MODAL ANALYSIS INSPECTION FOR DAMAGE EVALUATION IN STRUCTURES

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1. INTRODUCTION

Many time-domain and frequency-domain analytical methods have been developed to identify structural parameters and to evaluate structural damage through analyzing the time-history of motions in structures. However, it is noticed that sometimes the accuracy of identification is greatly influenced by noise. To avoid this shortcoming, a various modal analysis inspections are recently develpoed to evaluate structural damage. In this study, based on our previous study[1] for undamped structures, a modal analysis method is presented to identify structutal degradation for damped structures by using lower measured modes. Moreover, a steel truss structure has been analyzed in order to demonstrate the availability of the method.

2. Basic formulations

A equation of motion for damped free vibration system with N degrees of freedom is described as

$$[M]\{X\}_p\mu_p^2+[C]\{X\}_p\mu_p+[K]\{X\}_p=\{0\};\ p=1,\ 2,\cdots,\ N \qquad \cdots\cdots(1)$$

where [M]=mass matrix, [K] =stiffness matrix, μ_p is the pth complex natural frequency and $\{X\}_p$ is pth eigenvector corresponding to μ_p. For structures, if L lower modes have been measured, those modes can be expressed as follows

$$[\Omega]_{L\times L}=\mathrm{diag}\ (\mu_1^2,\mu_2^2,\ldots\mu_L^2)\ =[\Omega]_a+i[\Omega]_b;\ [\Phi]_{N\times L}=(\ \{X_1\}\ ,\ \{X_2\}\ ,\ldots\ \{X_L\}\ ) \cdots\cdots(2)$$

Replacing the μ_p and $\{X\}_p$ by $[\Omega^2]$ and $[\Phi]$ respectively, Eq. (1) becomes

$$[M][\Phi][\Omega^2]+[C][\Phi][\Omega]+[K][\Phi]=[0] \qquad \cdots\cdots(3)$$

Traditionally, Rayleigh damping has been assumed in most dynamic structural analysis because it is convenient for mathematical treatment. The damping matrix for Rayleigh damping may be written as

$$[C]=\beta_1[M]+\beta_2[K] \qquad \cdots\cdots(4)$$

in which β_1, β_2=scalars. For Rayleigh damping, substituting Eq. (4) into Eq. (3), yields[3]

$$[K]\{X\}_p=[M]\{X\}_p\ \{-(\mu_p^2+\beta_1\mu_p)/(\beta_2\mu_p+1)\} \qquad \cdots\cdots(5)$$

The relation between pth natural frequencies ω_p(undamped) and ω'_p (damped) are[2]

$$\mu_p=-\xi_p\omega_p+i\omega'_p;\ \omega_p=\omega'_p/\sqrt{1-\xi_p^2}\ ;\ i^2=-1 \qquad \cdots\cdots(6)$$

in which ξ_p=the pth damping ratio.

Particularly for Rayleigh damping, because of

$$2\xi_p\omega_p=\beta_1+\beta_2\omega_p^2 \qquad \cdots\cdots(7)$$

the coefficients β_1 and β_2 can be obtained as follows

$$\beta_1=\omega_1\omega_2(2\xi_2\omega_1-2\xi_1\omega_2)/(\omega_1^2-\omega_2^2);\ \beta_2=(2\xi_1\omega_1-2\xi_2\omega_2)/(\omega_1^2-\omega_2^2) \qquad \cdots\cdots(8)$$

Denote $[K]=[K_0]+[\triangle K]$, in which $[\triangle K]$ is the change of stiffness matrix before and after the damage. Substituting Eqs. (2), (4) and $[K](=[K_0]+[\triangle K])$ into Eq. (3), the arranged equations for real-part and imaginary-part respectively are

$$[\triangle K][\Gamma_1]=[\Xi_1];\ [\triangle K][\Gamma_2]=[\Xi_2]; \qquad \cdots\cdots(9)$$

where $[\Gamma_1]_{N\times L}=(\beta_2[\Phi][\Omega]_a+[\Phi])$ and $[\Xi_1]_{N\times L}=-[M][\Phi]([\Omega^2]_a-[\Omega^2]_b)-\beta_1[M][\Phi][\Omega]_a-[K_0][\Gamma_1]$; $[\Gamma_2]_{N\times L}=\beta_2[\Phi][\Omega]_b$ and $[\Xi_2]_{N\times L}=-2\times[M]([\Phi][\Omega]_a[\Omega]_b-\beta_1[M][\Phi][\Omega]_b-[K_0][\Gamma_2]$.

The Eq. (9) could be further expressed as

$$[\triangle K][[\Gamma_1],[\Gamma_2]]=[[\Xi_1],[\Xi_2]] \qquad \cdots\cdots(11)$$

Therefore the least-square estimation[4] of $[\triangle K]$ is

$$[\triangle K][[\Gamma_1],[\Gamma_2]]=[[\Xi_1],[\Xi_2]]^+ \qquad \cdots\cdots(12)$$

in which $[[\Xi_1],[\Xi_2]]^+$ is pseudoinverse matrix[4] of $[[\Xi_1],[\Xi_2]]$.

The change ratio $\triangle k_{pp}/k_{0pp}$($p=1,2,\cdots,N$) for diagonal stiffness coefficients can be used to detect the location of damage according to the magnitude of the change ratio. When the node with a remarkable change ratio is detected, each non-zero stiffness coefficient in the column (or row) corresponding to this node in matrix [K] is multiplied by an unknown coefficient α_k respectively. Therefore the [K] is represented as $[K(\alpha)]$, in which α is the set of α_k. Substituting the $[K(\alpha)]$ into Eq. (3), the α could be solved by the use of arranged equations. The details has been given in reference 1).

3. NUMERICAL EXAMPLE

Fig. 1 shows a steel truss structure which is described as a FEM model with 12 nodes, 25 elements and 21 degree-of-freedom. The cross-sectional areas of the structural members are as follows: Bottom chords 0.06 m^2; top chords 0.0312 m^2; verticals 0.024 m^2; diagonals 0.024 m^2. The damping ratios ξ_1 and ξ_2 are equal to 2%. In this example, it is assumed that P_y(see Fig. 1) is constant load of 20 tf and P_x increases monotonically, also the stress-strain relation of all materials are elastic-perfectly-plastic model. While full areas of cross-sections of elements ① (nodes 1-2) and ② (nodes 2-3) have been yielded, the vibrational modes are calculated as the modes of damaged state of this structure. The calculated frequencies are: $f_1=1.008$ Hz and $f_2=5.614$ Hz. Moreover, the coefficents(see Eq. (8)) are: $\beta_1=0.034$ and $\beta_2=0.00604$. Furthermore, the locations of damage are detected from Fig. 2 in which nodes 2 and 3 are remarkable. On the other hand, as shown in Fig. 3 for our previous method[1], even if 4 modes are used, it seems there are stiffness changes in almost all the nodes. As the location of damage has been detected, the damaged matrix $[K(\alpha)]$ can be identified by the use of aforementioned procedure. Also the identified result and error ER_{ii} are shown in Table 1, in which the identified result is well satisfied. As for the effect of measurement error of modal parameters on identification accuracy, similarly mode shapes has a strong effect[2] on identification accuracy and frequencies and damping ratio have comparatively weak ones.

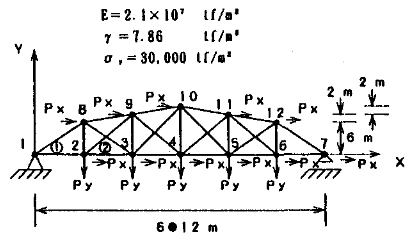

Fig. 1 A steel truss.

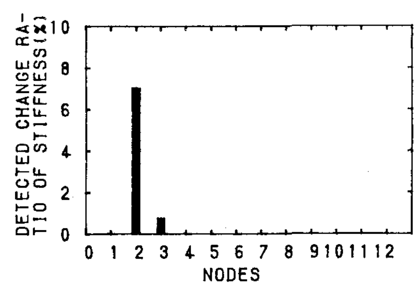

Fig. 2 Damage location detection of this method by using 1~2th modes

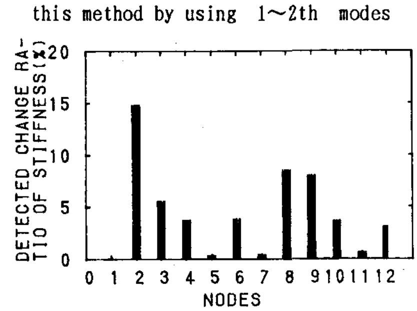

Fig. 3 Damage location detection of previous method[1] by using 1~4th modes

Table. 1 Identified result

NODE	UNDAMAGED	DAMAGED		
		IDENTIFIED	EXACT	E_{Rii}
2-X	232168.1	24161.8	24481.9	1.3 %
2-Y	95737.8	94928.6	95310.7	0.4 %
3-X	258938.7	154568.5	154073.6	0.3 %
3-Y	84575.2	84094.3	84563.3	0.6 %

$ER_{ii}=100\times(k_{iiEXACT}-k_{iiIDEN.})/k_{iiEXACT}$

4. CONCLUSIONS

A modal analysis method is presented to identify both the location and severity of damage for damped structures. This method have better sensitivity of damage location detection than previous study[1] for undamped structures.

REFERENCES : 1) Yuan, H.Y. and Hirao, K., et. al : Detection of stiffness degradation of structural elements from measurement of natural frequencies and mode shape, Journal of Struct. Engrg. Mech., Vol. 39A, JSCE, pp. 759-771, March, 1993. 2) Yuan, H.Y. and Hirao, K., et. al : Stiffness degradation identification of structures using modal analysis, J. Struct. Engrg., Vol. 40A, JSCE, pp. 795-805, March, 1994. 3) Togawa, H.:Vibrational analysis for finite element method, Science Book Company, 1975(in Japanese). 4) Lowson, C.L. and Hanson, R.J. : Solving least squares problems, Prentice Hall, Englewood Cliffs, N.J., 1974.