375, 1376, 1376, 1377, 1377, 1378, 1378, 1379, 1379, 1380, 1380, 1381, 1381, 1382, 1382, 1383, 1383, 1384, 1384, 1385, 1385, 1386, 1386, 1387, 1387, 1388, 1388, 1389, 1389, 1390, 1390, 1391, 1391, 1392, 1392, 1393, 1393, 1394, 1394, 1395, 1395, 1396, 1396, 1397, 1397, 1398, 1398, 1399, 1399, 1400, 1400, 1401, 1401, 1402, 1402, 1403, 1403, 1404, 1404, 1405, 1405, 1406, 1406, 1407, 1407, 1408, 1408, 1409, 1409, 1410, 1410, 1411, 1411, 1412, 1412, 1413, 1413, 1414, 1414, 1415, 1415, 1416, 1416, 1417, 1417, 1418, 1418, 1419, 1419, 1420, 1420, 1421, 1421, 1422, 1422, 1423, 1423, 1424, 1424, 1425, 1425, 1426, 1426, 1427, 1427, 1428, 1428, 1429, 1429, 1430, 1430, 1431, 1431, 1432, 1432, 1433,

$\delta_{11m}, M_{11m}, \delta_{12m}, \delta_{21m}, \delta_{22m}$ をもつためには、これら諸式の係数行列式が 0 とならない
ために座屈条件式が成立し、これより所要の座屈荷重が決定される。

3. 計算例

図-3 のごとく 4 周辺が単純支持された二方向連続板が周辺直压力 N_x および N_y をうける場合の座屈荷重を求めてみる。端辺条件を考慮して中間節線における連続条件式(1)を立てば次のようになる。

$$\left. \begin{aligned} & ((1-u)A_{21m} + uA_{1m}) \frac{\partial}{\partial x} M_{11m} + \sum_{n=1}^{\infty} (-1)^m v E_{2mn} \frac{\partial}{\partial x} M_{21m} + 0 + \sum_{m=1}^{\infty} (-1)^m (-1)^m v E_{1mn} \frac{\partial}{\partial x} M_{41m} = 0 \\ & - \sum_{m=1}^{\infty} (-1)^m (1-u) K_{2mn} \frac{\partial}{\partial x} M_{11m} + ((1-v)G_{11mn} + vG_{21mn}) \frac{\partial}{\partial x} M_{21m} + \sum_{n=1}^{\infty} (-1)^m K_{4mn} \frac{\partial}{\partial x} M_{31m} + 0 = 0 \\ & 0 + \sum_{m=1}^{\infty} (-1)^m E_{4mn} \frac{\partial}{\partial x} M_{21m} + ((1-u)A_{13m} + uA_{31m}) \frac{\partial}{\partial x} M_{31m} + \sum_{n=1}^{\infty} (-1)^m (-1)^m v E_{3mn} \frac{\partial}{\partial x} M_{41m} = 0 \\ & \sum_{m=1}^{\infty} (-1)^m (-1)^m u K_{3mn} \frac{\partial}{\partial x} M_{11m} + 0 + \sum_{m=1}^{\infty} (-1)^m u K_{3mn} \frac{\partial}{\partial x} M_{31m} + ((1-v)G_{31m} + vG_{41m}) \frac{\partial}{\partial x} M_{41m} = 0 \end{aligned} \right\} \quad (12)$$

座屈条件式は、式(12)の係数行列式 = 0 とおいてえられる。

すなはち $u = v = 0.5$ で二方向圧縮力 N_x, N_y が作用する場合を考え、上記座屈条件式を満足する N_x, N_y の相関関係をプロットすれば図-4 の結果をうる。すなはち N_x, N_y の無次元量 P, Q に対して $P+Q=16$ の直線式が成立し、特例として $P=0$ または $Q=0$ の一方圧縮に対する最小座屈荷重が、それが $Q=16$ または $P=16$ として求められる。

次に v を 0.5 に固定、 u を 0.3 ～ 0.7 の範囲で変化させた場合の一方圧縮 ($N_y = 0$) に対する最小座屈荷重を算定し、結果を表-1 にまとめて示した。同表より明らかなるごとく $u = 0.5$ すなはち連続板のスパンが相等しい場合に座屈荷重は最小となり、上例で述べた $P = 16$ に一致する。

u	$P \left(\frac{N_x Q^2}{D x^2} \right)$
0.3	18.3
0.4	17.2
0.5	16.0
0.6	17.2
0.7	18.3

表-1

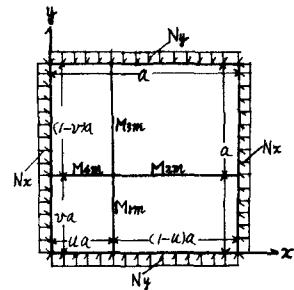


図-3

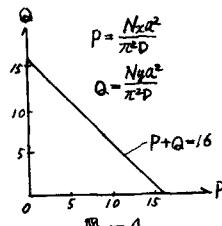


図-4

結語 本研究は端辺が単純支持、固定もしくは自由で中間節線が単純に支持され 3 二方向連続等方性矩形板が二方向からの周辺直圧力を受ける場合の座屈荷重算定式を誘導し、簡単な計算例を示したものである。座屈条件式たる多元行列式の計算は旧来の手計算では到底不可能であるが、電子計算機を利用すれば予め準備された行列式計算のサブルーチンに連結させて迅速かつ高精度に座屈荷重を求めることができる。なお二方向連続板の特例として一方圧縮板の解析が可能であることはいうまでもなく、たとえば $M_{11m}=M_{21m}=\delta_{11m}=\delta_{21m}=0$ とおけば一方辺単純支持、 $\theta_{11m}=\theta_{21m}=\delta_{11m}=\delta_{21m}=0$ とおけば一方辺固定、さらに $M_{11m}=M_{21m}=V_{11m}=V_{21m}=0$ とおけば一方辺自由の一方圧縮板の座屈荷重がそれ算定される。

(参考文献) (1) 宮田宗章: “多経間連続板の座屈荷重計算法” 土木学会論文集 第6号 昭和26年8月

(2) 山崎彦坂: “一方辺自由33一方圧縮矩形板の安定” 第21回土木学会年次学術講演会講演概要 昭和41年