An Analytical Solution of a Cavity Expansion Problem in a Fixed Finite Soil Mass

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1. Introduction

Analysis of the expansion of cavities in an ideal soil mass provides a direct approach to study the stress and the displacement fields around cavities with a number of important geotechnical problems. There are many theories aiming at certain features and based on some assumptions. However, most of them have assumed the soil as infinite media or have considered the free expansion of the cavity while there are several instances in practice where soil is treated by a group of foundations or we need to apply geotechnical remedies under existing structures. In such cases, effectively the cavity is bounded to expand freely. So here, we are concern with a spherical cavity problem in a fixed finite soil mass. The soil is assumed to be homogenous and an isotropic elasto–plastic material. The solution in basic follows the methodology given by Yu (1993) but with different boundary conditions.

2. Problem definition and theory

Fig. 1 depicts the problem. A hydrostatic pressure, p_o, acts throughout the soil. As the internal pressure increases to value p from its initial value p_o, the inner boundary, a_o, expands to radius, a, while the outer boundary, b_o, is bounded to expand. The soil behaves elastically and obeys Hooks law until the onset of yielding which is determined by the Mohr-Coulomb criteria: $\alpha\sigma_\theta - \sigma_r = Y$ (1)

where

$\alpha = (1+\sin\phi)/(1-\sin\phi)$ and

$Y = 2C\cos\phi/(1-\sin\phi)$

The equilibrium of an element at distance, r is

$$2(\sigma_\theta - \sigma_r) = r\frac{\partial\sigma_r}{\partial r} \qquad (2)$$

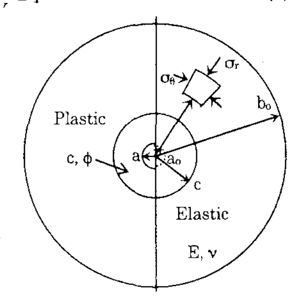

Fig. 1 Definition Sketch

which is subject to two boundary conditions: $u|_{r=b_o} = 0$; $\sigma_r|_{r=a} = -p$ (3)

2.1 Elastic solution

As the cavity pressure increases from its initial value, p_o, the deformation of the soil is first purely elastic. Under conditions of radial symmetry the elastic stress–strain relationship may be expressed as:

$$\dot\varepsilon_r = \frac{\partial\dot u}{\partial r} = \frac{1}{E}\left[\dot\sigma_r - 2\nu\dot\sigma_\theta\right]; \quad \dot\varepsilon_\theta = \frac{\dot u}{r} = \frac{1}{E}\left[-\nu\dot\sigma_r(1-\nu)\dot\sigma_\theta\right] \qquad (4)$$

Integrating the above equation for the initial conditions, $\sigma_r = \sigma_\theta = -p_o$ and $u = 0$ (5)

We have $\varepsilon_r = \frac{\partial u}{\partial r} = \frac{1}{E}\left[\sigma_r - 2\nu\sigma_\theta\right] + \frac{p_o}{E}(1-2\nu)$;

$$\varepsilon_\theta = \frac{u}{r} = \frac{1}{E}\left[-\nu\sigma_r + (1-\nu)\sigma_\theta\right] + \frac{p_o}{E}(1-2\nu) \qquad (6)$$

Solution of Eqs. 2 and 6, subject to the boundary conditions (Eq. 3) can be shown to be

$$\sigma_r = -p + \frac{(1-2\nu)(p-p_0)\left\{1-\left(\frac{a}{r}\right)^3\right\}}{0.5(1+\nu)\left(\frac{a}{b_0}\right)^3 - (1-2\nu)};$$

$$\sigma_\theta = -p + \frac{(1-2\nu)(p-p_0)\left\{1+0.5\left(\frac{a}{r}\right)^3\right\}}{0.5(1+\nu)\left(\frac{a}{b_0}\right)^3 - (1-2\nu)} \qquad (7)$$

Initial yielding occurs first at the cavity wall at the stage when cavity pressure, p, reaches to value

$$p = p_{1y} = \frac{1.5\alpha(1-2\nu)p_0 + Y\left\{0.5(1+\nu)\left(\frac{a}{b_0}\right)^3 - (1-2\nu)\right\}}{1.5\alpha(1-2\nu) + (1-\alpha)\left\{0.5(1+\nu)\left(\frac{a}{b_0}\right)^3 - (1-2\nu)\right\}} \qquad (8)$$

2.2 Elasto–plastic solution

A plastic zone of radius c (Fig. 1) forms around the inner cavity wall with an increase in the applied pressure, p.

2.2.1 Stresses in the plastic region

The stress components, which satisfy the equilibrium condition (Eq. 2) and the yield criteria (Eq. 1), are found to be

$$\sigma_r = \frac{Y}{(\alpha-1)} + Ar^{-k}; \qquad \sigma_\theta = \frac{Y}{(\alpha-1)} + \frac{A}{\alpha}r^{-k} \qquad (9)$$

where $k = \alpha/(1-\alpha)$ and A is constant of integration.

2.2.2 Stresses in the elastic region

The stress components in the elastic region can be obtained from Eqs. 2, 3 and 6 as follows

$$\sigma_r = \frac{2(1-2\nu)(p_0+B)}{(1+\nu)\frac{r}{b_0^3}} + B; \qquad \sigma_\theta = \frac{-(1-2\nu)(p_0+B)}{(1+\nu)\frac{r}{b_0^3}} + B \tag{10}$$

where B is second constant of integration.

Applying the continuity of stress components at the elastic–plastic interface, constants come out to be

$$A = -\left[\left\{\frac{3\alpha}{(\alpha-1)(\alpha+2)} + \frac{(1+\nu)\left(\frac{c}{b_0}\right)^3}{(\alpha+2)(1-2\nu)}\right\}\mathrm{Y} + p_0\right]c^k;$$

$$B = \frac{-\mathrm{Y} - p_0\dfrac{(1-2\nu)(2+\alpha)}{(1+\nu)\left(\frac{c}{b_0}\right)^3}}{\dfrac{(1-2\nu)(2+\alpha)}{(1+\nu)\left(\frac{c}{b_0}\right)^3}} \tag{11}$$

3. Results and Discussion

Summary of the results is given in Table 1. For all the values of injection pressure, p, in the elastic range, σ_r decreases with distance, r/a_o, from the value p at the inner boundary, a/a_o, and becomes almost of value p_o, at $r/a_o = 3.0$. In the same manner, σ_θ increases with r/a_o and also becomes of value p_o at the same distance (Fig. 2). If p, is more than the yielding pressure, p_{1y}, the solution is elasto–plastic (Fig. 3). For $c/b_o = 0.5$, σ_r decreases with r/a_o from the value p at a/a_o to value 408.7 at elastic–plastic boundary, c/a_o, and further decreases to value 246.4 at the outer boundary. σ_θ decreases with r/a_o from the value 594.4 at a/a_o to value 130.5 at c/a_o while further increases to value 211.6 at the boundary.

4. Conclusions

Based on the elasto–plastic theory, a general analytical solution has been developed for a cavity expansion problem in a finite soil mass. It requires less number of design parameters (Table 1) to plot pressure–expansion curve. This solution may have many practical applications. Among them are the determination of the end bearing capacity of deep foundations and the quantification of the confinement effect of compaction grouting method. The analysis can be more elaborated to have solution for cylindrical cavity problem including a geometry parameter in the basic equations (Eqs. 2 and 4).

Table 1 Summary of the Results

Preprocessor		Elastic solution			Elasto–plastic solution				
Input:		Input: Injection pressure, p;			Input: c/b_o ($c/a_o = c/b_o \times b_o/a_o$)				
Elasticity parameters, E & ν;	1000, 0.2	$p_o \le p \le p_{1y}$			Outputs: Injection pressure,p, a/a_o,				
C & φ parameters of the soil;	5, 30	Outputs: a/a_o, σ_r & σ_θ vs. r/a_o (Fig. 2)			σ_r & σ_θ vs. r/a_o (Fig. 3)				
Dilation angle, ψ;	10°	SN	p (kPa)	a/a_o	SN	c/b_o	c/a_o	a/ao	p (kPa)
Initial value of hydrostatic pressure, p_o;	200	1	200.0	1.000	1	0.1	1.25	1.02	484.5
Initial sphere geometry b_o/a_o;	12.5	2	241.8	1.003	2	0.3	3.75	1.40	1424.6
Note: All the stresses are in kPa.		3	283.5	1.005	3	0.5	6.25	2.08	1800.6
Outputs: Initial yielding pressure, p_{1y}; 367.1		4	325.3	1.008	4	0.7	8.75	3.02	2131.6
Geometry, a/a_o; 1.0101		5	367.1	1.010	5	0.9	11.25	4.62	2941.2

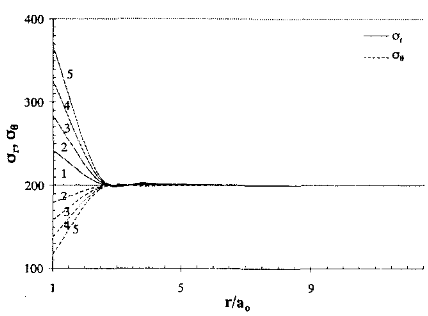

Fig. 2 σ_r & σ_θ vs. r/a_o (Elastic solution)

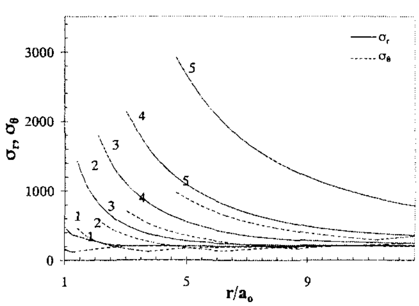

Fig. 3 σ_r & σ_θ vs. r/a_o (Elasto–Plastic solution)

Yu, H.S. (1993). Finite elastoplastic deformation of an internally pressurized hollow sphere. *Acta Mechanics Solida Sinica.*, 6(1), 81-97.