

## DYNAMIC SOIL-STRUCTURE INTERACTION ANALYSIS BY THE BOUNDARY ELEMENT METHODS

Takemiya H. and C. Y. Wang

Department of civil engineering, Okayama University, Okayama, Japan

## ABSTRACT

Soil-structure interaction problem as characterized by impedance functions is studied by applying Green functions. The physical explanation, the accuracy and the efficiency of various boundary element methods (the indirect boundary element method, the direct boundary element method) are discussed.

Because of the singularity of the Green functions, the displacements and tractions on the interface between the foundation and the soil can hardly be represented accurately, no matter offset is introduced or not. Generally a large number of sources are required. This is the major difficulty in the application of the boundary element methods. In the case of a rigid foundation, the  $6 \times 6$  impedance matrix and the input motion can be effectively obtained by introducing an equivalent rigid-body motion suggested in this paper.

## 1. PHYSICAL EXPLANATION OF BOUNDARY ELEMENT METHODS

Refer to Fig.(1). The boundary element methods may be explained as:

Select a set of loads to act in the free field in such a way that the boundary conditions over surface  $S_g$  can be well represented. By the knowledge of Green functions, the impedance functions can be obtained by calculating the tractions and displacements over surface  $S_g$  produced by the selected loads.

The direct boundary element method and the indirect boundary element method differ in how to represent the boundary conditions. In the direct boundary element method, the boundary conditions are satisfied at a certain number of points; In the indirect boundary element method, the boundary conditions are satisfied in an average sense connecting with work.

## 2. FORMULATIONS OF THE IMPEDANCE MATRIX

Discretizing the displacements and tractions over surface  $S_g$ , the boundary integral equations can be written in matrix form. The well-known formulations of the impedance matrix  $K$  for direct boundary element method and indirect boundary element method can be written respectively as

$$K = G^{-1} H^T \quad (1)$$

$$K = H (G^T H)^{-1} H^T \quad (2)$$

where  $G$  and  $H$  correspond to Green functions for displacements and consistent forces respectively.

## 3. FORMULATIONS OF INPUT MOTION

Using the impedance matrix the equivalent driving force vectors  $F_p$  (forces for keeping the foundation fixed under seismic incidences) can be obtained.

$$F_p = K U_f - F_f \quad (3)$$

where  $U_f$  and  $F_f$  are displacements and consistent forces in the free field caused by the incident waves defined on surface  $S_g$ .

A relevant formulation is (in this case the offset is no permissible)

$$F_p = G U_f \quad (4)$$

## 4. RIGID FOUNDATION

In the case of a rigid foundation, we are interested in the  $6 \times 6$  impedance matrix  $K^*$  connecting the equivalent displacements  $U^* = (U_x, U_y, U_z, Q_x, Q_y, Q_z)$  and forces  $F^* = (F_x, F_y, F_z, M_x, M_y, M_z)$ .  $K^*$  can be obtained by

$$K^* = T K T \quad (5)$$

where  $T$  represents a rigid-body motion influence matrix.

## 5. EQUIVALENT RIGID-BODY MOTION

On the fact that the prescribed rigid body displacements on surface  $S_g$  can hardly be represented by limited number of singular solutions, an equivalent rigid-body motion is suggested here to modify the results. Introducing a rigid-body displacement  $U_e$  as

$$U_e = (T^T T)^{-1} T^T U \quad (6)$$

We can prove that the equivalent force vectors  $F^*$  corresponding to  $U$  are identical to those corresponding to  $U_e$ , here  $U$  may not be rigid-body displacements. As  $K$  in Eq (2) is symmetric, then

$$\begin{aligned} & T^T K U - T^T K T U_e \\ &= T^T K U - T^T K T (T^T T)^{-1} T^T U \\ &= T^T K U - T^T T (T^T T)^{-1} T^T K U \\ &= 0 \end{aligned} \quad (7)$$

Similarly the input motion vectors  $F_p^*$  can be calculated as

$$F_p^* = K^* U_n - F_f^* \quad (8)$$

where  $U_n = (T^T T)^{-1} T^T U_f$ .  $F_p^* = T F_p$ ,  $F_f^* = T F_f$ . Equation (8) is obtained by Iguchi(1982), Luco(1986) by different approaches.

## 6. NUMERICAL INVESTIGATION

In order to conduct the numerical calculation effectively we will discuss three variables: the sources (selected loads), observations (meshes on surface  $S_g$ ) and offset (distance from sources to surface  $S_g$ ). See Fig 3.

In Fig 2, the displacements on the surface of a cylinder calculated in the case of offset=0.0 and 1.0 meter are plotted. It is understood that the boundary is distorted in both cases. An offset may increase the distortion in two and three dimensional problems, especially when there are corners in the foundation. Because of the singularity of Green functions, reducing the offset will cause difficulties in numerical calculation.

Depending on the singularity of the Green functions, the observation meshes near sources should be small enough, say less than  $1/2$  of the offset; while may be rough in other places.

The modeling of sources concerns with the behavior of the Green functions and the whole geometry of the foundation. Also

because of the singularity of Green functions, in order to get reliable results generally a large number of sources are required.

All the disadvantages of the boundary element method come from the singular Green functions. They may be overcome by using linear distributed loads instead of concentrated ones; this requires further improving of Green functions. So in the application of boundary element method, it is important to consider the physical characteristics of each certain problem.

## 7. RESULTS AND CONCLUSIONS

Refer to Fig. (3), numerical calculations are conducted in frequency domain for a cylinder rigid foundation, embedded in a layered viscoelastic media with a rigid rock at a certain depth. Modeling of sources and observations are presented in Fig. (4)

The Green functions for ring loads obtained by E. Kausel are used. These solutions are based on a discretization of the medium in the direction of layering, which results in a formulation yielding algebraic expressions whose integral transforms can readily be evaluated (no numerical integration necessary).

The five complex impedance functions calculated by both direct boundary element method and indirect boundary element method are plotted in Fig. (5) with frequency from 0 to 10 Hz. Results are compared to those obtained by the finite element method with use of the so-called transmitting boundary method.

It may be seen in Fig. (5) that there is a close agreement between the finite element method and the boundary element methods. The only significant differences appear at high frequencies where the finite element method results are slightly lower.

The boundary element methods are reliable and efficient comparing to the finite element methods in the soil-structure interaction problems. When an offset is taken, the indirect boundary element method has advantage over the direct boundary element method. The difficulties in calculating Green functions and the demand of a large number of sources state the need of further improving.

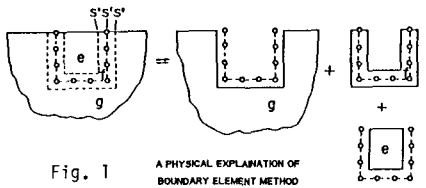


Fig. 1

A PHYSICAL EXPLANATION OF BOUNDARY ELEMENT METHOD

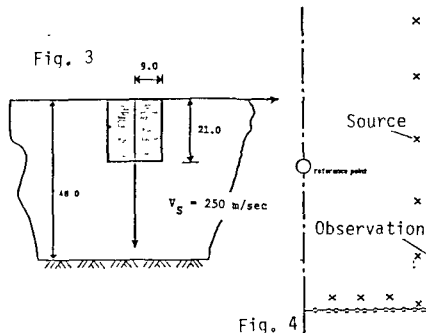


Fig. 3

Fig. 4

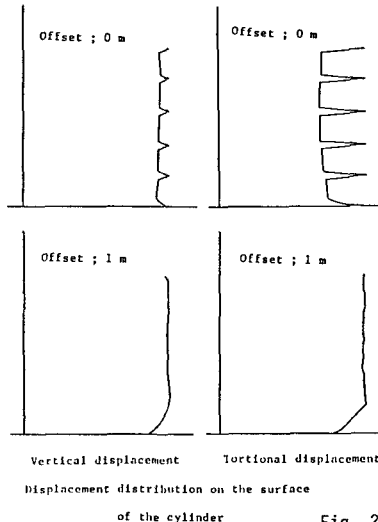


Fig. 2

## REFERENCES

1. E. Kausel (1981) An explicit solution for the Green functions for dynamic loads in layered media; Research report R81-13, Publication No. 699, Dep. of civil engineering, M.I.T. Cambridge, 1743-1761
2. G. Waas, H. R. Riggs and H. Werkle (1985) Displacement solutions for dynamic loads in transversely isotropic stratified media; Earthquake engineering and structural dynamics Vol. 13, 177-193
3. R. J. Apse (1979) Dynamic Green's functions for layered media and applications to boundary value problems; PH.D. Thesis, University of California at San Diego, San Diego.
4. J. E. Luco (1986) On the relation between radiation and scattering problems for foundations embedded in an elastic half-space; Soil dynamics and earthquake engineering, Vol. 5 No. 2

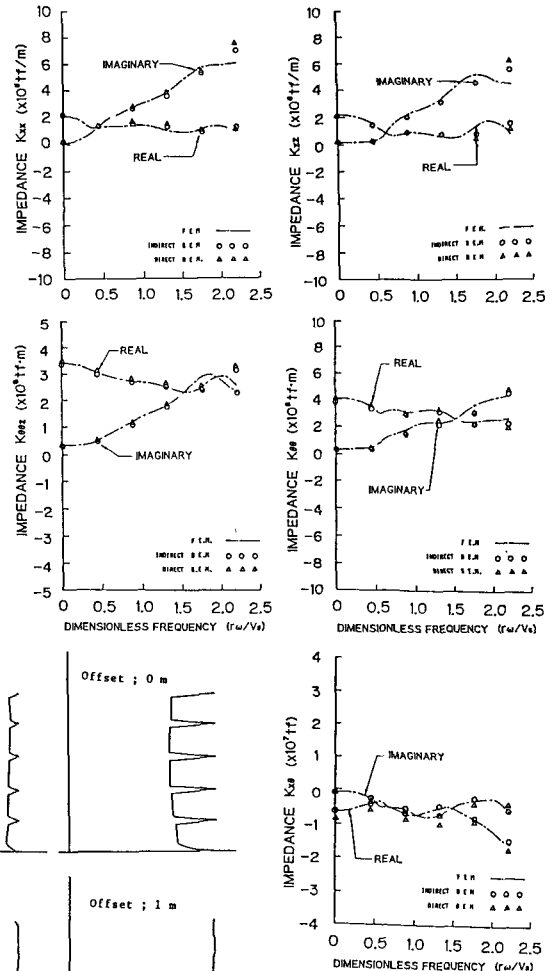


Fig. 5